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Journal of Food Engineering 39 (1999) 285–287

JOURNAL OF FOOD ENGINEERING

Heat transfer coefficient in laminar flow of non-Newtonian fluid in tubes

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Received 9 June 1998; accepted 17 November 1998

Abstract

Heating of non-Newtonian fluids is of particular importance to food processing systems. However, excessive pumping demand generally precludes development of turbulence in tubular arrangements. So, laminar flow is all that can be developed in heat transfer equipment handling non-Newtonian materials. The paper presents a theoretical model of heat transfer, which is dependent only on the flow behaviour index of the fluid. Increase in flow behaviour index produces a lower convective heat transfer coefficient and for real non-Newtonian fluids Nusselt's number approaches a fixed value corresponding to a flow behaviour index of 4.

Nomenclature

A	heat transfer area, m^2
B	integration constants
C	specific heat capacity, J kg^{-1} K^{-1}
h	convective heat transfer coefficient, W m^{-2} K^{-1}
k	thermal conductivity, W m^{-1} K^{-1}
K	consistency index, Pa s^n
L	tube length, m
m	mass flow rate, kg s^{-1}
n	flow behaviour index, dimensionless
N	dimensionless number
Q	heat flow rate, W
r	radial distance from the centre of the tube, m
R	radius of cylindrical tube, m
T	temperature of liquid, K
u	velocity of laminar layer, m s^{-1}
x	axial distance along the length of the tube, m

Greek symbols

α	thermal diffusivity, $m^2.s^{-1}$
δ	differential operator
Δ	infinitesimal
ρ	density

Subscripts

1,2	integration constants
net	total
axial	along the axis
radial	along the radius
in	heat inflow
out	heat outflow
b	bulk
Nu	Nusselt number
o	central axis
p	constant pressure
w	wall

1. Introduction

Fluids like tomato paste, condensed milk, sugar solution, apple sauce, banana puree', etc. are non-Newtonian in behaviour and heat transfer in non-Newtonian fluid is generally analysed by empirical equations relating dimensionless groups. This is acceptable in turbulent flow conditions but non-Newtonian fluids put severe pumping demand if turbulent flow is desired. For example, banana puree' having a consistency index of 7.28 Pa-s^n and a flow behaviour index of 0.5 will require about 23 m s^{-1} of average velocity in a tube of 9 mm inside diameter (smallest size available) to develop turbulence. This means a mass flow rate of 95 kg min^{-1}. To raise the temperature of this fluid by 50°C requires a straight tube of 57 m length. The pressure drop in such situation is close to 450 bar, which is impractical. Hence, laminar flow has to be depended upon for more reasonable pressure drop and length. But in laminar flow as fluid moves in sliding layers, it is necessary to develop a heat transfer model, which relates convective heat transfer coefficient to thermal conductivity, density, and viscosity of fluids. Besides, due to laminar flow, the

PII: S0260-8774(98)00162-9

fastest moving regime is also the slowest heating (at the centre of a cylindrical tube). Therefore a heat balance model is also necessary which can predict the temperature of the fluid at the slowest heating region.

2. Model development

Taking a small ring element of length Δx and thickness Δr at a radial distance r from the centre the heat balance in the element can be expressed as (Fig. 1):

$$Q_{\text{net}} = Q_{\text{in-axial}} + Q_{\text{in-radial}} - Q_{\text{out-axial}} - Q_{\text{out-radial}} \tag{1}$$

or

$$0 = 2\pi r \Delta r u \rho C_{\text{p}} T|_x + 2\pi r k|_{r+\Delta r} \Delta x \frac{(\delta T/\delta r)|_{r+\Delta r} \Delta r}{\Delta r} - 2\pi r \Delta r u \rho C_{\text{p}} T|_{x+\Delta x} - 2\pi r k|_r \Delta x \frac{(\delta T/\delta r)|_r \Delta r}{\Delta r}. \tag{2}$$

In Eq. (2) it should be noted that $Q_{\text{radial}} = +kA(\Delta T/\Delta r)$ as T increases with r.

Eliminating $2\pi\Delta r$ from Eq. (2) we get

$$ru\rho C_{\text{p}} \frac{T|_{x+\Delta x} - T|_x}{\Delta x} = k \frac{r(\delta T/\delta r)|_{r+\Delta r} - r(\delta T/\delta r)|_r}{\Delta r}$$

$$\text{or } ru\rho C_{\text{p}} \frac{\delta T}{\delta x} = k \frac{\delta}{\delta r}\left(r \frac{\delta T}{\delta r}\right)$$

$$\text{or } \frac{1}{\alpha}\frac{\delta T}{\delta x} = \frac{1}{ru}\frac{\delta}{\delta r}\left(r\frac{\delta T}{\delta r}\right). \tag{3}$$

It can be shown from Prasad (1987) that $\delta T/\delta x$ is independent of r for a straight tube heat exchanger excluding the entrance and exit regions. Similarly, α can be assumed constant for a finite region of heat exchange. Laminar velocity u can be expressed for non-Newtonian fluid as:

$$u = u_0\left[1 - (r/R)^{(1/n)+1}\right]. \tag{4}$$

From Eq. (3) we can write

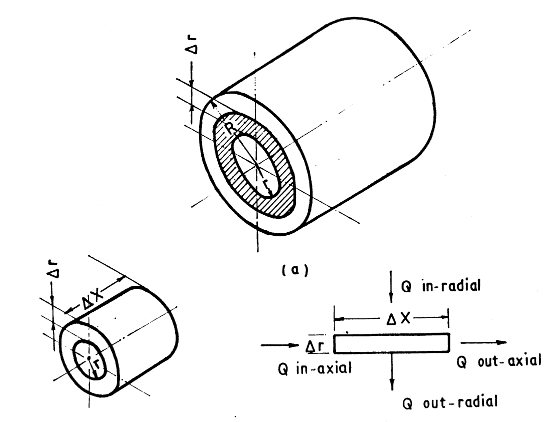

Fig.1. Heat balance in tube flow: (a) shell element in tube; (b) shell element isolated; (c) heat flow in element.

$$\frac{\delta}{\delta r}\left(r\frac{\delta T}{\delta r}\right) = \left(\frac{1}{\alpha}\frac{\delta T}{\delta x}\right) ru = \frac{1}{\alpha}\frac{\delta T}{\delta x} u_0 \left(r - \frac{r^{(1/n)+2}}{R^{(1/n)+1}}\right)$$

$$\text{or } r\frac{\delta T}{\delta r} = \frac{1}{\alpha}\frac{\delta T}{\delta x} u_0\left[\frac{r^2}{2} - \frac{r^{(1/n)+3}}{((1/n)+3)R^{(1/n)+1}}\right] + B_1$$

$$\text{or } T = \frac{1}{\alpha}\frac{\delta T}{\delta x} u_0 \left[\frac{r^2}{4} - \frac{r^{(1/n)+3}}{((1/n)+3)^2 R^{(1/n)+1}}\right] + B_1 \ln r + B_2. \tag{5}$$

Since T is finite at $r = 0, B_1 = 0$. Hence,

$$T = T_0 + \frac{1}{\alpha}\frac{\delta T}{\delta x} u_0 \left[\frac{r^2}{4} - \frac{r^{(1/n)+3}}{((1/n)+3)^2 R^{(1/n)+1}}\right] \tag{6}$$

and

$$T_w = T_0 + \frac{1}{\alpha}\frac{\delta T}{\delta x} u_0 R^2 \left[\frac{1}{4} - \frac{n^2}{(3n+1)^2}\right]. \tag{7}$$

Average bulk temperature of the fluid can be obtained by using the following expression (Holman, 1986):

$$T_{\text{b}} = \frac{\int_0^R \rho 2\pi r \Delta r u C_{\text{p}} T}{\int_0^R \rho 2\pi r \Delta r u C_{\text{p}}} \quad \text{or} \quad T_{\text{b}} = \frac{\int_0^R ruT \, \mathrm{d}r}{\int_0^R ru \, \mathrm{d}r}, \tag{8}$$

$$\int_0^R ruT \, \mathrm{d}r = \int_0^R ru \left[T_0 + \frac{1}{\alpha}\frac{\delta T}{\delta x} u_0 \left\{\frac{r^2}{4} - \frac{r^{(1/n)+3}}{((1/n)+3)^2 R^{(1/n)+1}}\right\}\right] \mathrm{d}r$$

$$= u_0 \int_0^R r\left(1 - \frac{r^{(1/n)+1}}{R^{(1/n)+1}}\right)\left[T_0 + \frac{1}{\alpha}\frac{\delta T}{\delta x} u_0 \times \left\{\frac{r^2}{4} - \frac{r^{(1/n)+3}}{((1/n)+3)^2 R^{(1/n)+1}}\right\}\right] \mathrm{d}r$$

$$= \frac{1}{2} u_0 R^2 \left[T_0 \frac{n+1}{3n+1} + \frac{1}{\alpha}\frac{\delta T}{\delta x} u_0 R^2 \frac{n+1}{5n+1}\left\{\frac{1}{8} - \frac{n^3}{(3n+1)^3}\right\}\right], \tag{9}$$

$$\int_0^R ru \, \mathrm{d}r = u_0 \int_0^R r\left(1 - \frac{r^{(1/n)+1}}{R^{(1/n)+1}}\right) \mathrm{d}r = \frac{1}{2} u_0 R^2 \frac{n+1}{3n+1}. \tag{10}$$

Substituting the expressions from Eqs. (9) and (10) in to Eq. (8) we get

$$T_{\text{b}} = T_0 + \frac{3n+1}{5n+1}\frac{u_0}{\alpha}\frac{\delta T}{\delta x} R^2 \left[\frac{1}{8} - \frac{n^3}{(3n+1)^3}\right]. \tag{11}$$

Using convection–conduction model at the wall of the heat exchanger, we can write

Table 1
N_{Nu} as a function of n from Eq. (15)

n	0.2	0.5	0.6	0.7	0.8	0.9	1.0	1.5	2.0	3.0	4.0
N_{Nu}	5.52	4.75	4.63	4.54	4.47	4.41	4.36	4.21	4.13	4.05	4.01

Table 2
$\left({}_{Nu}|_n/{}_{Nu}|_{n=1}\right)$ for various values of n

n	0.2	0.5	0.6	0.7	0.8	0.9	1.0
Model (15)	1.149	1.08	1.097	1.074	1.05	1.025	1.0
Eq. (16)	1.266	1.09	1.062	1.041	1.025	1.011	1.0

$$Q_w = hA_w(T_w - T_b) = kA_w \frac{\delta T}{\delta r}\bigg|_{r=R},$$

$$N_{Nu} = 2R\frac{(\delta T/\delta r)_{r=R}}{T_w - T_b},$$

$$T_w = T_0 + \frac{u_0}{\alpha}\frac{\delta T}{\delta x}R^2\left[\frac{1}{4} - \frac{n^2}{(3n+1)^2}\right]. \quad (12)$$

So,

$$T_w - T_b = \frac{u_0}{\alpha}\frac{\delta T}{\delta x}R^2\left[\left\{\frac{1}{4} - \frac{n^2}{(3n+1)^2}\right\} - \frac{3n+1}{5n+1}\left\{\frac{1}{8} - \frac{n^3}{(3n+1)^3}\right\}\right] \quad (13)$$

and

$$\frac{\delta T}{\delta r}\bigg|_{r=R} = \frac{u_0}{\alpha}\frac{\delta T}{\delta x}\left[\frac{r}{2} - \frac{r^{(1/n)+2}}{((1/n)+3)R^{(1/n)+1}}\right]_{r=R} = \frac{1}{2}\frac{u_0}{\alpha}\frac{\delta T}{\delta x}R\frac{n+1}{3n+1}. \quad (14)$$

Combining Eqs. (13) and (14) we get

$$N_{Nu} = \{(n+1)/(3n+1)\}/\left\{\left[1/4 - n^2/(3n+1)^2\right] - ((3n+1)/(5n+1))\left[1/8 - n^3/(3n+1)^3\right]\right\}. \quad (15)$$

3. Conclusion and discussion

Table 1 obtained from Eq. (15) suggests that increase in n results in decrease in N_{Nu}. Since most liquid foods are pseudoplastic, i.e. $n < 1$ it can be pointed out that N_{Nu} increase with non-Newtonian behaviour. N_{Nu} involves the parameter thermal conductivity in the denominator. So, the convective heat transfer coefficient is directly proportional to thermal conductivity of the liquid times N_{Nu}. Flow behaviour index of liquid food decreases from 1 with increase in total solids (Reddy & Datta, 1994) Thermal conductivity also decreases with increase in lipids and carbohydrates for their non-polar composition. But the increase in N_{Nu} due to lowering of the value of n is generally outweighed by the decrease in thermal conductivity value of liquid foods having large total solid concentration. Hence, convective heat transfer coefficient of liquid foods decrease with increasing non-Newtonian behaviour.

An attempt was made to validate the proposed model with published literature data. Equation of Metzner and Gluck (Geankoplis, 1993) was found to be semi-empirical and applicable to "highly viscous" non-Newtonian fluids:

$$N_{Nu} = 1.75\left(\frac{3n+1}{4n}\right)^n \frac{mC_p}{kL}\left(\frac{K_b}{K_w}\right)^{0.14}. \quad (16)$$

Using Eq. (16) and taking N_{Nu} for $n = 1$ as unity the ratio (N_{Nu} for any n/N_{Nu} for $n = 1$) were computed. These and the same ratio, obtained from Eq. (15), are compared in Table 2. There is some discrepancy but the overall trend is identical.

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