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Analysis of Flow Behavior of Non-Newtonian Fluids Based on a Concept of Traveling Force

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The non-Newtonian flow behavior of a culture fluid with suspended adhesive particles of microbes and polysaccharides is analyzed in this study based on a new concept. The concept assumes that the force generated by the contact between particles under shearing flow due to mixing is decreasingly transmitted through the fluid in radial directions. A viscosity equation that includes the degree of force transmisson is derived by considering the shear stress to the force. On the other hand, the shear stress dependence on the concentration of bound particles is expressed in an equation by introducing an effective shear stress that works on the contact sites of the bindings and varies the concentration. Relating the degree of force transmission to the concentration derives a non-Newtonian viscosity equation in terms of shear stress (or shear rate), in which zero-shear viscosity is correlated with both the particle concentration and molecular weight of polymers. It is confirmed that calculations based on the equation are in good agreement with experimental results previously reported for aqueous solutions of several polysaccharides.

[**Key words:** non-Newtonian fluid, contact particles, traveling force, shear stress, viscosity equation]

In most fermentation processes, the viscosity of culture broths in a fermentor increases with the growth of microbes, resulting in a significant reduction in agitation efficiency which is important in the fermentation industry. The flow of culture broths containing filamentous microbes and/or polysaccharides exhibits non-Newtonian flow properties (1–6). It has been expressed approximately by a power law model. In previous studies, we investigated the steady-shear viscosity and dynamic viscoelasticity of concentrated cell suspensions of bacteria which did not produce soluble polysaccharides, and explained the complicated flow behavior of the cell suspensions: strong viscoelastic properties are caused by the formation of microbial flocks, which are fragile under applied stresses, and the network structure of linked microbes (7–9). In the present paper, the flow behavior of non-Newtonian fluids consisting of monodispersed adhesive particles of polymeric substances, that is, having noncoagulated structure of constituent particles is analyzed, to determine the basis of rheological properties of some microbial culture system producing polysaccharides.

A large number of experimental and theoretical studies on the rheological properties of such polymeric fluids and suspensions of particles have been carried out. The flow behavior of non-Newtonian fluids has been thoroughly investigated from various points of view, in which considerable effort has been focused on analyzing that of the viscous fluids (10–13).

In an ordinary treatment on viscosity of a fluid, theoretical considerations have mainly focused on a single flow plane independent of other flow planes even in the case of non-Newtonian fluids. When suspended particles collide with one another (hereafter, the term 'particle' is used even for solute) under shearing flow by an external action, force caused by a contact action between the particles arises in a flow plane and is succeedingly transmitted to the neighboring flow planes through bindings made by the contact. Therefore, it is considered that the force generated in every part of the fluid travels through it as a transmission medium. The force should consequently be included in the estimate of shear stress to derive a viscosity equation for the non-Newtonian flow behavior.

Since the transmission of force in this kind of mechanisms, in which the bindings that compose the transmission medium are in a state of makings and breakings, is generally accompanied by loss as it travels, the force decreases during the transmission process. With a shear stress against the total forces, which is the sum of the force transmitted from every place in the fluid to a flow plane chosen for consideration, a viscosity equation can be given as a function of a transmission coefficient (or a reciprocal of an attenuation coefficient) of the force.

On the other hand, in a fluid containing adhesive particles, under the shearing flow, both making and breaking of the binding sites between particles are occurring concurrently. At a steady state the concentration of the bindings remains constant, depending on shear stress, binding forces between the particles, and other factors. In the present study, the dependence of the concentration on the shear stress is expressed in an equation by introducing a newly defined effective shear stress that works on the contact sites of the bindings and varies the concentration. Since the contact between suspended particles enables the transmission of the force, it is surmised that the degree of force transmission can be correlated with a concentration proportional to the number of bindings in a line. By relating the transmission coefficient to the concentration, we have a non-Newtonian viscosity equation in terms of shear stress (or shear rate), which will express the flow behavior of various kinds of fluids consisting of adhesive particles.

For concentrated polymer solutions, it has been determined experimentally that zero-shear viscosity is propor-

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tional to around 3.4 power of both polymer concentration and molecular weight (14–16), and the relation has been studied theoretically (17, 18). In this study, this tendency is elucidated in the course of the analysis. A general nature of non-Newtonian flow exhibiting shear-thinning behavior of viscosity, which has been examined on several polymer solutions (25–28), is also indicated by the viscosity equation.

A relation between viscosity and shear rate is obtained by a parametric method in the equation. It was verified that the viscosity equation was applicable to experimental results measured in various aqueous polysaccharide solutions (15, 19).

METHODS AND RESULTS

Shear stress in the Couette flow Usually, special shearing flow that resembles the Couette flow with a constant shear rate in every part of a fluid has been used for the analysis of non-Newtonian flow behavior (10–13). Theoretical treatment in this study is also based on the assumption that the flow was imposed the same shear rate throughout the whole fluid.

First, the shearing flow of a constant rate ($\dot{\Upsilon}$) is expressed in spherical coordinates. A volume element and flow velocity at coordinates (r, θ, ϕ) are expressed by $d\upsilon = r^2 \sin\theta dr d\theta d\phi$ and $u = \dot{\Upsilon} r \cos\theta$, respectively, where r, θ, and ϕ represent the radial length, latitude, and longitude, respectively. Here, θ represents the angle between the radial axis and the line perpendicular to the flow which is drawn from the origin of the coordinate system.

For homogeneous shearing flow of a non-Newtonian fluid in which adhesive particles (the concentration of which is represented by c) are suspended, the force produced through the contact action between the particles arises in every part of the fluid and is propagated through the fluid as a transmission medium in radial direction. It is surmised that the force is reduced during each transmission process due to the contact sites between the suspended particles, because the bindings of contact particles are in a state of making and breaking. The attenuation rate of the traveling force σ is, on assuming uniformity of the rate of reduction, expressed by the equation:

$$-\frac{d\sigma}{dr} = \alpha\sigma \qquad (1)$$

where α is an attenuation coefficient of the traveling force. When the initial value of σ is σ_0, integration of Eq. 1 gives: $\sigma = \sigma_0 \exp(-\alpha r)$. A factor analogous to the form appeared in the discussion of the report on screened hydrodynamic interactions in entangled polymer solutions (20), but it is not directly related to that of the present paper.

In order to estimate the traveling force in a proportional relation, we choose a small volume ($\Delta\upsilon$) at coordinates (r, θ, ϕ), in which the velocities of suspended particles are nearly the same, and flow planes are separated by a distance of one particle diameter. The contact between the particles within the volume and those on the neighboring flow planes produces a force which is directly proportional to both fluid momentum at the position and collision frequency of the particles. The same proportional relation except for the magnitude of the force can be formed for flow planes separated for an arbitrarily chosen distance within the range of the particle diameter. In other words, the velocity of the fluid and a function of the particle concentration c, which is generally represented as c^l (the exponent l is estimated later), are concerned and the volume $\Delta\upsilon$ is used in relation to the force caused by the particles passing through the volume. The force is expressed, with the introduction of a proportional coefficient β, as

$$\sigma(\Delta\upsilon) = \beta c^l \exp(-\alpha r)\dot{\Upsilon} r \cos\theta \Delta\upsilon \qquad (2)$$

Equation 2 indicates the traveling force produced in $\Delta\upsilon$ and reached a unit area of the flow plane located at the origin. The coefficient β is the efficiency of the force generated from a pair of colliding particles. It is necessary for the estimation of viscosity to include the stress due to the transmission of the force generated in every part of the fluid that is expressed by Eq. 2.

The traveling force transmitted throughout the fluid, which is acting on the flow plane, is the difference between the forces generated from positive and negative sides of the coordinates. Shear stress against the force is added to a shear stress in the absence of the effect. The total shear stress emerging on the face of a unit area surrounding the origin is expressed by

$$p = p_\infty - 2\beta c^l \int_0^\infty \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \exp(-\alpha r)\dot{\Upsilon} r \cos\theta d\upsilon \qquad (3)$$

where $d\upsilon = r^2 \sin\theta dr d\theta d\phi$, and p_∞ represents a shear stress without the effect of the traveling force. Integrating Eq. 3 on the condition that the shear rate $\dot{\Upsilon}$ is constant throughout the fluid gives

$$p = p_\infty - 12\pi\beta c^l \dot{\Upsilon}/\alpha^4 \qquad (4)$$

A non-Newtonian viscosity equation In the case of the Couette flow of Newtonian fluids, a shear stress is proportional to a shear rate. Viscosity is defined as

$$\eta = -\frac{p}{\dot{\Upsilon}} \qquad (5)$$

where η, p, and $\dot{\Upsilon}$ represent the viscosity, shear stress, and shear rate, respectively. Even for the case of non-Newtonian fluids where viscosity varies depending on the magnitude of shear rate, the viscosity defined in Eq. 5 can be used on condition that the shear rate holds a constant value throughout the fluid.

Substituting p expressed in Eq. 4 into Eq. 5 gives the following viscosity equation:

$$\eta = \eta_\infty + 12\pi\beta c^l \xi^4 \qquad (6)$$

where $\xi = 1/\alpha$ is a transmission coefficient, and is a function of shear stress (or shear rate) which varies the number of contact particles. The term $\eta_\infty (= -p_\infty/\dot{\Upsilon})$ represents the viscosity neglecting the effect due to the traveling force.

Variation of the concentration of bindings by shear stress In non-Newtonian fluids, many sites between constituent adhesive particles are bound by attractive force. Under a state of shearing flow, some of the bindings are broken by shear stresses acting on the faces of flow planes and are simultaneously made by contact between the particles. Meanwhile, since the contact between the suspended particles results in the transmission of force, it is surmised that the transmission degree of the traveling force is related to the concentration of the bindings. The dependence of the concentration on the shear stress would be expressed in an equation on the assumption that contact areas between the particles in pair and adhesive forces per unit contact area are given.

In the following analysis we restrict our discussion to states under steady shearing flow in which structural and characteristic changes of a fluid do not occur except for the number of bindings.

We introduce the new concept of an effective shear stress, which is the ratio of shear stresses working on the contact sites of the bindings to the binding forces between the particles. In a case when the shear stress increases by $\Delta\tau$ ($\tau=|p|=\eta|\dot{\Upsilon}|$), the increment ($\Delta\tau_{\mathrm{R}}^{X}$) of the effective shear stress for the bindings is expressed by

$$\Delta\tau_{\mathrm{R}}^{X}=\frac{X\cdot\bar{s}\cdot\Delta\tau}{X_0\cdot\bar{s}\cdot\bar{\tau}_{\mathrm{c}}} \tag{7}$$

where, X is the concentration of the bindings, which is the number of bindings on a unit length of the line between two points that the force travels. The concentration is constant under the condition of steady shearing flow, as the bindings are in a dynamic equilibrium state of making and breaking. X_0 represents a value of X at a limiting case of $\tau\rightarrow 0$; $\bar{\tau}_{\mathrm{c}}$ and $\bar{s}$ represent the average values of the binding forces and the contact areas between the paired particles, respectively, which are projected on planes in parallel to the direction of the flow. The average contact area $\bar{s}$ in the numerator and the denominator of Eq. 7 cancel each other, and do not enter explicitly in the following discussion.

It is clear from the definitions that as the shear stress increases by a small amount ($\Delta\tau$), the concentration X decreases in proportion to both concentration and increment of the effective shear stress varying it directly. The relation is described by $-\Delta X \backsim X\cdot\Delta\tau_{\mathrm{R}}^{X}$, which, on substituting Eq. 7, offers the dependence of the concentration on the shear stress in equation

$$-\frac{\mathrm{d}X}{\mathrm{d}\tau}=\frac{a_1}{X_0}X^2 \tag{8}$$

where a_1 is a proportional constant ($a_1=z/\bar{\tau}_{\mathrm{c}}$), in which z is a constant independent of states of the steady shearing flow and shapes, sizes, and species of constituent particles of the fluid.

Integrating Eq. 8 on the condition that $X\rightarrow X_0$ at $\tau\rightarrow 0$ gives a relation of X to τ in the following form:

$$X=\frac{X_0}{1+a_1\tau} \tag{9}$$

Equation 9 expresses the dependence of X on the shear stress in states under steady shearing flow.

A viscosity equation for non-Newtonian flow behavior In Eq. 6, viscosity has been given as a function of the transmission coefficient of traveling force. At that stage, the dependence of the equation on shear stress (or shear rate) emerging in the non-Newtonian fluid is not explicitly shown. The transmission coefficient ξ is confidentially depended on the shear stress (or shear rate) from what the coefficient is correlated with the concentration of bindings that is varied by the magnitude of shear force affecting the binding texture.

Since the contact between suspended particles enables the transmission of force as previously mentioned, it is reasonable to correlate the transmission coefficient directly with the concentration of bindings which are along the line that the force travels. With a proportional correlation between ξ and X expressed by Eq. 9, the following relation in terms of the shear stress τ is obtained, where X_0 is represented as m-th power of the particle concentration c (m is estimated later), that is,

$$\xi=\frac{a_0c^m}{1+a_1\tau} \tag{10}$$

where a_0 is a proportional constant.

Substituting Eq. 10 into Eq. 6 transforms the viscosity equation as a function of the transmission coefficient to that of the shear stress and the particle concentration. On the expressions of $l+4m=n$ and $12\pi\beta a_0{}^4=\beta_1$, we have

$$\eta=\eta_\infty+\frac{\beta_1c^n}{(1+a_1\tau)^4} \tag{11}$$

With the definition of $\eta=\eta_0$ at $\tau\rightarrow 0$, Eq. 11 gives

$$\eta_0=\eta_\infty+\beta_1c^n \tag{12}$$

From Eqs. 11 and 12, it follows:

$$\eta=\eta_\infty+\frac{\eta_0-\eta_\infty}{(1+a_1\tau)^4} \tag{13}$$

The constant a_1 relates to the binding force only and does not depend on the particle concentration as indicated in Eq. 8.

Another form of viscosity equation Substituting $\tau=\eta\dot{\Upsilon}$ ($\dot{\Upsilon}>0$) for τ in Eq. 13 transforms the viscosity equation, shown as a function of the shear stress, to an equation expressed in terms of the shear rate,

$$(\eta-\eta_\infty)(1+a_1\eta\dot{\Upsilon})^4=\eta_0-\eta_\infty \tag{14}$$

When η_0 and η_∞ are known, the viscosity at a given shear stress or shear rate can be calculated from Eq. 13 or Eq. 14, provided that the constant a_1 is given.

The equations derived above express the non-Newtonian flow behavior of viscosity in states under constant shear rate flow. The equations are applicable to various fluids that are solutions of polymers, monodispersive suspensions of adhesive particles, and their mixtures, on condition that the shear-thickening effect is negligible; this effect has been observed often in states of high shear rate flow of concentrated suspensions consisting of particles with weak adhesive properties (21–24). Though empirically, viscosity is affected by temperature, the relations in the analysis ought to obey the same forms as the temperature is constant.

An estimate of the constant n In a range of concentration where monodispersive condition for particles in a fluid holds under a state of steady shearing flow, a frequency at which the particles passing through a differential volume element in the fluid collide with those on the neighboring flow plane is proportional to the two-thirds power of the particle concentration c, since the number of particles in a line is in proportion to the one-third power of c. Thus, it is inferred that values of l and m appearing in Eqs. 2 and 10 should be 2/3. Consequently, a value of n in Eq. 12 is 10/3 ($\fallingdotseq 3.33$). An argument which resembles the above statement for the estimate also holds for a correlation between the zero-shear viscosity η_0 and molecular weight of polymers, if each polymer in solution is regarded as a mass of adhesive particles the number of which is proportional to its molecular weight. The value obtained in the above argument for n agrees closely with experimental values (around 3.4) obtained in relation to zero-shear viscosity (corresponding to η_0 in Eq. 12 neglecting the term η_∞) to both concentration and molecular weight in concentrated solutions of certain kinds of polymers (14–16). Studies were conducted to explain the dependence quantitatively in molecular dynamics based on a tube model and in a molecular flow theory for entangled polymer solutions (17, 18).

A characteristic feature of the viscosity equation at a limiting case In a shear rate region where the limiting conditions of $\eta \gg \eta_\infty$ and $a_1\eta\dot{\gamma} \gg 1$ are realized, a logarithmic expression of Eq. 14 is in the form:

$$\log \eta = 0.2 \log (\eta_0/a_1{}^4) - 0.8 \log \dot{\gamma} \qquad (15)$$

Therefore, it can be shown from the equation that the maximum (negative) slope of a curve representing viscosity plotted as a function of shear rate in double-logarithmic coordinates takes 0.8; this significant feature of the non-Newtonian flow expressed by the viscosity equation agrees well with that of the tendency measured for several polymer solutions (25–28). It is surmised that the dispersing states of the polymers in these solutions allow the basic conditions imposed in this study.

VERIFICATION OF THE THEORETICAL VISCOSITY EQUATION

The applicability of the viscosity equation (Eq. 14) derived theoretically to some experimental data cited from literature was examined to confirm the above analysis on the mechanism of non-Newtonian flow.

Simulation of experimental data The correlation between viscosity and shear rate is obtained by means of a parametric method in Eq. 14. The parameter adopted to solve the equation is

$$t = a_1\eta\dot{\gamma} \qquad (16)$$

From Eqs. 14 and 16, the following relation is obtained:

$$\begin{cases} \eta = \eta_\infty + (\eta_0 - \eta_\infty)/(1+t)^4 \\ \dot{\gamma} = t/a_1\{\eta_\infty + (\eta_0 - \eta_\infty)/(1+t)^4\} \end{cases} \qquad (17)$$

The characteristic constants a_1, η_0, and η_∞ of a fluid concerning the flow can be determined by fitting Eq. 17 to some experimental data of η vs. $\dot{\gamma}$ with varying t under the restriction of $t>0$ (Eq. 16).

In Fig. 1, the open circles denote the data obtained by Woods and Krieger (19) using 0.7% (w/v) aqueous solution of hydroxyethyl cellulose at 30°C. The solid line represents a simulation curve calculated by Eq. 17 with $a_1 = 2.0 \times 10^{-2}$ (1/Pa), $\eta_0 = 0.08$ (Pa·s), and $\eta_\infty = 0.012$ (Pa·s). The simulation using these characteristic constants fits well to the experimental data; the data were obtained using various types of instrument, and were superposed on each other, which would indicate that those data were intrinsic values independent of the measurement methods. That is, under approximately the same shearing flow; the condition for the flow enables the comparison with calculated values from the viscosity equation derived based on the Couette flow.

The results presented in Fig. 1 show that the viscosity equation (Eq. 14) can express the experimental data over a wide range of shear rates.

Dimensionless expression of the viscosity equation Next, we transform Eq. 14 to a dimensionless form to generally express the non-Newtonian flow behavior, as carried out by Morris (29). The method was based on an equation proposed by Cross (11) to express the flow behavior.

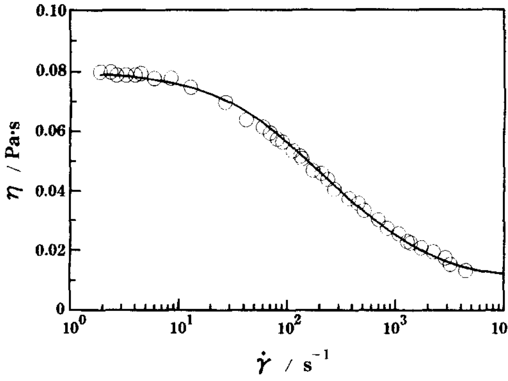

FIG. 1. Shear rate dependence of viscosity of an aqueous solution of 0.7% (w/v) hydroxyethyl cellulose. The data are cited from the paper reported by Woods and Krieger (19). The solid line represents the result of simulation according to Eq. 17.

Dividing both sides of Eq. 14 by η_0 yields the equation:

$$(\eta_R - \eta_{R\infty})(1 + a_1\eta\dot{\gamma})^4 = 1 - \eta_{R\infty} \qquad (18)$$

where η_R denotes η/η_0.

By denoting the shear rate in the case of $\eta = \eta_f$ (*i.e.*, $\eta_R = \eta_{Rf}$) as $\dot{\gamma}_f$, we write Eq. 18 as

$$(\eta_{Rf} - \eta_{R\infty})(1 + a_1\eta_f\dot{\gamma}_f)^4 = 1 - \eta_{R\infty} \qquad (19)$$

i.e.,

$$a_1 = \left\{\left[\frac{1-\eta_{R\infty}}{\eta_{Rf} - \eta_{R\infty}}\right]^{1/4} - 1\right\}/\eta_f\dot{\gamma}_f \qquad (20)$$

By substituting Eq. 20 into Eq. 18, we have

$$(\eta_R - \eta_{R\infty})(1 + A_f\eta_R\dot{\gamma}_R)^4 = 1 - \eta_{R\infty} \qquad (21)$$

where

$$A_f = \left\{\left[\frac{1-\eta_{R\infty}}{\eta_{Rf} - \eta_{R\infty}}\right]^{1/4} - 1\right\}/\eta_{Rf} \text{ and } \dot{\gamma}_R = \dot{\gamma}/\dot{\gamma}_f$$

Equation 21 is a dimensionless form of Eq. 14, which is considered to generally express the non-Newtonian flow behavior.

In a case where $\eta_{R\infty}$ is negligible in comparison with η_R and $\eta_{R\infty} \ll 1$, Eq. 21 reduces to

$$\eta_R(1 + A_f\eta_R\dot{\gamma}_R)^4 = 1 \qquad (22)$$

where a reduced form of A_f is expressed by

$$A_f = \left\{\left[\frac{1}{\eta_{Rf}}\right]^{1/4} - 1\right\}/\eta_{Rf} \qquad (23)$$

In Fig. 2, various data of viscosity against shear rate, which were obtained by Morris *et al.* (15) with the introduction of dimensionless values of η_R and $\dot{\gamma}_R$ by adopting η_{Rf} value as 0.1, are replotted. The data points for the concentrated aqueous solutions of guar gum, λ-carrageenan (1.5–5% (w/v)), locust bean gum, 'high mannuronate' alginate, and hyaluronate (M_r: 1.6×10^6, 0.16–1 g/dl) at 25°C all converge to a single curve. The solid line in Fig. 2 was calculated using Eqs. 22 and 23 at $\eta_{Rf} = 0.1$ ($A_f = 7.78$). The polysaccharides used in the experiments are conformationally mobile, and have been expected to adopt disordered or 'random coil' chain geometry. The deducible formations of the polymers in the solutions meet the dispersive condition required in the present analysis.

The theoretically derived dimensionless viscosity equa-

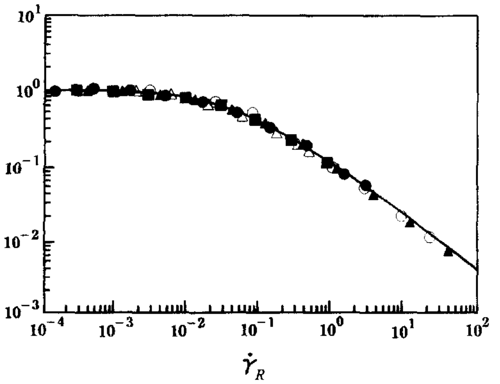

FIG. 2. Nondimensional expression of a correlation between viscosity and shear rate. The data are for the concentrated aqueous solutions of polysaccharides (15): guar gum, △; λ-carrageenan (1.5–5% (w/v)), ▲; locust bean gum, ●; 'high mannuronate' alginate, ■; hyaluronate (M_r: 1.6×10^6, 0.16–1 g/dl), ○. The solid curve represents the results of calculation according to Eqs. 22 and 23.

tions (Eqs. 22 and 23) which were neglected the term $\eta_{R\infty}$ can express the experimental tendency for the viscosity to decrease with increasing shear rate over a wide range of shear rates for several polysaccharide solutions.

Conclusion The flow behavior of fluids with suspended particles of adhesive substances, which simulate culture broths dispersing microbes and polysaccharides uniformly, was analyzed based on a concept of force and its transmission resulting from contact between the suspended particles, under the condition of constant shear rate flow by an external action. The force generated in every part of the fluid travels through it in radial direction, and is attenuated during each transmission process accompanied by making and breaking of the binding sites between particles. A non-Newtonian viscosity equation related to a transmission coefficient of the force was derived on adopting a shear stress against the total force summed up the traveling forces. It is surmised that the transmission degree of the force is directly proportional to the concentration of bindings, which are pairs of particles that lie along the line that the force travels, since the contact between the suspended particles causes the transmission of the force. On the other hand, the shear stresses acting on the faces of flow planes affect the formation of bound particles. On the assumption that adhesive forces between the contact particles were given, the dependence of the concentration on the shear stress was expressed in an equation by introducing an effective shear stress that affected the binding texture and varied the number of bindings. On the basis of the above-mentioned analysis of the fluid flow, a non-Newtonian viscosity equation in terms of the shear stress (or shear rate) was obtained, and, from it, relations of zero-shear viscosity to both the particle concentration and molecular weight of polymers were inferred. The equation is also characterized by the maximum slope of a curve representing the relation of viscosity to shear rate in double-logarithmic coordinates having the value of −0.8.

It is considered from the good agreement between experimental results and calculations according to the viscosity equation, that the analysis based on the concept of the force traveling through fluid under shearing flow elucidated an essential mechanism of the flow behavior of various polymeric fluids representing microbial culture broths with dissolved polysaccharides.

NOMENCLATURE

A_f : constant expressed with Eqs. 21 and 23
a_0 : proportional constant in Eq. 10
a_1 : proportional constant $(=z/\bar{\tau}_c)$
c : concentration of particles
l, m, n : power constants
p : shear stress
p_∞ : shear stress neglecting the effect of traveling force
r : radial length in spherical coordinates
$\bar{s}$: average value of contact areas
t : parameter defined with Eq. 16
u : velocity of fluid
v : chosen volume in fluid
X : concentration of bindings
X_0 : value of X at the limiting case of $\tau\to0$
z : proportional constant
<Greek letters>
α : attenuation coefficient of traveling force
β : proportional coefficient in Eq. 2
β_1 : proportional coefficient $(=12\pi\beta a_0{}^4)$
$\dot{\gamma}$: shear rate
$\dot{\gamma}_f$: shear rate in the case of $\eta=\eta_f$
$\dot{\gamma}_R$: shear rate ratio $(=\dot{\gamma}/\dot{\gamma}_f)$
η : viscosity of fluid
η_0 : zero-shear viscosity
η_∞ : viscosity neglecting the effect of traveling force
η_f : value of viscosity
η_R : viscosity ratio $(=\eta/\eta_0)$
$\eta_{R\infty}$: viscosity ratio $(=\eta_\infty/\eta_0)$
η_{Rf} : viscosity ratio $(=\eta_f/\eta_0)$
θ : latitude in spherical coordinates
ξ : transmission coefficient $(=1/\alpha)$
σ : traveling force
σ_0 : initial value of traveling force
τ : shear stress (absolute value)
$\bar{\tau}_c$: average value of binding forces
τ_R^X : effective shear stress for bindings X
ϕ : longitude in spherical coordinates

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