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ANALYTICAL MODELLING OF INORGANIC CONTAMINANTS IN LEACHATE

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The need for estimating the contaminant concentration of leachates from landfills has rapidly become important because of the increased demand for landfilling, causing serious pollution of water bodies in some places. The present study examines the movement of inorganics from waste into leachate in the course of time. Inorganics are discussed in two groups according to their solubilities in water. The first group of inorganics dissolve immediately in water (i.e. at time $t = 0$). The inorganic compounds of the second group transfer into water over a longer period of time. The mathematical model is based on a porous and well-mixed medium, and on assumptions of dissolution/dilution and mass transfer according to their infiltration rate. The agreement between theoretical and experimental results from laboratory and literature are investigated. After experimental verification, some mathematical relationships to predict the mass transfer coefficient (k) and maximum concentration (C_{max}) were also investigated.

Key Words—Landfills, leachate, mathematical modelling, inorganics, chloride, sodium, potassium, total solids, seasonal wastes.

Introduction

Landfilling is the ultimate facility in any solid waste disposal system. Leachate production with the application of this disposal technique becomes a very important problem for the environment. Changes in leachate pollutant characteristics with fill age cause difficulties in estimating the leachate hazards to groundwater and selection of the appropriate leachate treatment. For this reason, it is important to determine the changes in leachate quality with fill age in modelling studies. For this purpose, many experimental studies have been made under natural and laboratory conditions. In all these experimental studies, results have varied depending on ambient conditions and the physical and chemical compositions of the experimental materials used. Mathematical modelling studies which have been made by several investigators (Qasim & Burchinal 1970, Raveh & Avnimelech 1970, Wigh 1979, Lu *et al.* 1982) offer empirical and semi-empirical simulations obtained from experimental results. In these works, particular mathematical equations dealing with only their own experimental data were obtained. A comprehensive study was made by Straub & Lynch in 1982. They theoretically simulated various experimental data found in literature. Their study was composed of two different models. The first model was based on the assumption of a well mixed single reactor. These investigators have recommended an analytical solution and have used this solution to obtain approximate simulated curves. In the other model, the governing equation, which is widely used for unsaturated porous media by soil physicists, has been numerically examined, and good agreement with data from the literature has been

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obtained. Other investigators have followed this study by using different numerical methods (Demetracopolos *et al.* 1986).

In the present study different mathematical formulations, which are analytical solutions developed to improve the application of the single reactor model, are applied to data from laboratory scale studies and literature.

Mathematical model

A solid waste landfill is assumed to be a well mixed reactor. Solid wastes are filled uniformly in a column (reactor) of volume V. Volumetric moisture content (θ) and contaminant concentration (C) are uniform in the column. The contaminant concentration of water entering into the column (C_i) can be neglected. In the beginning of leachate production, the volumetric moisture content reaches the field capacity, θ_f. The model is developed by means of the following moisture balance as:

$$\frac{d(V\theta)}{dt} = Q_i - Q \tag{1}$$

where t is time, Q_i is moisture flow rate entering into the column, and Q is moisture flow discharging from the column. In the model, it is considered that solid waste constituents pass into moisture in the column by mass transfer and by their solubilities. The settling of waste in the column in the course of time is assumed to be negligible. The model is divided into two parts according to the transferability of solid waste constituents into the liquid phase.

Readily dissolvable inorganics

In this case, the transfer of constituents into the liquid phase occurs immediately, that is to say, there is no mass transfer and the initial concentration is a peak concentration. Neglecting influx of contaminant, the mass balance for these constituents is:

$$\frac{d(CV\theta)}{dt} = C_i Q_i - CQ \tag{2}$$

Equation 2 may be solved, with $C_i = 0$, as

$$C(t) = C_o Exp[-G(t)] \tag{3}$$

where C_o is initial concentration and $G(t)$ is a parameter which is a function of cumulative local precipitation as expressed below (Straub & Lynch 1982):

$$G(t) = \int_{t=0}^{\bar{t}} \frac{Q(\bar{t})}{V\theta_f} d\bar{t} \tag{4}$$

To demonstrate this part of the model, sodium, potassium and chloride simulations were made.

Inorganics transferable into the liquid phase over time

The transfer of constituents in this second case occurs according to (Straub & Lynch 1982):

$$R(t)=k\frac{S(t)}{S_o}[C_{max}-C(t)] \tag{5}$$

where $R(t)$ is the mass transfer rate having a gradient of present leachable constituent, k is the mass transfer rate coefficient, $S(t)$ is the leachable amount at time t that is given by:

$$S(t)=S_o-A\int_{t=0}^{t}Q(t)C(t)dt \tag{6}$$

S_o is the initial leachable amount, and C_{max} is the maximum contaminant concentration. However, a formulation for estimating average mass transfer rate, R is obtained as:

$$R=\frac{C^*}{T} \tag{7}$$

where C^* is average contaminant concentration and T is hydraulic retention time in the column. Therefore, a mass balance for inorganics having significant mass transfer can be established as:

$$\frac{d(CV\theta)}{dt}=C_iQ_i-CQ+V\theta R \tag{8}$$

This governing equation may be solved for $C_i=0$ to yield:

$$C(t)=Exp[-G(t)]\{C_o+\int_{t=0}^{t}RExpG(t)dt\} \tag{9}$$

However, Straub & Lynch had offered the following different solution to Eqn. 8:

$$C(t)=C_oExp[-G(t)]+\frac{RV\theta_f}{Q}\{1-Exp[-G(t)]\} \tag{10}$$

While offering this solution, they considered that contaminant concentration decreases to an asymptotic concentration value with time, which does not physically exist in a visible time, since that value should actually be $C_\infty=0$. Besides, the solution does not provide acceptable seasonal fluctuations.

In order to simulate and compare this part of the model, the pollutant parameter of Total Solids (TS) dried at 103–105°C will be used. The selection of TS parameter for monitoring inorganic constituents transferable into present moisture in the course of time has been made because of negligible amounts of biologically decomposable matter, compared to the total amount of solid matter found in the leachate in the short term, and also existing toxic constituents in solid waste mixtures.

Experimental

In order to apply the mathematical model, two simulated landfills with details shown in Fig. 1 were set up in the laboratory. Variation of the pollutant parameters with time in the model developed were monitored for nearly one year.

Synthetic mixtures representing Metropolitan İstanbul City's seasonal solid waste features are shown in Table 1 (Basturk 1980). Dimensions, physical characteristics and operational features of the columns are shown in Table 2.

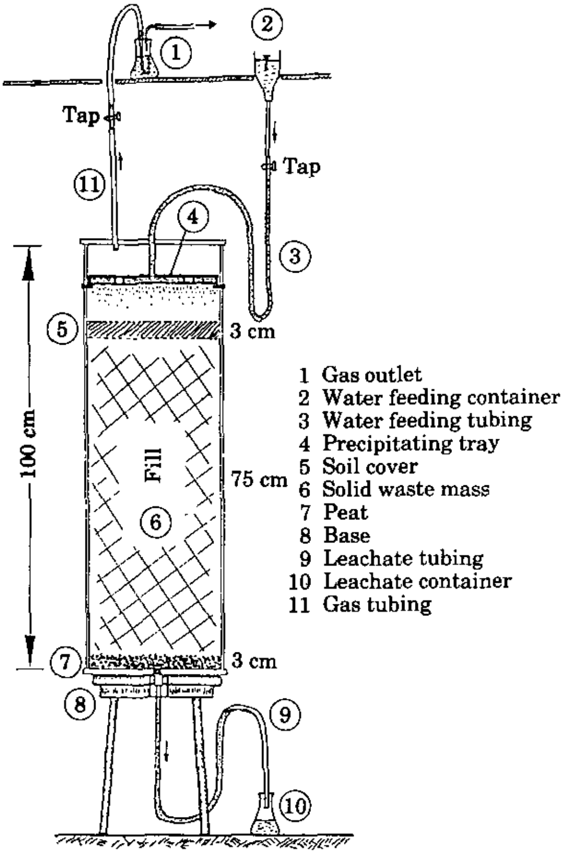

Fig. 1. Experimental pilot plant.

Daily water additions, after a high rate water addition to determine the field capacity for each mixture, were made according to local rainfall statistics covering a period of 50 years from reports of the Turkish State Meteorological Works Department.

TABLE 1
Compositions of mixtures filled into columns according to local seasonal waste features

Material groups	Summer waste (%)	Winter waste (%)
Organic materials	56	37
Paper	14	10
Textile	4	3
Plastic	5	3.5
Glass	3	3
Metals	1.5	1.5
< 10 mm in dia (ash, soil etc)	16.5	42

TABLE 2
Constructional and operational features of columns

Features	Summer	Winter
Column surface area, A, m^2	0.073	0.073
Column height, H, cm	100	100
Waste height, h, cm	75	70
Waste volume, V, litre	55	51
Waste wet density, D_w, kg/m^3	840	680
Waste dry density, D_d, kg/m^3	504	498
Waste moisture content, %	40.1	26.8
Waste weight, kg	46	35
Water amount added for waste saturation, litre	10.3	13.9
Daily mean water application, mm/m^2	1.878	1.878
Ratio of leachable matter, γ_l, %	17.3	11.5

Estimation of model parameters

While field capacities of wastes filled into columns are determined, both initial moisture content of wastes and amount of water added for saturation are used. In addition, surface area, height and weight of wastes in columns have been taken into account. Hence, field capacities of summer and winter wastes have been found, respectively, as 0.526 and 0.454 cm/cm.

Average moisture detention times in columns are estimated by:

$$T=\frac{V\theta_f}{Q} \tag{11}$$

and determined as 210 and 169 days, respectively.

The variations of C_{max} and C^* values for TS are shown to be in the ranges of 15 000 to 100 000 mg/l and 1000 to 20 000 mg/l, respectively, from numerous experimental simulation and case studies in literature. For both summer and winter wastes, C^* values will be taken as 10 000 mg/l. Accordingly, average mass transfer rates in Eqn 6, for wastes, are determined as 50 and 60 mg/l/day. Amounts of dry leachable waste in columns are taken as 8 and 4 kg because all food waste materials may be estimated as unique leachable material in the leaching period without putrification of paper, textile and plastic. And also, the amount of leachable inorganics (ions) in ash and soil are neglected. Mass transfer rate coefficient, k in Eqn 5 may be found as 0.003 day^{-1} for both wastes by taking $S(t)/S_0=0.5$ and $C_{max}=70\,000$ mg/l and $C(t)=C_{max}/2=35\,000$ mg/l, for TS parameter.

Simulation results

The simulation results for sodium, potassium and chloride in the first part of the present model for the readily dissolvable constituents (Eqn 3) are shown in Figs 2 to 4. From these simulation figures, good fitted curves are obtained. In this part of the model, there is no mass transfer into water from waste ($R=0$), and initial concentration, C_0 and a parameter of cumulative daily water application rates, $G(t)$ are the major factors that affect the curves of simulation. It can be seen that, there are different C_0 values for each

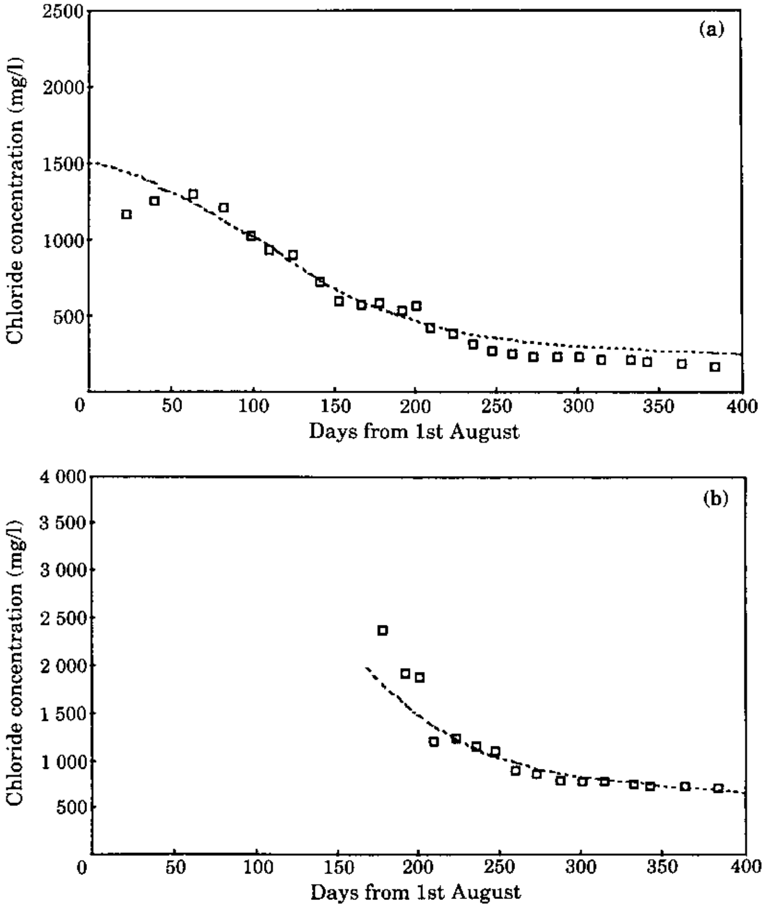

Fig. 2. Simulation of chloride data from laboratory. (a) summer waste: (□) experimental, (— —) $C_o = 1500$ mg/l; (b) winter waste: (□) experimental, (— —) $C_o = 2000$ mg/l.

kind of waste (e.g., in sodium, 1100 and 3000 mg/l for summer and winter wastes). These differences result from inorganic contents of wastes.

The simulation results of the second part of the present model (that is to say, the part of the model for constituents transferable into water from solid waste in the course of time, Eqn 9) and the single reactor model recommended by Straub & Lynch (Eqn 10) are plotted in Figs 5 and 6 for comparison. In these simulation figures, TS is taken as a pollutant parameter which roughly represents the inorganic strength of the leachate. In Fig. 5a, in which the Straub–Lynch analytical model is applied to the summer experimental column for the values of $C_{max} = 70\,000$ mg/l, $C_o = 36\,700$ and $45\,000$ mg/l, and $k = 0.003$ day^{-1}, it is seen that, despite different initial concentration values such as 36 700 and 45 000 mg/l, discordant simulation curves with observed data were obtained. This discordance is essentially due to unrepresentative calculated mass transfer rates. While observed data follow the seasonal strength variations, simulated results have not followed sufficiently. On the other hand, Fig. 6a shows the behaviour of the present model being most consistent with observed data which is affected by seasonal conditions, taking $k = 0.003$ day^{-1}, $C_o = 36\,700$ and $45\,000$ mg/l, and $C_{max} = 70\,000$ mg/l. Even

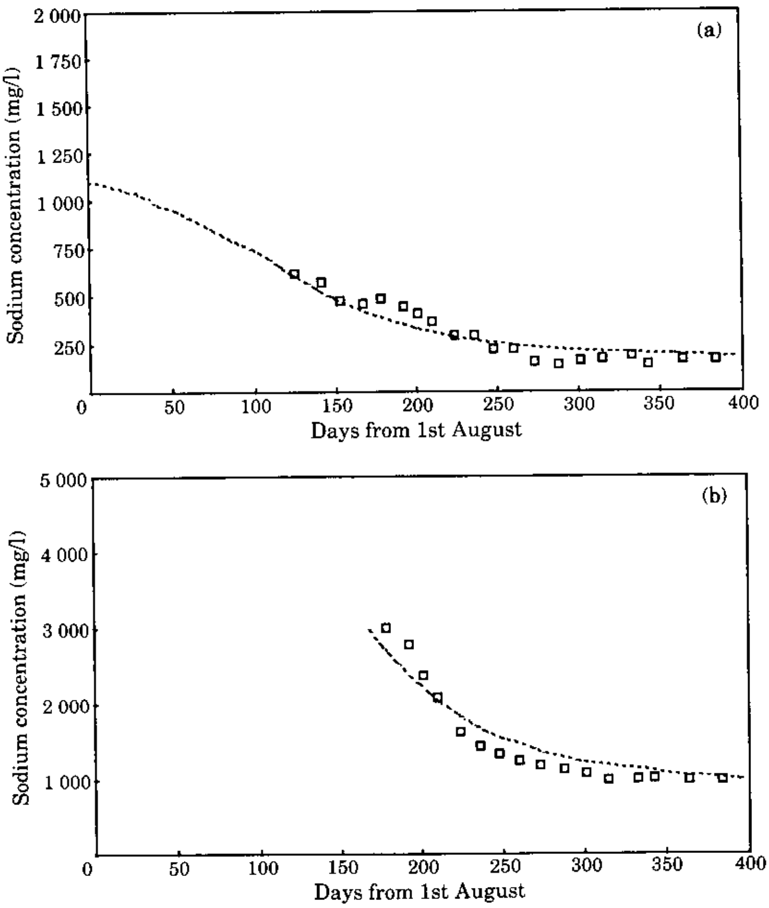

Fig. 3. Simulation of sodium data from laboratory. (a) summer waste: (□) experimental, (– –) $C_o = 1100$ mg/l; (b) winter waste: (□) experimental, (– –) $C_o = 3000$ mg/l.

though the two models show a contrast in application to the summer column data, they fit closely for winter column data, but it must be noticed that leaching in the winter column was performed for a shorter period (see Figs 5b and 6b).

The present model was also applied in order to simulate and compare the leachate TS concentration data observed from three pilot-scale experimental plants reported by Qasim & Burchinal (1970) and Pohland (1980). Simulation results of these experimental data are shown in Figs 7 to 9. These simulations or comparisons were realized for physical sizes of pilot plants set up and reported by researchers and model variables estimated (Table 3). Consequently, by means of these last comparisons, a reasonable verification of the present model was provided.

During all simulations made with present model, expected maximum contaminant concentrations (C_{max}), initially observed contaminant concentrations (C_o) and mass transfer rate coefficients (k) were predicted by trial and error. These sorts of model parameters should be principally determined according to waste and filling characteristics. The estimation of C_{max} and k parameters is very important in use of the model. It can be presumed that factors such as the initial amount of leachable matter in the waste,

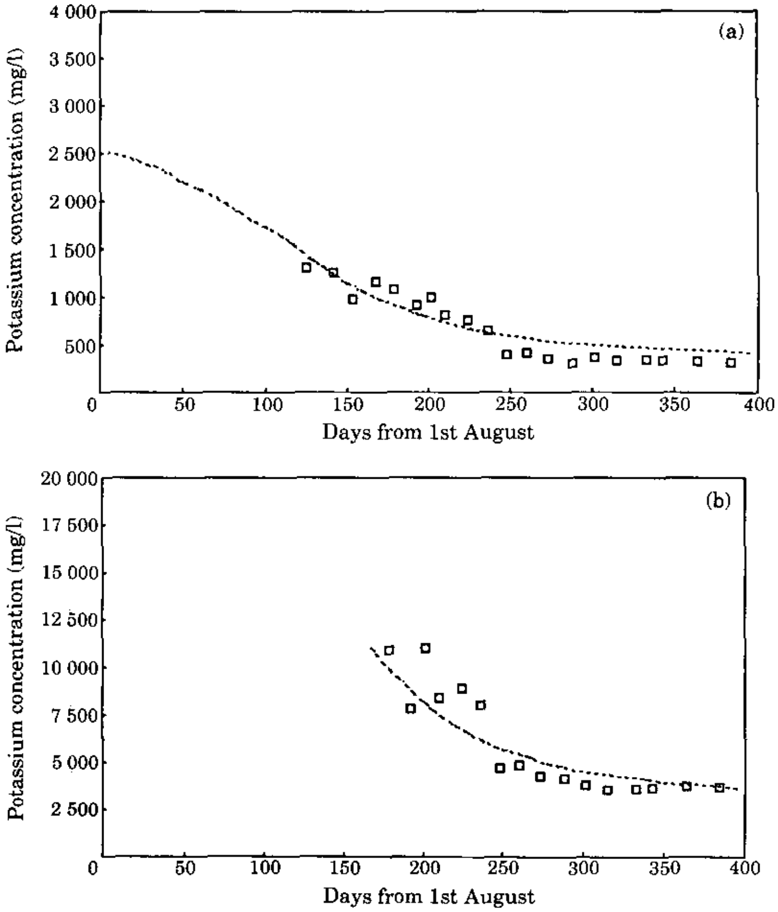

Fig. 4. Simulation of potassium data from laboratory. (a) summer waste: (□) experimental, (– –) $C_o = 2500$ mg/l; (b) winter waste: (□) experimental, (– –) $C_o = 11\,000$ mg/l.

TABLE 3
Variables used for the simulation of experimental data (TS) taken from several studies in the literature

Variable	Unit	Qasim–Burchinal Cylinder B	Qasim–Burchinal Cylinder C	Pohland
V, volume	Litre	1240	1960	2000
θ_f, field capacity	cm/cm	0.400	0.400	0.471
A, cross sectional area	m^2	0.636	0.636	0.658
S_o, leachable solids	kg	227.8	336.2	158.5
T, average retention time	day	151	239	167
k, mass transfer rate coefficient	day^{-1}	0.005	0.005	0.0005
C_o, initial concentration	mg/l	55 000	55 000	8800
C_{max}, maximum concentration	mg/l	70 000	70 000	15 000
Q, daily mean flow rate	mm/day/m^2	5.430	5.430	5.641
D_d, dry density	kg/m^3	219	204	317
H, waste height	m	1.95	3.07	3.07
γ_l, the ratio of leachable matter (dry) to total matter	%	80.0	80.0	25.0

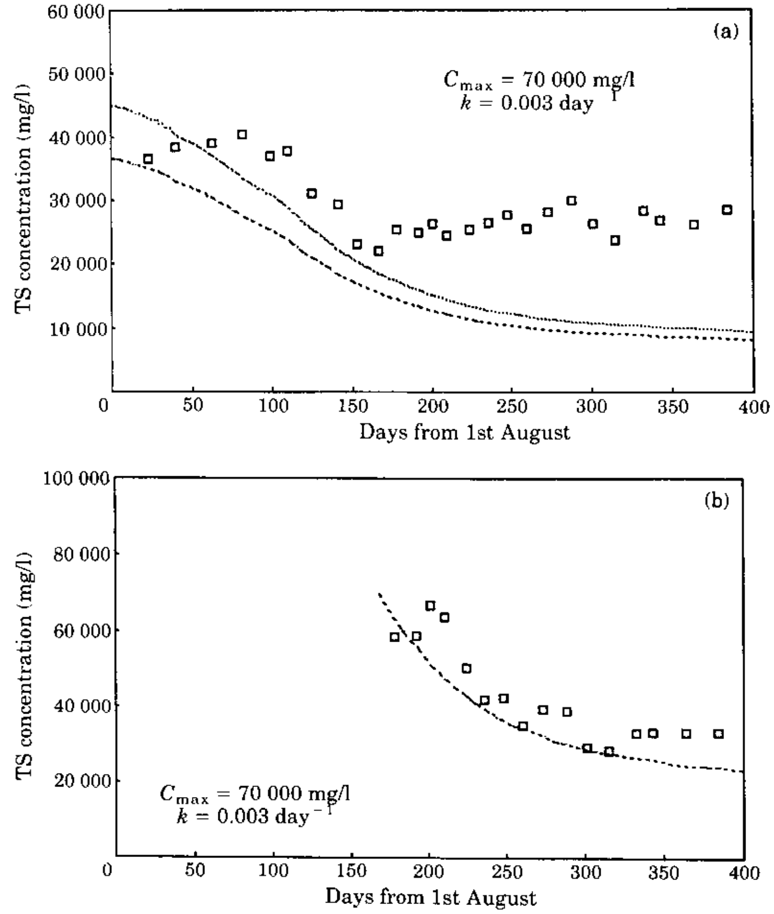

Fig. 5. Comparison of Straub-Lynch's Model with laboratory data obtained for total solids. (a) summer waste: (□) experimental, (– –) $C_o = 36\,700$ mg/l, (····) $C_o = 45\,000$ mg/l; (b) winter waste: (□) experimental, (– –) $C_o = 70\,000$ mg/l.

the fill compaction rate, the percolation rate, the fill height, the retention time and so on will play a role in the determination of those parameters. After an examination of all affecting factors, it was concluded that only two factors, the initial amount of leachable matter in the waste and the fill compaction rate, were the main factors affecting the estimation of leaching rates. In order to provide relationships among parameters and affecting factors, multiple regression analysis using the least squares fit was performed. The independent variables in the multiple regression analysis were the ratio of the leachable mass (γ_l) and the dry density of fill (D_d). The following equations were obtained for the Total Solids (TS) pollutant parameter:

$$k = -8.526 \times 10^{-3} + 1.959 \times 10^{-5} D_d + 1.170 \times 10^{-4} \gamma_l \tag{12}$$

$$C_{max} = -147849.6 + 385.1 \times D_d + 1700 \times \gamma_l \tag{13}$$

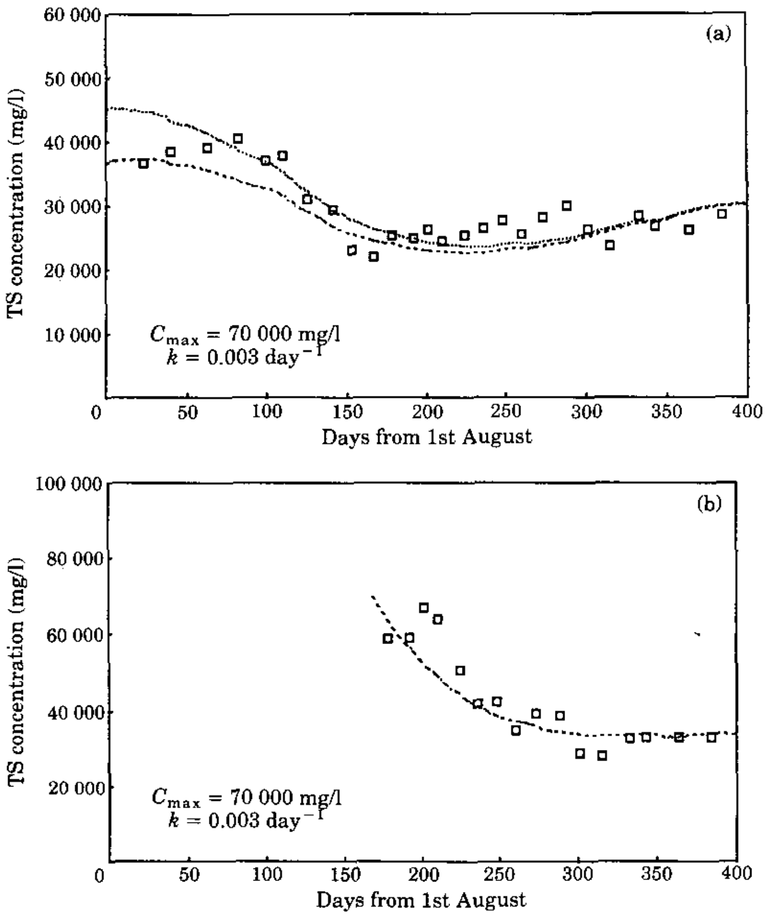

Fig. 6. Comparison of the present model with laboratory data obtained for total solids. (a) summer waste: (□) experimental, (– –) C_o = 36 700 mg/l, (····) C_o = 45 000 mg/l; (b) winter waste: (□) experimental, (– –) C_o = 70 000 mg/l.

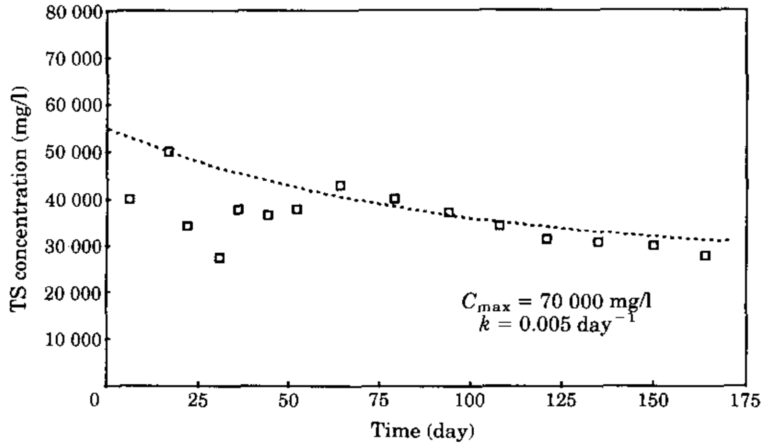

Fig. 7. Comparison of the present model with cylinder B data from Qasim–Burchinal. (□) experimental; (– –) C_o = 55 000 mg/l.

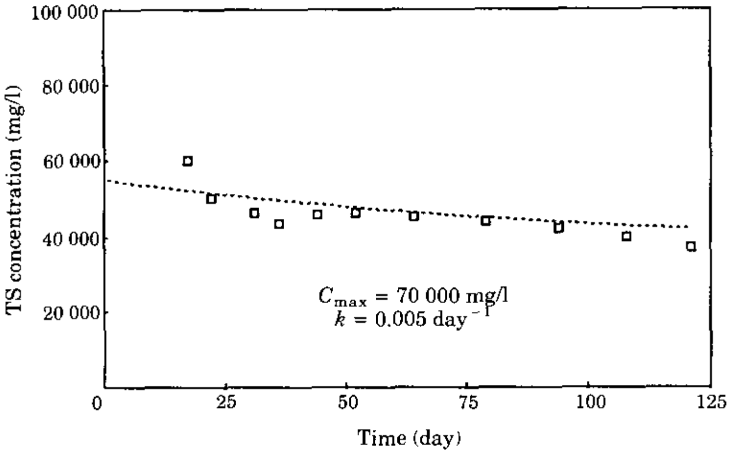

Fig. 8. Comparison of the present model with cylinder C data from Qasim-Burchinal. (□) experimental; (– –) C_o = 55,000 mg/l.

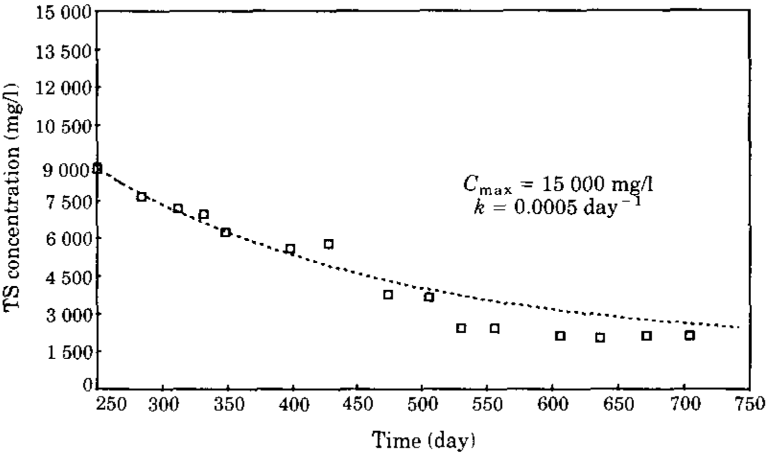

Fig. 9. Comparison of the present model with experimental data from Pohland. (□) experimental; (– –) C_o = 8800 mg/l.

Conclusions

A mathematical model which reflects the variations of concentration by means of seasonal rainfall and fill ages is developed. A good agreement between the model and experimental results was found. The present model enables the possibility of prediction of the chemical quality of leachates, and therefore more consistent design and control during and after the construction of the landfills should be possible. However, due to the existing need for the determination of the factors affecting the leaching process of different solid waste and filling characteristics, a multiple regression analysis was conducted on factors such as the filling dry density (D_d) and the ratio of the leachable mass (γ_1).

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