THE APPLICATION TIMING OF NITROGEN FERTILIZER

WEN-YUAN HUANG, LEROY HANSEN, and NOEL D. URI[1]

(Received February 11, 1992; accepted January 19, 1993)

Abstract. This paper is concerned with factors affecting a farmer's decision concerning the timing of nitrogen ferilizer application. These factors include the expected nitrogen loss associated with different application times, the expected seasonal fluctuations in nitrogen fertilizer prices and operating costs, and the perceived risk of not being able to apply nitrogen fertilizer during the growing season. This paper shows that a split application of nitrogen fertilizer is an optimal strategy for both risk-neutral and risk-averse cotton farmers in the United States if there is a possibility that they may be unable to apply nitrogen fertilizer after planting. Furthermore, a risk-averse cotton farmer relative to a risk neutral farmer will apply more nitrogen fertilizer prior to planting.

Introduction

High concentration levels of nitrates in ground water have become a public concern because of the real and suspected risks to human health through drinking water with elevated nitrate levels (Cantor, 1988; Nielsen and Lee, 1987). Recent results from the Environmental Protection Agency's (EPA) National Survey of Pesticides in Drinking Water Wells indicate that about 1.2% of all community water system wells nationwide and 2.4% of rural domestic wells have nitrate concentration levels in excess of the EPA's Maximum Contaminant Level of 10 mg L^{-1} (EPA, 1990).

Application of nitrogen fertilizer by the agricultural sector has been identified as a major contributor to the elevated nitrates concentration levels in ground water (Freshwater Foundation, 1988; Office of Technology Assessment, 1987; and Nielsen and Lee, 1987). A farmer applies nitrogen fertilizer to enhance crop yield. By doing so, he or she insures that there will be an adequate concentration of nitrates in the root zone during the growing season to achieve maximum crop yield, all other things given. In general, however, not all of the nitrogen applied in the form of fertilizer is taken up by the plants (Bock, 1984; Fertilizer Institute, 1976). Thus, the amount of nitrogen applied in excess of the amount taken up by plants and/or removed from the field at harvest will be ultimately lost to the atmosphere, dissipated into the surface water, and/or leached into the ground water (White, 1989).

A farmer can minimize this excess nitrogen emitted into the environment through a variety of practices. For example, applying nitrogen fertilizer during the times when plant uptake is greatest and when the potential for nitrogen losses due to

---

[1] The authors are with the Economic Research Service, U.S. Department of Agriculture, Washington, DC. The views expressed are those of the authors and do not necessarily represent the policies of the U.S. Department of Agriculture or the views of other U.S. Department of Agriculture staff members. They would like to thank the anonymous referees for useful suggestions. Senior authorship is not assigned.

Water, Air, and Soil Pollution 73: 189–211, 1994.
© 1994 *Kluwer Academic Publishers. Printed in the Netherlands.*

soil erosion, rainfall, and other environmental factors are minimized will mitigate nitrogen losses. Thus, the application of nitrogen fertilizer after planting in some areas can be more effective than the pre-planting application in reducing nitrogen losses (Bock, 1984; Kanwar *et al.*, 1988).

Why then would a farmer apply nitrogen prior to planting if applying nitrogen fertilizer after planting can reduce nitrogen losses, and therefore, costs? The objective of the following analysis is to evaluate the factors that affect a farmer's decision concerning the timing of nitrogen fertilizer application and, in turn, answer this question. The key elements considered are the expected nitrogen loss associated with application timing, seasonal variations in nitrogen fertilizer application costs (including the nitrogen fertilizer price and operating costs),[2,3] and a farmer's perceived risk of being unable to apply nitrogen after planting because of weather and field conditions.

In assessing the issue, behavioral models are employed using the analytical framework of Feinerman *et al.* (1990) to explain why most farmers might practice the split application of nitrogen fertilizer whereby some nitrogen fertilizer is applied before planting and some after planting. Results of the 1989 Cotton Water Quality Survey conducted by the U.S. Department of Agriculture are used to verify the integrity of the behavioral models considered. The data reflect farming practices with regard to the use and timing of nitrogen fertilizer application in the production of cotton in the United States.

This paper is organized into four sections. In the first section, two different farm-level decision models for production are formulated that include factors that affect a farmer's decision concerning the timing of nitrogen fertilizer application. The models differ based on the assumed risk preference of a farmer. In the second section, the 1989 Cotton Water Quality Survey data are used to estimate nitrogen fertilizer losses due to timing of application and irrigation. These estimates are combined with the farm-level production decision models in the third section to determine an optimal timing for nitrogen fertilizer application to explain the timing decisions of the cotton farmers in the United States. The final section offers the conclusions.

A Farm-Level Decision Model of Production

In this section a farm-level decision model for production will be developed in order to examine the factors affecting the decision concerning the timing of nitrogen fertilizer applied for both irrigated and dryland (non-irrigated) cotton production.[4] Consider the cotton production function

---

[2] Nitrogen fertilizer prices are, in general, lower in the fall than in the spring. For example, from 1987 to 1991, the spring price of anhydrous ammonia was, on average, 10% higher than the fall price of the preceding year (National Agricultural Statistics Service, 1991).

[3] Operating costs include such things as hired labor costs, energy expenses to operate farm machinery, etc.

$$Y(N_a, W \mid V) \tag{1}$$

where N_a is the nitrogen fertilizer available during the growing season, W is the irrigation rate and V is a vector of site specific variables (including such things as the slope of the cropland, soil permeability, and soil organic matter content). It is assumed that $\partial Y/\partial N_a \geqslant 0$ and $\partial^2 Y/\partial^2 N_a \leqslant 0$. Both N_a and W are a farmer's decision variables while V represents the characteristics of the cropland over which a farmer has little control.[5] The variable N_a is defined as[6]

$$N_a = N_f(d_1-e_1) + N_s(d_2-e_2) + N_g(1-e_3) \tag{2}$$

where N_f is the amount of nitrogen fertilizer applied in the fall, N_s is the amount of nitrogen fertilizer applied in the spring, and N_g is the amount of nitrogen fertilizer applied during the growing season. The parameters d_1 and d_2 represent the portions of the nitrogen fertilizer applied in the fall and spring, respectively, that are available for plant uptake during the growing season. The parameters e_1, e_2, and e_3 represent, respectively, the portions of fall, spring, and growing season nitrogen fertilizer losses associated with irrigation. Thus, e_1, e_2, and e_3 are zero when there is no irrigation. Faced with an expected cotton price of p_c, water costs for irrigation of r_w, and fall, spring, and expected growing season nitrogen fertilizer prices of r_f, r_s, and r_g, respectively, and imputed operating costs of OC,[7] a farmer will endeavor to maximize his or her expected utility of net farm income, π:

$$\begin{aligned} Z &= E[U(\pi)] \\ &= E[U(p_c\, Y(N_a, W \mid \mathrm{V}) - r_f N_f - r_s N_s - r_g N_g - r_w W - \mathrm{OC})] \end{aligned} \tag{3}$$

where E is the expectation operator and $U(\pi)$ is a monotonically increasing and concave von Neumann-Morgenstern utility function where $\partial Y/\partial N_a \geqslant 0$ and $\partial^2 Y/\partial^2 N_a \leqslant 0$ (Anderson *et al.*, 1977).

Next, assume that P represents a farmer's perceived probability of having weather and field conditions that prohibit nitrogen fertilizer from being applied after planting.

---

[4] Both irrigated and dryland cotton production are considered coincidentally in order to fully utilize the 1989 Cotton Water Quality Survey data. The survey questionnaire makes a distinction between the two different production practices.

[5] Obviously, the irrigation rate will equal zero in both the fall and spring. To keep the notation as simplified as possible, however, the form of the production will not be changed between periods.

[6] An alternative specification for N_a was considered where

$$N_a = N_f(d_1-e_1 W) + N_s(d_2-e_2 W) + N_g(1-e_3\, W),$$

where the $e_i s$ (i=1, 2, 3) represent nitrogen losses associated with irrigation and the other terms are as defined in the text. None of the e_is, however, were statistically significantly different than zero at the 95% level in preliminary analyses. Consequently, this specification is not used in the exposition.

[7] Imputed operating costs are defined as the computed marginal labor, energy and machinery costs for each unit of irrigation water and nitrogen fertilizer applied. Operating costs vary with the fertilizer application method. For example, the cost of applying fertilizer will be less when the fertilizer is applied coincidentally with cultivation (or irrigation) than when it is applied separately (Taylor, 1991).

A risk neutral farmer maximizes his or her expected utility of net farm income by maximizing expected profit (Arrow, 1971; Borch, 1968).[8] Consequently, the objective function for a risk neutral farmer becomes

$$\begin{aligned} Z = {} & (p_c\, Y(N_a, W \mid V) - r_f N_f - r_s N_s - r_g N_g - r_w W - \mathrm{OC})\,(1\text{–}\mathrm{P}) \\ & + (p_c\, Y(N_a, W \mid V, N_g{=}0) - r_f N_f - r_s N_s - r_w W - \mathrm{OC})\,\mathrm{P} \end{aligned} \tag{4}$$

where $Y(N_a, W \mid V, N_g{=}0)$ is the yield function when N_a includes no growing season nitrogen fertilizer application.[9]

The first order conditions for the optimal levels of N_f, N_s, N_g and W that maximize Z are

$$\begin{aligned} \partial Z/\partial N_f = {} & p_c(d_1\text{–}e_1)\,(\partial Y(N_a, W \mid V)/\partial N_a \\ & - (\partial Y(N_a, W \mid V)/\partial N_a - \partial Y(N_a, W \mid V, N_g = 0)/\partial N_a)\ \mathrm{P}) \\ & - r_f \leqslant 0 \\ \partial Z/\partial N_s = {} & p_c(d_2\text{–}e_2)\,(\partial Y(N_a, W \mid V)/\partial N_a \\ & - (\partial Y(N_a, W \mid V)/\partial N_a - \partial Y(N_a, W \mid V, N_g = 0)/\partial N_a)\ \mathrm{P}) \\ & - r_s \leqslant 0 \\ \partial Z/\partial N_g = {} & (p_c(1\text{–}e_3)\,\partial Y(N_a, W \mid V)/\partial N_a - r_g(1 - P) \leqslant 0 \\ \partial Z/\partial W = {} & p_c\,\partial Y(N_a, W \mid V)/\partial W(1 - P) \\ & + (\partial Y(N_a, W \mid V, N_g = 0)/\partial W - r_w\, P \leqslant 0 \end{aligned} \tag{5}$$

Note that the term $(\partial Y(N_a, W \mid V)/\partial N_a - \partial Y(N_a, W \mid V, N_g = 0)/\partial N_a)P$ found in the first and second relationships is the expected (marginal) productivity gains due to a split application of nitrogen fertilizer.

From these first order conditions, the optimal timing of nitrogen fertilizer application can be determined (1) when growing-season application is not possible (i.e., $P = 1$), (2) when growing-season application is not restricted (i.e., $P = 0$), and (3) when growing-season application may be possible (i.e., $0 < P <$). These are considered in turn.

A. NITROGEN FERTILIZER APPLICATION WHEN GROWING-SEASON APPLICATION IS NOT POSSIBLE

When the growing-season application of nitrogen fertilizer is not possible, $N_g = 0$ and $P = 1$. Thus, the first order conditions relating to nitrogen fertilizer application

---

[8] A risk neutral farmer will not change his or her behavior in response to a variation in profit. That is, for a risk neutral farmer, $\partial^2 U(\pi)/\partial^2 \pi = 0$ (Deaton and Muellbauer, 1980).

[9] Note that there is an implicit assumption here that the prices of the factors of production remain unchanged between the two scenarios. Assuming they are different – which they might or might not be depending on a variety of economic considerations – needlessly complicates the analysis.

from the relationships given in (5) can be written as

$$\begin{aligned} &p_c(d_1 - e_1)\,(\partial Y(N_a, W \mid V, N_g = 0)/\partial N_a) \leqslant r_f \\ &p_c(d_2 - e_2)\,(\partial Y(N_a, W \mid V, N_g = 0)/\partial N_a) \leqslant r_s\,. \end{aligned} \tag{5a}$$

Solving these two relationships will give an optimal nitrogen fertilizer application rate either for the fall or the spring, or for an optimal fall/spring split application. Moreover, these relationships will give the necessary condition for preferring a fall application over a spring application. This is given by[10]

$$(d_1 - e_1)/(d_2 - e_2) \geqslant r_f/r_s\,. \tag{6}$$

Spring application is preferable to a fall application if the converse relationship holds. Note that the relationship given in (6) holds even if P is not equal to 1. The selection of either a fall or a spring application is not affected by the value of P. Proof of this statement is straightforward and hence left to the reader.

B. NITROGEN FERTILIZER APPLICATION WHEN GROWING-SEASON APPLICATION IS NOT RESTRICTED

When growing season application of nitrogen fertilizer is not restricted (i.e., $P = 0$), the first order conditions of (5) relating to nitrogen fertilizer application can be written as

$$\begin{aligned} &p_c(d_1 - e_1)\,(\partial Y(N_a, W \mid V)/\partial N_a) \leqslant r_f \\ &p_c(d_2 - e_2)\,(\partial Y(N_a, W \mid V)/\partial N_a) \leqslant r_s \\ &p_c(1 - e_3)\,(\partial Y(N_a, W \mid V)/\partial N_a) \leqslant r_g\,. \end{aligned} \tag{5b}$$

The solution to this system of relationships gives an optimal application rate for nitrogen fertilizer for the fall, the spring, or the growing-season. It can be shown[11] that fall application is preferred to growing season application when

$$(d_1 - e_1)/(1 - e_3) \geqslant r_f/r_f \tag{7}$$

and that spring application is preferred to growing season application when

$$(d_2 - e_2)/(1 - e_3) \geqslant r_s/r_g \tag{8}$$

---

[10] Using the first order conditions in (5a), fall nitrogen fertilizer application is preferable to spring application if the adjusted fall nitrogen fertilizer cost is less than the adjusted spring fertilizer cost, assuming all other things equal. That is, fall application is preferred if

$$r_f/(d_1 - e_1) \leqslant r_s/(d_2 - e_2).$$

The relationship can be rewritten as

$$(d_1 - e_1)/\,(d_2 - e_2) \geqslant r_f/r_s.$$

[11] The mechanics are left to the reader. The details are similar to those found in the previous footnote.

If the goal is to promote growing season nitrogen fertilizer application,[12] then relationships (7) and (8) can be used to determine a tax on nitrogen fertilizer to be applied in the fall or spring to discourage a farmer from applying fertilizer before planting. Alternatively, the relationships can be used to determine a subsidy to nitrogen fertilizer applied during the growing season to provide a farmer with an incentive not to apply nitrogen fertilizer before planting. Thus, for example, a tax on nitrogen fertilizer applied in the fall, t_f, may be imposed to discourage fall application. This tax must be sufficient so that when it is added to r_f, the following relationship holds:

$$(d_1 - e_1)/(1 - e_3) \leqslant (r_f + t_f)/r_g. \tag{9}$$

This guarantees that a farmer endeavoring to maximize his or her expected utility of net farm income (profit) will not apply nitrogen fertilizer in the fall.

C. NITROGEN FERTILIZER APPLICATION WHEN GROWING-SEASON APPLICATION MAY BE POSSIBLE

Uncertainty as to the condition of the field at the time of planting appears to play a significant role in a farmer's decision concerning nitrogen fertilizer application timing (Feinerman *et al.*, 1990; Stan and Hinkle, 1987). A farmer practices a fall or spring application because he or she assigns a nonzero probability to the likelihood that nitrogen fertilizer application will not be possible after planting. It is assumed that the farmer assigns the probability P that he or she will be unable to apply nitrogen fertilizer after planting. This P is used as the basis for determining application timing and the amount of nitrogen fertilizer applied to maximize expected net farm income.

When growing-season application of nitrogen fertilizer may be possible (i.e., $0 < P < 1$), the first order conditions from (5) can be used to illustrate that a split nitrogen fertilizer application can be optimal whereby some nitrogen fertilizer is applied prior to planting and some after planting. In what follows, the necessary condition for a split application to be optimal will be developed. Fall and growing-season split application is used as the example. The result is easy to generalize.

To develop the necessary condition for a split application of nitrogen fertilizer to be optimal, first move r_f in the first relationship in (5) to the right-hand-side and also move the term involving r_g in the third relationship in (5) to the right-hand-side. Then divide the first relationship by the third to obtain the following:

$$\begin{aligned} &((d_1 - e_1)/(1 - e_3))\,(1 - P \\ &+ (P\,(\partial Y(N_a, W \mid V, N_g = 0)/\partial N_a)/(\partial Y(N_a, W \mid V)/\partial N_a)\,)\,) \leqslant r_f/r_g. \end{aligned} \tag{10}$$

This relationship suggests the optimal nitrogen fertilizer application rates for

---

[12] It has been suggested that this policy be promoted in order to minimize nitrogen fertilizer runoff into surface water and to reduce nitrate leaching into the ground water. Blackmer (1987), among others, discusses this in more detail.

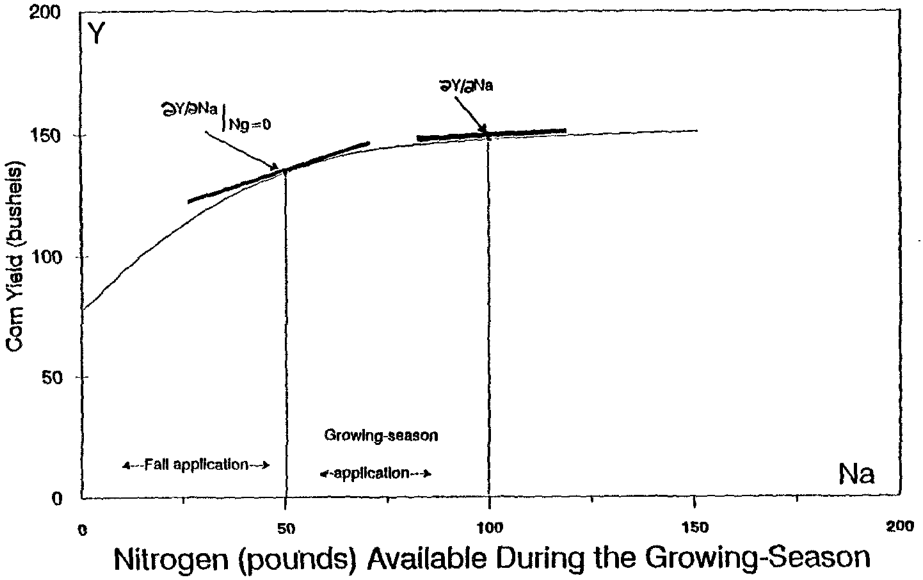

Fig. 1. Marginal products of fall and growing-season split nitrogen fertilizer application.

the fall and growing-season. If a fall/growing-season split application is optimal, then the ratio of the marginal physical product for fall applied nitrogen fertilizer and growing-season nitrogen fertilizer applied, $(\partial Y(N_a, W \mid V, N_g = 0)/\partial N_a)/(\partial Y(N_a, W \mid V)/\partial N_a)$), will be greater than or equal to 1. Note that $\partial Y(N_a, W \mid V, N_g = 0)/\partial N_a)$ is equal to $(d_1 - e_1)\ \partial Y(N_a, W \mid V, N_g = 0)/\partial N_f)$, the marginal physical product of nitrogen fertilizer applied in the fall discounted by nitrogen fertilizer losses. Figure 1 shows the relationship between the marginal physical products of nitrogen fertilizer applied in the fall and during the growing-season under a fall/growing-season split application.

Next, for given values of P, d_1, d_2, e_1, e_2, and e_3, the necessary condition for a fall/growing-season split fertilizer application to be optimal is for the adjusted fall nitrogen fertilizer cost to be greater than the adjusted growing-season nitrogen fertilizer cost. That is, a fall/growing season split application will be optimal when

$$r_f/(d_1 - e_1) \geqslant r_g/(1 - e_3). \qquad (11)$$

This is demonstrated as follows: In relationship (10), let $K_1 = ((d_1 - e_1)/(1 - e_3)$ which is less than 1[13] and let $K_2 = (r_f/r_g)$ which also less than or equal to one (National Agricultural Statistics Service, 1991). Also, let $K_3 = (1 - P + (P\ (\partial Y(N_a, W \mid V, N_g = 0)/\partial N_a)/(\partial Y(N_a, W \mid V)/\partial N_a)))$. Then relationship (10) can be expressed as $K_1\ K_3 \leqslant K_2$. If a split application is optimal, K_3 will be greater than one for P not equal to zero because $(\partial Y(N_a, W \mid V, N_g = 0)/\partial N_a)/(\partial Y(N_a, W \mid V)/\partial N_a)$ is always greater than or equal to one as noted above. For K_3 is greater

---

[13] This is empirically verified below.

than one, K_1 will have to be less than K_2 in order for a split fertilizer application to occur. That is, for $K_3 > 1$, $(d_1 - e_1)/(1 - e_3) \leqslant r_f/r_g$. Rearranging terms yields relationship (11) which implies that the adjusted nitrogen cost (adjusted for nitrogen losses) for nitrogen fertilizer applied during the growing season must be less than the adjusted cost of nitrogen applied in the fall when the perceived probability that the farmer will not be able to apply nitrogen fertilizer during the growing season, P, is greater than 0 and less than 1 in order for a split application of nitrogen fertilizer to be optimal. Intuitively, if the adjusted cost of nitrogen fertilizer applied during the growing season is greater than the price for fall applied nitrogen fertilizer, the farmer would plan to apply all his or her nitrogen fertilizer in the fall to ensure that nitrogen would be available for plant uptake.

Furthermore, it would be increasingly likely that a farmer would apply nitrogen fertilizer only during the growing season as the adjusted price of nitrogen fertilizer applied during the growing season becomes smaller relative to the adjusted price of fall applied nitrogen fertilizer. In fact, using the first and the third relationships in (5), it can be shown that a farmer would not apply nitrogen fertilizer in the fall if

$$\begin{aligned}&(\partial Y(N_a, W \mid V, N_g = 0)/\partial N_a)\, P \\ &< (r_f/(p_c\,(d_1 - e_1)) - (r_g/p_c\,(1 - e_3)))\,(1 - P).\end{aligned} \tag{12}$$

That is, when the perceived probability of not being able to apply nitrogen fertilizer during the growing season is P for $0 < P < 1$, nitrogen fertilizer will not be applied in the fall by a risk neutral, profit maximizing farmer when the expected marginal physical product of nitrogen fertilizer applied in the fall is less than the expected adjusted price of nitrogen fertilizer applied in the fall after subtracting the expected adjusted cost of growing-season applied nitrogen fertilizer.

A Farm-Level Decision Model with Risk Aversion

The foregoing section analyzed the timing of the application of nitrogen fertilizer assuming a farmer endeavoring to maximize expected utility of net farm income is risk neutral. Now, the analysis will be altered by assuming that the farmer is risk averse. With P defined as the perceived probability that field conditions will not allow nitrogen fertilizer to be applied after planting, the expected utility function of a risk averse, profit maximizing farmer can be expressed as:

$$\begin{aligned}Z &= E\,[U(\pi)] \\ &= U(\pi_1)\,(1 - P) + U(\pi_2)\,P\end{aligned} \tag{13}$$

where $\pi_1 = p_c\, Y(N_a, W \mid V) - r_f N_f - r_s N_s - r_g N_g - r_w W - \mathrm{OC}$, $\pi_2 = p_c\, Y(N_a, W \mid V, N_g = 0) - r_f N_f - r_s N_s - r_g N_g - r_w W - \mathrm{OC}$, and the other terms are as previously defined.

In this formulation of the problem as in the risk neutral case, it is assumed that the farmer is endeavoring to maximize expected utility of net farm income where net farm income is dependent on the timing of nitrogen fertilizer application. The formulation does not consider the impact on net farm income due to uncertainty in the price of nitrogen fertilizer associated with seasonal variations.

The first order conditions for the optimal nitrogen fertilizer application rates and irrigation, N_f, N_s, N_g, W - which are variables over which the farmer has control - are [14]

$$\begin{aligned}
\partial Z/\partial N_f = {} & U'(\pi_1)\,(p_c\,(d_1 - e_1)\,(\partial Y(N_a, W \mid V)/\partial N_a) - r_f)\,(1 - P) \\
& + U'(\pi_2)\,(p_c\,(d_1 - e_1)\,(\partial Y(N_a, W \mid V, N_g = 0)/\partial N_a) - r_f)\,P \\
& \leqslant 0 \\
\partial Z/\partial N_s = {} & U'(\pi_1)\,(p_c\,(d_2 - e_2)\,(\partial Y(N_a, W \mid V)/\partial N_a) - r_s)\,(1 - P) \\
& + U'(\pi_2)\,(p_c\,(d_2 - e_2)\,(\partial Y(N_a, W \mid V, N_g = 0)/\partial N_a) - r_s)\,P \\
& \leqslant 0 \\
\partial Z/\partial N_g = {} & p_c\,(1 - e_3)\,(\partial Y(N_a, W \mid V)/\partial N_a) - r_g \leqslant 0 \\
\partial Z/\partial W = {} & U'(\pi_1)\,(p_c\,\partial Y(N_a, W \mid V)/\partial W) - r_w)\,(1 - P) \\
& + U'(\pi_2)\,(p_c\,\partial Y(N_a, W \mid V)/\partial W) - r_w)\,P \leqslant 0\,.
\end{aligned} \tag{14}$$

Note that if the after-planting application of nitrogen fertilizer is not possible (i.e., $P = 1$), the first order conditions for the optimal nitrogen fertilizer application rates for a risk averse farmer are the same as the conditions for a risk neutral farmer. Similarly, if the after-planting application is always possible (i.e., $P = 0$), the first order conditions for the optimal nitrogen fertilizer application rates are same as the conditions for a risk neutral farmer. Also note that the third relationship in (14) indicates that the optimal amount of nitrogen fertilizer that should be applied during the growing season is not affected by the nature of the utility function specified for a farmer maximizing the expected utility of net farm income.

When growing-season application of nitrogen fertilizer may be possible (i.e., $0 < P < 1$), the optimal rates for the fall and growing-season application of nitrogen fertilizer can be derived. (As before, in order to simplify the exposition, just the fall/growing season split is considered. The generalization of the results to include spring application is straightforward). The optimal application rates for split fall/growing-season applications are given by

$$\begin{aligned}
& ((d_1 - e_1) / (1 - e_3))\,((1 - P + P\,(U'(\pi_2)/U'(\pi_1)) \\
& ((\partial Y(N_a, W \mid V \mid N_g = 0) / \partial N_a) / (\partial Y(N_a, W \mid V) / \partial N_a)))\,/\,(1 - P \\
& + P\,(U'\,(\pi_2) / U'\,(\pi_1))) \leqslant r_f/r_g\,.
\end{aligned} \tag{15}$$

---

[14] Note that U′ denotes the first derivative of the farmer's utility function with respect to the decision variable.

In order to give the relationship an empirical measure, it is necessary to introduce a functional form for the utility function. A utility function that is frequently used and one that is mathematically tractable is one with constant relative risk aversion – $U(\pi) = \pi^{(1-R)}/(1-R)$ where R is a measure of risk aversion and $0 \geqslant R$.[15] This utility function has been suggested as one providing an appropriate structure for investigating risk in supply response analyses (Newbery and Stiglitz, 1981; Pope and Just, 1991).

Given this functional specification for the farmer's utility function, the ratio $U'\pi_2)/U'(\pi_1)$ is equal to $(\pi_1/\pi_2)^R$. Substituting this into relationship (15), the relationship can be rewritten as

$$\begin{aligned}&((d_1 - e_1) / (1 - e_3)) (((1 - P + P (\pi_1/\pi_2)^R \\ &(\partial Y(N_a, W \mid V, N_g = 0) / \partial N_a) / (\partial Y(N_a, W \mid V) / \partial N_a)) / \qquad (16) \\ &(1 - P + P (\pi_1)/\pi_2)^R \leqslant r_f/r_g .\end{aligned}$$

Note that relationship (16) becomes relationship (10) when the farmer is risk neutral (i.e., $R = 0$). Using an argument similar to that for when a farmer is risk neutral, it can be shown that relationship (11) is also a necessary condition for an optimal split fall/growing-season nitrogen fertilizer application for a risk averse farmer when P is greater than 0 and less than 1.[16]

So far it has been shown that if the necessary condition (relationship (11)) holds, a split application of nitrogen fertilizer is the optimal strategy for both risk neutral and risk averse farmers when $0 < P < 1$. Next, the question of whether a risk averse farmer will apply more nitrogen fertilizer in the fall than a risk neutral farmer if both risk averse and risk neutral farmers have the same values for d, e, P and yield response to N_a will be addressed. If this is the case, will the term $(\partial Y(N_a, W \mid V, N_g = 0)) / (\partial N_a/\partial Y(N_a, W \mid V) / \partial N_a))$ be larger for a risk neutral farmer than for a risk averse farmer at their respective optimal level of N_a? In what follows, it will be shown that a risk averse farmer will apply more nitrogen fertilizer in the fall than will a risk neutral farmer.

To simplify the exposition, assume that each of the relationships in (11) holds with equality. From the third relationship, $\partial Y(N_a, W \mid V)/\partial N_a = r_g/(p_c\,(1 - e_3))$. Substitute this expression for $\partial Y(N_a, W \mid V)/\partial N_a$ in the first relationship. Rearranging terms, the first relationship can be expressed as

$$\begin{aligned}\partial Y(N_a, W \mid V, N_g = 0) / \partial N_a = {} & r_f/(p_c(d_1 - e_1)) \\ & - (\pi_2/\pi_1)^R\, (r_g/(p_c\,(1 - e_3)) \\ & - r_f/(p_c\,(d_1 - e_1)))\,(1 - P)/P. \qquad (17)\end{aligned}$$

---

[15] The farmer is risk neutral when $R = 0$.

[16] Let $K_1 = (d_1 - e_1)/(1 - e_3)$, $K_2 = r_f/r_g$, and $K_3 = (((1 - P + P(\pi_1/\pi_2)^R\,((\partial Y(N_a, W \mid V, N_g = 0)/(\partial Y(N_a, W \mid V)/\partial N_a)))/(1 - P + P(\pi_1/\pi_2)^R))$. Then, $K_1\,K_3 = K_2$. If a split nitrogen fertilizer application is optimal, then the term $(\partial Y(N_a, W \mid V, N_g = 0)/(\partial Y(N_a, W \mid V)/\partial N_a))$ will be greater than 1 which implies that K_3 is greater than 1. If K_3 is greater than 1, then K_1 must be less than K_2.

Observe that Equation (17) employs the utility function with constant relative risk aversion. Since $R > 0$ for a risk averse farmer and $R = 0$ for a risk neutral farmer, the difference in the marginal physical product of fall applied nitrogen fertilizer, for a given level of N_a, between a risk averse farmer and a risk neutral farmer can be computed. This difference is given as

$$\begin{aligned}&(\partial Y(N_a, W \mid V, N_g = 0) / \partial N_a)_n - (\partial Y(N_a, W \mid V, N_g = 0) / \partial N_a)_a \\ &= -(1 - (\pi_2/\pi_1)^R)((r_g/[p_c(1 - e_3)) - r_f/(p_c(d_1 - e_1)))(1 - P)/P.\end{aligned} \tag{18}$$

where subscripts n and a denote a risk neutral and risk averse farmer, respectively. From Equation (18), it can be seen that if the adjusted fall nitrogen fertilizer cost, $r_f/(d_1 - e_1)$, is greater than the adjusted growing-season nitrogen fertilizer cost, $r_g/(1 - e_3)$, for a given p_c, then $(\partial Y(N_a, W \mid V, N_g = 0)/\partial N_a)_n - (\partial Y(N_a, W \mid V, N_g = 0)/\partial N_a)_a$ is greater than 0. This is because $(\pi_2/\pi_1)^R < 1$. Therefore, a risk averse farmer will tend to allocate relatively more nitrogen fertilizer for fall application than will a risk neutral farmer, all other things equal.

This result has an intuitive explanation by using the fact that a risk neutral farmer maximizes expected net farm income while a risk averse farmer simultaneously maximizes expected net farm income and minimizes the variance of net farm income (Newbery and Stiglitz, 1981). When the adjusted fall nitrogen fertilizer cost increases relative to the adjusted growing-season nitrogen fertilizer cost, a risk neutral farmer in attempting to maximize the expected utility of net farm income will reduce his or her fall nitrogen fertilizer application and increase the growing-season application. As the fall nitrogen fertilizer application rate is reduced and more nitrogen fertilizer is applied during the growing season, the difference between net farm incomes π_1 and π_2 increases. Consequently, the variance of net farm income, $P(1 - P)(\pi_1 - \pi_2)^2$ increases.[17] Since a risk averse farmer would also attempt to minimize this variance, he or she will be more reticent than the risk neutral farmer to reduce the application of nitrogen fertilizer in the fall and increase growing-season application. Therefore, a risk averse farmer is likely to apply more nitrogen fertilizer in the fall than a risk neutral farmer when the adjusted fall nitrogen fertilizer price increases relative to the growing-season price. Furthermore, if both risk neutral and risk averse farmers are able to apply nitrogen fertilizer during the growing season, each will equate $p_c(1 - e_3)(\partial Y(N_a, W \mid V)/\partial N_a$ with r_g (from the third relationship in (14)). Assuming N_a is the same between farmers, a risk averse farmer would apply less nitrogen fertilizer during the growing season than would a risk neutral farmer.

Next, from Equation (18), if the adjusted fall nitrogen fertilizer cost is less than the adjusted growing-season nitrogen fertilizer cost, $(\partial Y(N_a, W \mid V, N_g = 0)/\partial N_a)_n$

---

[17] Given the net farm income random variable π_1 with a probability of occurring of P and the net farm income random variable π_2 with a probability of occurring of $(1\text{-}P)$, expected net farm income $E[\pi] = \pi_1 P + \pi_2 (1 - P)$. The variance of expected net farm income, $V(\pi) = E[\pi^2] - (E[\pi])^2 = P(1 - P)(\pi_1 - \pi_2)^2$. Newbery and Stiglitz (1981) develop this and other relationships.

is less than $(\partial Y(N_a, W | V, N_g = 0)/\partial N_a)_a$. Thus, a risk averse farmer, in contrast to a risk neutral farmer, would tend allocate relatively less nitrogen fertilizer for a given less of N_a for fall application.

Additionally, as P approaches 1, the term $(1 - P)/P$ in Equation (18) approaches 0. Consequently, the difference in the fall nitrogen fertilizer application by a risk neutral farmer and by a risk averse farmer will become small. Alternatively, as P increases, π_2/π_1 approaches 1 because more nitrogen fertilizer will be applied in the fall and less during the growing season. This implies that the term $(\pi_2/\pi_1)^R$ in Equation (18) will approach 1. As a result, the difference in nitrogen fertilizer application rates between seasons by a risk averse and by a risk neutral farmer becomes insignificant regardless of the value of R.

The foregoing two sections have developed the necessary conditions for the optimal timing of nitrogen fertilizer application for a risk neutral and a risk averse farmer. In what follows, these necessary conditions will be used to explore the timing decisions of cotton farmers in the United States.

Estimating Nitrogen Fertilizer Losses

(a) BACKGROUND

In order to empirically implement the farm-level decision models developed, estimates of nitrogen losses associated with the timing of nitrogen fertilizer application is needed. Before developing these estimates, however, the data used in the estimation will be discussed.

As part of the President's Water Quality Initiative, the U.S. Department of Agriculture's Economic Research Service (ERS) and National Agricultural Statistics Service (NASS) conducted a survey of cotton producers in 14 Southern and Western States in 1989 (Crutchfield *et al.*, 1990). The survey accounted for cotton production on 10.5 million acres (99% of the total planted cotton acreage in the United States). Information on eleven different aspects of cotton production was obtained. There were questions on yield, fertilizer use, herbicides and insecticides applied, irrigation practices employed, water availability, government program participation, value of the cropland on which cotton was produced, pest management practices employed, operator characteristics and soil characteristics. The fertilizer use section collected data on fertilizer application rates in the fall and spring (both pre-planting periods), coincident with planting and during the growing season (i.e., post-planting). The soil characteristics section collected information on the soil texture for each field surveyed classified according to the Soils-5 criteria (Soil Conservation Service, 1983). The survey yielded 1491 useable responses.

Cotton farmers in the United States practice single and split nitrogen fertilizer applications. Of the total number of cotton producers responding to the survey, 315 (21.1%) had a single nitrogen fertilizer application in the spring, 280 (18.8%) applied nitrogen fertilizer in the spring and during the growing season, and 213

(14.3%) of those surveyed applied no nitrogen fertilizer. Additionally, 67.4% of cotton farmers applied nitrogen fertilizer in the fall or in the spring but few (0.6%) applied it in both the fall and spring. Many cotton producers surveyed (32.4%) practice split applications whereby the application of nitrogen fertilizer was split before and after planting. Finally, most farmers applied some fertilizer during the growing season (78.1%). Note that these percentages sum to more that 100% since some farmers applied nitrogen fertilizer in more than one period.

The nitrogen fertilizer application rate varies significantly across application timing. For example, farmers applying nitrogen fertilizer during the fall as well as the planting and growing seasons applied an average of 253 pounds per acre (although the sample is small so any inferences are obviously suspect) while farmers applying nitrogen fertilizer during the planting and growing seasons applied an average of 120 pounds per acre.

(b) COTTON YIELD FUNCTION ESTIMATE

Estimates of nitrogen losses associated with the timing of nitrogen fertilizer application are needed. To this end, a production function approach will be used to estimate nitrogen losses for nitrogen fertilizer applied. The approach estimates a cotton yield function designed to show the relationship between the amount and timing of nitrogen fertilizer applied and cotton yield. Because the functional specification of the yield function can affect the estimates, several commonly used specifications were considered in order to test the robustness of the estimates (Cerrato and Blackmer, 1987; Frank *et al.*, 1990). The purpose in using several different functional forms was not to compare the performance of one against the others. Rather, it was to assist in deciding whether the integrity of the conclusions with regard to nitrogen fertilizer application timing are such as to withstand scrutiny if the estimates are subject to a robustness test. Also, different functional specifications permit a comparison of the magnitudes of the estimated nitrogen fertilizer losses from the various functional forms to see whether there is any consistency. Three functional forms were tested. These were a quadratic specification, Mitscherlich-Baule functional form, and a von Liebig-type specification. While each functional form is considerably different from the others (they were selected because of this), each is relatively parsimonious and imposes certain technical characteristics on the yield response curve. (Frank *et al.*, 1990 discusses these considerations in greater detail).

Nonlinear least squares was used to estimate the parameters for each cotton yield function specification. An examination of the matrix of correlation coefficients indicated no significant collinearity was present among the explanatory variables. Additionally, the variance-decomposition proportions (Belsley, Kuh, and Welsch, 1980) were uniformly relatively small. The proportions never exceed 0.30 and typically average around 0.01 to 0.05. Using the adjusted coefficient of determination (i.e., R^2) and the Davidson-MacKinnon J-test as evaluation criteria, the quadratic specification, among those considered, best characterizes the relationship between

TABLE I

Estimated quadratic cotton yield function[a]

$$
\begin{aligned}
\mathrm{Y} = {} & \underset{(63.21)}{315.20} + \underset{(39.43)}{118.58}\,\mathrm{IR} + \underset{(319.47)}{186.83}\,\mathrm{WC} - \underset{(691.11)}{1207.78}\,\mathrm{SL} - \underset{(22.10)}{13.66}\,\mathrm{PC} \\
& + \underset{(0.1203)}{0.1197}\,\mathrm{RF} - \underset{(4.54)}{3.95}\,\mathrm{SP} + \underset{(18.90)}{208.49}\,\mathrm{W} - \underset{(2.52)}{22.12}\,\mathrm{W}^2 - \underset{(14.03)}{16.47}\,\mathrm{OM} \\
& + \underset{(0.3732)}{0.8779}\,\mathrm{W}\,[\mathrm{N}_f(\underset{(0.1920)}{0.5792} - \underset{(0.2142)}{0.2821}\,\mathrm{IR}) + (\mathrm{N}_s + \mathrm{N}_p)(\underset{(0.1501)}{0.5920} - \underset{(0.1614)}{0.4041}\,\mathrm{IR}) + \mathrm{N}_g(1 - \underset{(0.1932)}{0.5763}\,\mathrm{IR})] \\
& + \underset{(0.8531)}{5.4724}\,[\mathrm{N}_f(\underset{(0.1904)}{0.5790} - \underset{(0.2112)}{0.2828}\,\mathrm{IR}) + (\mathrm{N}_s + \mathrm{N}_p)(\underset{(0.1515)}{0.5924} - \underset{(0.1604)}{0.4040}\,\mathrm{IR}) + \mathrm{N}_g(1 - \underset{(0.1930)}{0.5761}\,\mathrm{IR})] \\
& - \underset{(0.8544)}{0.02373}\,[\mathrm{N}_f(\underset{(0.1922)}{0.5792} - \underset{(0.2100)}{0.2824}\,\mathrm{IR}) + (\mathrm{N}_s + \mathrm{N}_p)(\underset{(0.1514)}{0.5921} - \underset{(0.1661)}{0.4040}\,\mathrm{IR}) + \mathrm{N}_g(1 - \underset{(0.1931)}{0.5761}\,\mathrm{IR})]^2
\end{aligned}
$$

Adjusted $R^2 = 0.5232$ MSE = 79960.0

[a] The values in parentheses are the standard errors of the estimates.

factor inputs and cotton output (yield) with the von Liebig function being slightly behind in explanatory power. (A complete discussion of the estimation procedure and results is given in Huang *et al.*, 1991).

The estimate of the quadratic yield function is given in Table I. Note that since cotton yield is determined not only by the amount of nitrogen fertilizer available for plant uptake during the growing season, N_a, but also by site specific characteristics, variables to reflect these site specific considerations are included in the yield function specification to reflect yield variation due to these factors. Site specific variables include IR which is a variable indicating whether irrigation occurs (i.e., IR = 1) or does not occur (i.e., IR = 0),[18] WC which measures the water holding capacity of the soil, SL which calibrates the slope of the field, PC which is a variable indicating whether the previous crop was cotton (i.e., PC = 1) or some other crop (PC = 0), RF which is an index of the rainfall as defined by Wischmeier and Smith (1978), SP which is a measure of the soil permeability, and OM which is a measure of the organic matter content of the soil. The other variables found in the table are defined as they were previously.

The quadratic yield function indicates the presence of diminishing marginal returns with respect to the amount of nitrogen fertilizer applied and with respect to irrigation. Moreover, the results suggest that irrigation and nitrogen fertilizer use are complementary inputs. All of the signs on the terms involving nitrogen fertilizer application rates and associated with application timing and those associated with irrigation are consistent with *a priori* expectations. For example, an increase in the nitrogen fertilizer application rate in any period will result in an expansion in the cotton yield while an increase in irrigation likewise will yield an increase

---

[18] This implies that with irrigation, the production function shifts from what it would be with no irrigation.

cotton output. Moreover, given that cross-section data are being used, the quadratic specification fits the data reasonably well.

Before examining the issue of application timing and nitrogen fertilizer losses, consider the impacts of site specific factors on cotton yield. To this end, consider first the effect of the presence or absence of irrigation. As expected, the quadratic functional specification shows that the irrigation variable is statistically significant at the 95% level indicating that irrigated cropland in general has a higher cotton yield than non-irrigated cropland. This is because irrigation promotes nutrient movement to the root by diffusion and mass flow (Rhoads, 1984).

The coefficient on water holding capacity is positive but not significantly different from zero (at the 95% level). A positive sign was anticipated on this variable suggesting that soil with a higher water holding capacity would produce a higher yield. This result is expected because the agronomy literature illustrates that a higher water holding capacity is linked to higher productivity (Metcalfe and Elkins, 1980). Also, a higher water holding capacity can increase nutrient concentration in the soil and enhance the rate of nutrient movement to the root by diffusion and mass flow (Rhoads, 1984).

The coefficient on the slope of the cropland variable is negative but not statistically significantly different from zero at the 95% level. A statistically significant negative sign was expected because a greater slope promotes nitrogen loss by surface runoff leaving less nitrogen fertilizer for plant uptake and a coincident reduction in yield (Gilliam and Hoyt, 1987).

A negative sign on the coefficient estimate for the previous crop planted variable indicates that a reduction in cotton yield would be realized if the previous planted crop were cotton. This result is expected because the continuous planting of cotton promotes the propagation of insect pests such as the boll weevil which causes a loss of cotton yield (Stan and Hinkle, 1947; Metcalfe and Elkins, 1980). Unfortunately, the coefficient estimate on this variable was not statistically significant at the 95% level.

The sign on the coefficient of the rainfall index variable should be negative because the index is designed to quantify the impact of rainfall on soil erosion (Wischmeier and Smith, 1978). A reduction in yield is associated with increased soil loss (Taylor and Frohberg, 1978). The estimate, however, is not statistically significant at the 95% level or better.

The coefficient on soil permeability has a negative sign which is expected although it is not statistically significant (at the 95% level). Soil with greater permeability has a larger potential for nitrate leaching thereby reducing the nitrogen available for plant uptake (Bock, 1984).

A relatively large amount of organic matter in the soil is often associated with elevated crop yields (Soil Conservation Service, 1983). Therefore, a positive sign is expected on the organic matter variable. The estimation results, however, yield a negative sign although the estimate is not statistically significant (at the 95% level). A further examination of this result is warranted in subsequent research.

The impact of irrigation on cotton yield is statistically significant (at the 95% level) for the quadratic functional specification. The estimated coefficients on both the linear term and the quadratic term have the expected sign and are statistically significantly different than zero at the 95% level. The negative sign on the quadratic term indicates the presence of decreasing marginal returns with regard to the quantity of water applied.

(c) NITROGEN FERTILIZER LOSSES AND THE TIMING OF APPLICATIONS

To assess the nitrogen fertilizer losses associated with the timing of application, the empirical estimates from the cotton yield function are used. Nitrogen fertilizer losses, as defined earlier in relationship (2) and estimated in the cotton yield function, will be indicated by the proportion of nitrogen fertilizer applied before planting, which will not be available for plant uptake during the growing-season. The estimation results indicate that for dryland cotton production, fall and spring applications losses are approximately the same as the amount lost during the growing season. For example, fall application losses are about 0.42 (1–0.58) for each pound of nitrogen fertilizer applied while the spring application losses are about 0.41 (1–0.59) for each pound of nitrogen fertilizer applied. The difference in nitrogen fertilizer losses associated with these two application timings is not statistically significantly different than zero at the 95% level.

For irrigated cotton, the loss of nitrogen fertilizer applied during the spring and growing season is also statistically significant. Thus, for each pound of nitrogen fertilizer applied during the growing season, about 43% is available for plant uptake relative to growing-season nitrogen fertilizer applied in dryland cotton production. For each pound of nitrogen fertilizer applied in the spring, about 20% (0.592–0.404) is available during the growing season. Results for the fall application are not statistically significantly different than zero at the 95% level. These results show that irrigation is likely to cause additional nitrogen fertilizer loss for both spring and growing season applications.

(d) NITROGEN FERTILIZER LOSSES AND IRRIGATION

Irrigation results in a significant loss of nitrogen fertilizer in cotton production. The estimation results indicate that, in the presence of irrigation, about 0.58 pounds of nitrogen fertilizer is lost for each pound of nitrogen fertilizer applied during the growing season. When spring application is used, 0.40 of each pound of nitrogen fertilizer applied is lost when irrigation is practiced relative to no such losses for dryland cotton production.

It should be noted, however, that if nitrogen fertilizer is leached into the ground water, some (indeterminant) portion of the estimated loss may appear in the irrigation water and therefore be available for plant uptake. This is indicated by the complementary relation between nitrogen fertilizer applied and irrigation as shown by the statistically significant (at the 95% level) coefficient estimate on the nitrogen application/irrigation cross product term. Also, some portion of the leached nitrogen fertilizer may contribute to the fertility of irrigated cropland (Spilker, 1991). This is evinced by the statistically significant (at the 95% level) coefficient estimate on the irrigation variable, IR. Unfortunately, given the way the information was collected on the Cotton Water Quality Survey, it is not possible to separate these effects from the estimated nitrogen fertilizer losses.

The combined timing and irrigation effects on nitrogen fertilizer losses show that spring nitrogen fertilizer application losses are no greater than fall application losses.

There are a variety of reasons why nitrogen fertilizer loss associated with fall application should be different than losses from a spring application. These include the fact that a relatively high carbon to nitrogen ratio during the fall favors immobilization of nitrates in the soil (Blackmer, 1987). Next, relatively high soil moisture content due to irrigation and spring rain leads to a relatively higher level of denitrification of newly applied nitrogen fertilizer. (Note that the denitrification process refers to the biological transformation of nitrogen in the nitrate or nitrite forms to gaseous forms that escape from the soil (Blackmer, 1987)). Finally, nitrogen fertilizer applied in the spring on the soil surface is more susceptible to runoff loss due rain, while nitrogen fertilizer applied in the fall is typically laid under the top soil (Rhoads, 1984). These factors suggest that the application timing between fall and spring should have differential effects on cotton production. The fact that the empirical results suggest that it does not will be the subject of a future investigation. In what follows, we attempt to explain the current mixed single and split nitrogen fertilizer applications in U.S. cotton production.

Estimation of the Optimal Application Timing of Nitrogen Fertilizer

Estimation of the optimal application timing using the cotton yield function estimate derived together with recent United States nitrogen fertilizer price data will now be presented. The results are based on the previously delineated farm-level decision models for a risk neutral and a risk averse farmer. Corresponding to the previous discussion concerning possible values of the perceived probability P, the discussion is divided into three sections.

A. NITROGEN FERTILIZER APPLICATION WHEN GROWING-SEASON APPLICATION IS NOT POSSIBLE

Relationship (6) is used to test the hypotheses that fall application is preferable to spring application assuming that the farmer is maximizing expected net farm income (utility) and that he or she is risk neutral (averse). For dryland cotton production, $d_1/d_2 = 0.58/0.59 = 0.98$. The price ratio is $r_f/r_s = 10.9/12.0 = 0.91$, where r_f is the United States average fall price in cents per pound of anhydrous ammonia in October 1990 and r_g is the United States average April 1991 price (National Agricultural Statistics Service, 1991).[19] Since the value of d_1/d_2 (= 0.98) is greater than the nitrogen fertilizer price ratio (= 0.90), a fall application is preferable to a spring application. However, since these two ratios are not statistically significantly different from each other (at the 95% level), an U.S. cotton farmer would likely be indifferent between a fall and spring application.[20]

For irrigated cotton, if irrigation has no effect on the loss of fall nitrogen (recall that e_1 was not statistically significantly different than from zero in the cotton yield function estimate), the value of $(d_1 - e_1)/(d_2 - e_2) = 0.58/0.19 = 3.05$. Since this value is much greater than the nitrogen fertilizer price ratio, a farmer would prefer a fall application to a spring application.

B. NITROGEN FERTILIZER APPLICATION WHEN GROWING-SEASON APPLICATION IS NOT RESTRICTED

The hypothesis that growing-season application of nitrogen fertilizer is preferable to fall application for both dryland and irrigated cotton production cannot be rejected, if weather conditions do not inhibit the application of nitrogen fertilizer during the growing season. Using relationship (7), for fall fertilizer application for dryland cotton production, the value of $(d_1 - e_1)/(1 - e_3) = 0.58$ while for irrigated cotton production, the value is $(d_1 - e_1)/(1 - e_3) = 0.58/0.43 = 1.35$. These values suggest that for dryland cotton production, a growing-season application is preferable to a fall application since 0.58 is less than the fall/growing season nitrogen fertilizer price ratio of 0.91, even though the price of nitrogen fertilizer in the fall is lower than the price during the growing season. The nitrogen fertilizer loss from the fall application outweighs the gain due to the relatively lower nitrogen fertilizer price in the fall. For irrigated cotton production, a fall application is preferable to a growing-season application since $(d_1 - e_1)/(1 - e_3)$ (= 1.35) is greater than the nitrogen fertilizer price ratio. This finding simply reflects the fact that if some nitrogen fertilizer applied during growing season is lost due to irrigation, a fall nitrogen fertilizer application or a split fall and growing-season application is preferable.

Also considered are the hypotheses that growing-season nitrogen fertilizer application is preferable to a spring application for irrigated and dryland cotton production. These hypotheses are tested using relationship (8). The numerical results (the details of which are left to the reader) suggest that a growing season application is preferable to a spring application for both irrigated and dryland cotton production.

---

[19] Only the nitrogen fertilizer cost is considered. Not included are operating costs. It is likely that operating costs differ between fall, spring and growing-season applications. Field operating costs are not included because reported estimates of fall, spring and growing-season operating costs are inexorably intertwined with nitrogen fertilizer costs since nitrogen fertilizer application frequently occurs coincidentally with field preparation (Mississippi Cooperative Extension Service, 1987). Any attempt to separate the costs (which run about $5.00 per acre) would be purely arbitrary.

[20] A further uncertainty not dealt with is how expectations concerning the spring nitrogen fertilizer price formulated prior to the fall application will impact a farmer's timing decision.

C. NITROGEN FERTILIZER APPLICATION WHEN GROWING-SEASON APPLICATION MAY BE POSSIBLE

In this section, the optimal timing of the application and amounts of nitrogen fertilizer to be applied are estimated when P is greater than 0 and less than 1. First, relationship (11) will be used to determine whether a split application is optimal. For dryland cotton production, the test is whether a fall and growing-season split application of nitrogen fertilizer is optimal. (Recall that in the previous section, a fall application was identified as optimal given the seasonal nitrogen price differential between the fall and spring.) Because $(d_1 - e_1)/(1 - e_3) = 0.58 < r_f/r_g = 0.91$, a fall/growing-season split application of nitrogen fertilizer is an optimal strategy for dryland cotton production. Furthermore, if there is no seasonal difference in the nitrogen fertilizer price, a cotton farmer would be indifferent between considering a fall/growing-season split and a spring/growing-season split application of nitrogen fertilizer as an optimal strategy. For irrigated cotton production, because $(d_2 - e_2)/(1 - e_3) = 0.44 < 1.0$,[21] a split application is the optimal strategy for both dryland and irrigated cotton production.

Next, the optimal nitrogen fertilizer application rates will be estimated. This will be accomplished by maximizing expected net farm income given by the objective function (4) of a risk neutral farmer. The estimated quadratic cotton yield function given in Table I is used in the computations. The optimal levels of N_f, N_s, and N_g are determined for various values of P. The results for dryland and irrigated cotton production are plotted in Figure 2a and Figura 2b, respectively.[22] For irrigated cotton production, it is assumed that irrigation water is applied at a constant 2.1 acre-feet which is the average rate based on the Cotton Water Quality Survey resuls. The results clearly indicate a split nitrogen fertilizer application for $0 < P < 1$ is the optimal strategy for cotton producers.

To investigate the timing of nitrogen fertilizer application for a risk averse farmer, the constant relative risk aversion utility function previously introduced is employed. It will be assumed that $R = 1.5$.[23] In estimating π_1 and π_2, production costs for Texas dryland cotton and California irrigated cotton are used to estimate other costs, OC (McElroy *et al.*, 1989). The optimal levels of N_f, N_s, and N_g are determined for various values of P. The results for the dryland cotton are given in Table II. The estimates in Table II lend credence to the contention that if the adjusted fall nitrogen cost $(r_f/d_1 = 10.9/0.57 = 19.1)$ is greater than the adjusted growing-season

---

[21] Spring and growing-seasons applications are used as an example because the e_2 of the spring application is significant, while the e_1 of the fall application is not significant.

[22] The optimal nitrogen fertilizer application rates determined by the model generally are larger than the application rates found by the Cotton Water Quality Survey. This discrepancy may be due to fact that only nitrogen fertilizer cost is considered in the model. Labor, energy and equipment and machinery costs associated with the application of nitrogen fertilizer are not included.

[23] There is nothing unique about this value. It is simply used for computational purposes. Alternative values can be considered by the interested reader.

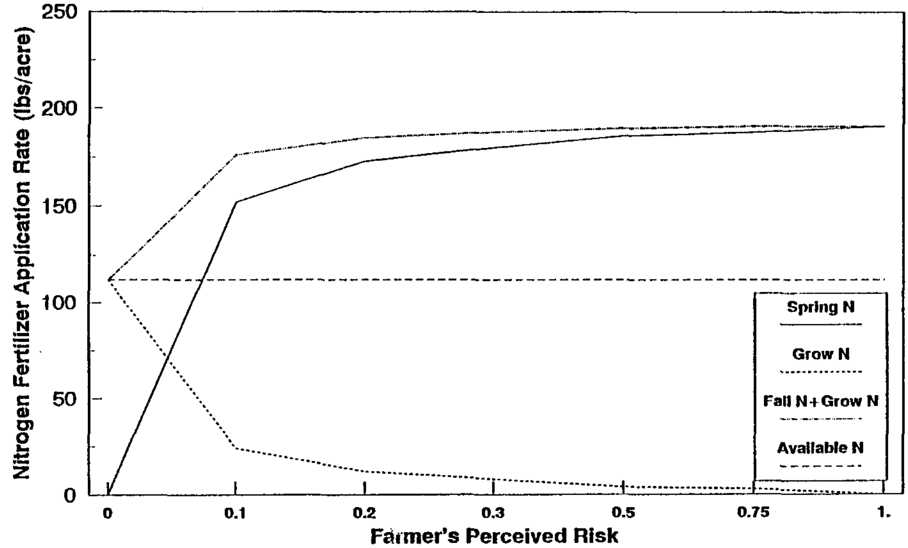

Fig. 2a. Nitrogen fertilizer application rates for dryland cotton.

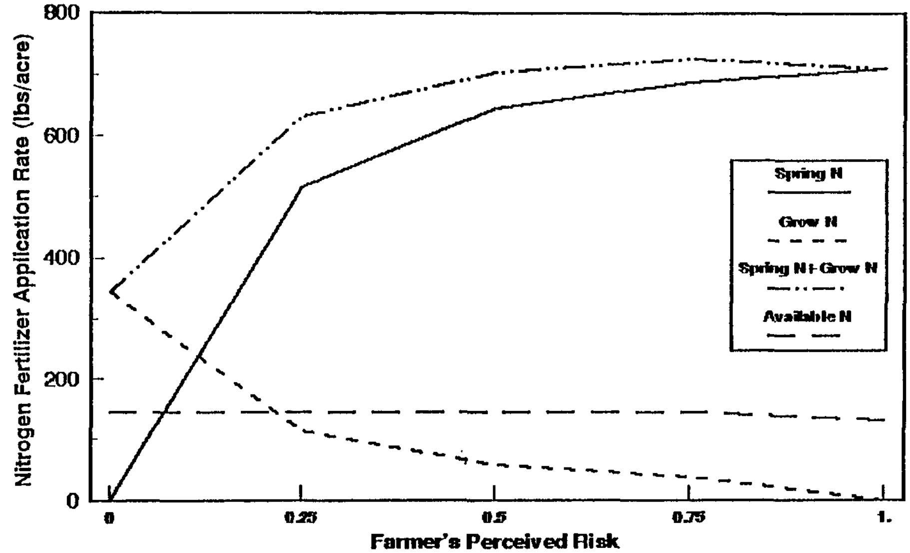

Fig. 2b. Nitrogen fertilizer application rates for irrigated cotton.

nitrogen fertilizer cost, r_g, a risk averse farmer, in contrast to a risk neutral farmer, tends to allocate more nitrogen fertilizer for fall application. The results are consistent with the finding of Feinerman *et al.* (1990).

In sum, the empirical results confirm the theoretical contentions that in the case of dryland cotton production, when growing-season application of nitrogen fertilizer is not restricted, growing-season application is optimal. When growing-season appli-

TABLE II

Nitrogen fertilizer application rates and timings in dryland cotton production under constant relative risk aversion $U = \pi^{(1-R)}/(1-R)$

Probability of failing to apply nitrogen fertilizer during growing-season		Risk-neutral $R = 0$	Risk averse $R = 1.5$
Timing			
0	Fall	0	0
	Growing	112	112
0.10	Fall	152	160
	Growing	24	20
0.20	Fall	173	174
	Growing	12	11
0.30	Fall	180	181
	Growing	8	7
0.50	Fall	185	186
	Growing	5	5
0.75	Fall	188	189
	Growing	3	3
1.00	Fall	190	190
	Growing	0	0

cation is not possible, then fall application is optimal. When growing-season application may be possible, a fall/growing-season split application is optimal. Analogous results hold for the case of irrigated cotton production. Next, the results validate the hypothesis that more nitrogen is applied for irrigated cotton production than for dryland cotton. This is done in order to compensate for the nitrogen losses due to irrigation.

The results also show that a farmer would plan to apply some nitrogen fertilizer after planting as long as growing-season application is possible. In practice, there is a minimum nitrogen fertilizer application rate needed for the activity to be economically justifiable. For example, for dryland cotton, if this minimum amount is 50 pounds per acre, a farmer would apply nitrogen fertilizer only in the fall (and probably would increase amount of nitrogen fertilizer applied to compensate for the expected losses) if P is greater than 0.1 (Figure 2a).

Similarly, a farmer would apply nitrogen fertilizer during the growing-season if P is less than 0.05. These findings explain why both single and split nitrogen fertilizer applications are common practices among cotton producers in the United States.

Cotton farmers who irrigate are likely to apply some nitrogen fertilizer during the growing season. Irrigated cotton is typically grown in areas where rainfall is limited so that the likelihood of having wet field conditions which limit after-planting nitrogen fertilizer application is extremely small. Consequently, a farmer can maximize his or her expected net farm income by applying nitrogen fertilizer during the growing season. The Cotton Water Quality Survey data show that about 70% of the farmers surveyed who irrigated applied nitrogen fertilizer after planting, as compared with only 50% of dryland cotton producers who applied nitrogen fertilizer after planting.

Conclusion

Based on farm-level production decision models for a cotton producer in the United States, if a farmer's perceived probability of not be able to apply nitrogen during the growing season is between 0 and 1 exclusively, then a split application of nitrogen fertilizer is the optimal strategy for both a risk neutral and a risk averse farmer. Additionally, a risk averse farmer will apply more nitrogen fertilizer prior to planting than a risk neutral farmer. Consequently, any policy designed to promote a single application of nitrogen fertilizer during the growing season to minimize nitrogen fertilizer runoff into surface water and to reduce nitrate leaching into ground water would cause a farmer to move away from his or her optimal nitrogen fertilizer application strategy. This being the case, the farmer would have to have a sufficient incentive to change. This incentive could take a variety of forms including compensation.

References

Anderson, J. R., Dillon, J. L., and Hardaker, B.: 1977, *Agricultural Decision Analysis*, The Iowa State University Press, Ames, Iowa.

Arrow, K.: 1971, *Essays in the Theory of Risk-Bearing*, Markham Publishing Company, Chicago.

Belsley, D., Kuh, E., and Welsch, R.: 1980, *Regression Diagnostics*. John Wiley and Sons, Inc., New York.

Blackmer, A. M.: 1987, 'Losses and Transport of Nitrogen from Soil'. Chapter 5 in F. D'itri and L. Wolfson (eds.), *Rural Groundwater Contamination*, Lewis Publishers, Inc., Chelsea, MI.

Bock, B. R.: 1984, 'Efficient Use of Nitrogen in Cropping Systems'. Chapter 18 in *Nitrogen in Crop Production*, American Society of Agronomy, Madison, Wisconsin.

Borch, K.: 1968, *The Economics of Uncertainty*, Princeton University Press, Princeton.

Cantor, P.: 1988, 'Health Effects of Agrichemicals in Groundwater: What Do We Know?' *Agricultural Chemicals and Groundwater Protection: Emerging Management and Policy*, Proceedings of a Conference held in St. Paul, Minnesota, October 22–23, pp. 21–32.

Cerrato, M. E. and Blackmer, A. M.: 1990, *Agronomy* **82**, 138.

Crutchfield, S. R., Ribaudo, M. O., Setia, P., Letson, D., and Hansen, L.: 1991, *Cotton Production and Water Quality: an Initial Assessment*, Staff Report AGES 9105, Resources and Technology Division, Economic Research Service, U. S. Department of Agriculture.

Davidson, R. and MacKinnon, J.: 1981, *Econometrica* **49**, 781.

Deaton, A. and Muellbauer, J.: 1980, *Economics and Consumer Behavior*, Cambridge University Press, Cambridge.

Environmental Protection Agency: 1990, *National Survey of Pesticides in Drinking Water Wells*, Phase I Report. EPA 570/9-90-015, Environmental Protection Agency, Washington.

Feinerman, E., Choi, E. K., and Johnson, S. R.: 1990, *American Journal of Agricultural Economics* **72**, 975.

Frank, M. D., Beattie, B. R., and Embleton, M. E.: 1990, *American Journal of Agricultural Economics* **72**, 598.

Freshwater Foundation: 1988, *Nitrates and Groundwater: A Public Health Concern*, The Freshwater

Foundation's Health and Environment Network, Navarre, MN.

Fertilizer Institute: 1976, *The Fertilizer Handbook*, Second Edition. The Fertilizer Institute, Washington, DC.

Gilliam, J. W. and Hoyt, G. D.: 19887, 'Effect of Conservation Tillage on the Fate and Transport of Nitrogen', Chapter 13 in *Effects of Conservation Tillage on Groundwater Quality*, Lewis Publishers, Inc., Chelsea, MI.

Huang, W., Hansen, L., and Uri, N. D.: 1991, 'The Effects of the Timing of Nitrogen Fertilizer Application and Irrigation on Yield and Nitrogen Loss in Cotton Production', Unpublished manuscript, Economic Research Service, United States Department of Agriculture, Washington.

Kanwar, R. S., Baker, J. L., and Baker, D. G.: 1988, *Transactions of the American Society of Agricultural Engineers* **31**, 453.

McElroy, R., Dismukes, R., and Ali, M.: 1989, *State-Level Costs of Production, 1987*, Agriculture and Rural Economy Division, Economic Research Service, U. S. Department of Agriculture.

Metcalfe, D. and Elkins, D.: 1980, *Crop Production: Principles and Practices*, Fourth Edition, Macmillan Publishing Co., New York.

Mississippi Cooperative Extension Service: 1986, *1987 Cotton Costs and Returns*, Agricultural Economics Report 20, Mississippi Agricultural and Forestry Experiment Station, Mississippi State University, Mississippi State, MS.

National Agricultural Statistics Service: 1991, *Agricultural Prices*. National Agricultural Statistics Service, U.S. Department of Agriculture, Washington.

Newbery, D. M. G. and Stiglitz, J. E.: 1981, *The Theory of Commodity Price Stabilization*. Clarendon Press, Oxford.

Nielsen, E. and Lee, L.: 1987, *The Magnitude and Cost of Ground Water Contamination from Agricultural Chemicals: A National Perspective*, AER-576, Economic Research Service, U. S. Department of Agriculture, Washington.

Pope, R. D. and Just, R. E.: *American Journal of Agricultural Economics* **73**, 743.

Rhoads, T.: 1984, 'Nitrogen or Water Stress: Their Interrelationships'. Chapter 20 in *Nitrogen in Crop Production*, American Society of Agronomy, Madison, WI.

Soil Conservation Service: 1983, *Application of Soil Information: Part 603*, 430-VI-NSH, Soil Conservation Service, U.S. Department of Agriculture, Washington.

Spilker, M.: 1991, 'Estimating Applied Nitrogen Equivalents with a Generalized Linear Response and Plateau Function', Paper resented at 1991 American Agricultural Economics Association Annual Meetings, Manhattan, KS.

Stan, G. and Hinkle, D.: 1987, *Maintaining Cotton Yields Through Fertilizer and Crop Rotations*, Bulletin 340, Agricultural Experiment Station, University of New Mexico, Albuquerque.

Taylor, C. R. and Frohberg, K. K.: 1978, *American Journal of Agricultural Economics* **60**, 560.

Taylor, H.: 1991, 'A Statistical Analysis of Fertilizer Application Rate Differences Between 1989 and 1990', *Agricultural Resources: Input Situation and Outlook Report* AR-21, Economic Research Service, U.S. Department of Agriculture.

Viets, F. G. and Hageman, R. H.; 1971, *Factors Affecting the Accumulation of Nitrates in Soil, Water, and Plants*, Agricultural Handbook No. 413. Agricultural Research Service, U. S. Department of Agriculture, Washington.

White, W.: 1989, *Sources of Nitrogen and Phosphorus*, Resource Washington, Inc., Washington, DC.

Wischemeier, W. and Smith, D.: 1978, *Predicting Rainfall Erosion Losses*, Agricultural Handbook Number 537, U.S. Department of Agriculture, Washington.