PREDICTING THE COOLING TIME FOR IRREGULAR SHAPED FOOD PRODUCTS

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ABSTRACT

A new method for calculating the cooling time for fresh fruits and vegetables and processed foods is presented. The method uses the truncated analytical solution of the governing partial differential equation to define a cooling curve with two parameters. One parameter is the lowest eigenvalue for the product. The second parameter is a constant multiplier similar to the one that occurs in the analytical solution. The lowest eigenvalue is evaluated using a finite element analysis. The multiplying constant is evaluated using a finite element solution in time. Cooling curves for a Rome apple and a Bartlett pear are presented and discussed.

INTRODUCTION

Cooling of a product is an important task in the food industry. The need for cooling includes fresh fruits and vegetables, the carcass of an animal after slaughter, and processed products in cans or jars. Cooling is usually accom-

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plished using forced air, hydrocooling or refrigerated room cooling. In each case, the heat moves to the surface by conduction and from the surface by convection. Heat loss by convection is dependent on the type of cooling process selected. The cooling time depends on whether the product is cooled individually or whether the product is stored in boxes or bulk bins. One engineering aspect of the cooling process is to predict the time required for a product to cool to its surrounding temperature. This calculation for food products is often complicated by irregular geometries. Cooling times are presently calculated by approximating the product as a slab, a cylinder or a spherical body because temperature response curves are available for these shapes (Singh and Heldman 1993).

The objective of this paper is to present a new procedure for estimating the cooling time for irregularly shaped food products. The method utilizes the irregular grid analysis capabilities of the finite element method to evaluate a pair of parameters that define a cooling curve for a product. The procedure for developing a cooling curve is presented using a pair of axisymmetric products, a Rome apple and a Bartlett pear.

REVIEW OF LITERATURE

Traditional methods to determine the time needed to cool agricultural products have used the Fourier equation for transient heat transfer. This equation is limited to homogeneous, isotropic substances with nice shapes such as slabs, cylinders and spheres. Heldman (1977) gives the infinite series solutions to the governing differential equation for the infinite slab, the infinite cylinder and a sphere and shows how to apply these solutions to solve heating and cooling problems. Singh and Heldman (1993) present the Fourier equation in one-dimension and solve heating and cooling problems using temperature response charts for the well defined shapes.

Smith *et al.* (1967) preformed a similitude study to develop a nomograph that could be used to predict the cooling time of anomalous shapes. They concluded that the ellipsoidal model was the most valid model and adapted well for replacing a large range of anomalous shapes for predictions in transient conduction heat transfer.

DeBaerdemaeker *et al.* (1977) used the finite element method to solve time dependent problems related to food materials. The temperature history was calculated for the heating of a cylindrical can, the heating of an infinite slab, the cooling of a pear, the cooking of a chicken leg and the cooking of a slice of ham that is turned over during the process. Predicted temperature histories were given for each case.

Misra and Young (1979) used the finite element method for a time dependent heat transfer problem, approximating apples as a spherical body. Their numerical results agreed well with an analytical solution and the authors concluded that the finite element method gave a very good approximation for the transient heat transfer problem in a sphere.

Hayakawa and Succar (1982) used the finite element method to determine thermal response and moisture loss during cooling of fresh potatoes and tomatoes. The surface heat conductance and transpiration coefficient were found to strongly effect the thermal response and moisture loss of spherical produce.

Bazan *et al.* (1989) predicted the temperature response during room cooling of a confined bin of spherical fruit. Experimental results agreed well with simulation studies. These investigators determined that small temperature gradients exist inside the fruit during cooling.

Pan and Bhownik (1991) used the finite element method to predict the temperature distribution in individual mature green tomatoes during forced air cooling. The calculated results were within 1C of the experimental results when cooling the tomatoes from 20C to 12C.

Fraser and Otten (1992) studied the cooling of peaches in well-vented containers. They measured temperature values and compared the results to the analytical solution for a sphere composed of peach flesh. Cooling times were slower for the peaches than predicted by the model. Fraser and Otten believed the slower cooling occurred because of the increase in air temperature as it flowed through the packed bed of peaches.

BASIC THEORY

The procedure for defining a cooling curve presented here starts with the well known Fourier equation for transient heat transfer. The idea can be illustrated by using the analytical solution to

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a}\frac{\partial T}{\partial t} \qquad (1)$$

for an insulated bar of unit length with the initial conditions of $T(x,0) = T_i$, boundary conditions of $T(0,t) = T(1,t) = T_\infty$ where T_∞ is a specified value. The parameters in (1) are the temperature, T, the time, t, the coordinate variable, x, and the thermal diffusivity, a. The units for temperature, time and space must be consistent with the units for the thermal diffusivity. The analytical solution of (1) for initial and boundary conditions similar to those given above is presented by several authors. A modification of the solution given by Smith (1978) is used here. The modification includes the dimensionless temperature ratio, (DTR)

$$DTR=\frac{T-T_\infty}{T_i-T_\infty} \tag{2}$$

and the thermal diffusivity, a. Most authors present the solution to (1) for the case where a = 1, T_i = 1 and T_∞ = 0.

The general solution to (1) is

$$DTR = \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}[\sin(2n+1)\pi x]\exp[-(2n+1)^2 a\pi^2 t] \tag{3}$$

The slowest point to cool occurs at the center of the bar, x = 1/2.

Substituting x = 1/2 reduces (3) to

$$DTR = \frac{4}{\pi}\sum_{n=0}^{\infty}(-1)^n\frac{e^{-\beta_n t}}{(2n+1)} \tag{4}$$

where the sequence of numbers

$$\beta_n=(2n+1)^2\pi^2 a \tag{5}$$

are called eigenvalues. The specific equations for a sequence of eigenvalues vary with the shape of the region, the boundary conditions and the material properties. Eigenvalues have units of s^{-1}.

The series in (4) can be simplified for two reasons. First, the cooling time for a product involves large values of time. Most of the exponential terms go to zero. Second, the eigenvalues are well spaced. The first three terms of (4) are

$$DTR = \frac{4}{\pi}(e^{-\beta_1 t} - \frac{1}{3}e^{-\beta_2 t} + \frac{1}{5}e^{-\beta_3 t} -) \tag{6}$$

where

$$\beta_1=a\pi^2,\ \beta_2=9a\pi^2,\ \text{and}\ \beta_3=25a\pi^2$$

The quantity $e^{-5} = 0.0067$ is negligible in the determination of the cooling time. When $\beta_2 t = 5$, which occurs when DTR = 0.73, the series in (4) reduces to

$$DTR = \frac{4}{\pi} e^{-\beta_1 t} \tag{7}$$

Every term in the series except the first disappears during the first 27% of the cooling process. The dimensionless temperature ratio can be represented by the two parameter equation

$$DTR = A\ e^{-\beta t} \tag{8}$$

The two parameters are the first or lowest eigenvalue, β, and a coefficient A.

The lowest eigenvalue in (8) can be determined from the system of ordinary differential equations generated by applying the finite element or finite difference method to (1). The parameter A varies with the dimension of the problem and the boundary conditions.

The derivation of (8) from (3) considered the one-dimensional step change problem. The analytical solutions for one-dimensional problems with convection boundary conditions and two- and three-dimensional problems are more complicated. Each solution, however, involves exponential terms and eigenvalues which are spaced such that the cooling process reduces to the solution of an equation similar to (8).

The value of the dimensionless temperature ratio at which all but the first term can be neglected is a function of the dimension and the boundary conditions. Calculations for one- and two-dimensional problems and cylindrical problems (not presented here) indicate that more than 50% of the cooling process is governed by an equation similar to (8) for the least favorable conditions. This fact is verified by observing the temperature response curves (Lienhard 1981; Singh and Heldman 1993). The straight line property of the curves on a semi-log plot indicate that only one term is left in the infinite series solution. Analysis of the temperature response curves indicates that the lowest starting point for the straight line relationship occurs for the cylindrical problem and starts with DTR = 0.65.

EVALUATION OF THE LOWEST EIGENVALUE

One reason a method similar to the procedure presented here has not been used is the difficulty involved in calculating the lowest eigenvalue for food products with irregular shapes. Analytical evaluations of the lowest eigenvalue are not possible. The evaluation of β must be done numerically and requires a significant amount of computer software.

Application of the finite element or finite difference method to time dependent heat transfer problems defined by (1) produces a system of ordinary differential equations

$$[C]\left\{\frac{dT}{dt}\right\} + [K]\{T\} - \{F\} = \{0\} \tag{9}$$

where [C] is the capacitance matrix, [K] is the stiffness or conductance matrix and {F} is a vector that contains a part of the convection boundary condition and point source or sink values. The details of the finite element formulation for (9) are given by Segerlind (1984). The analytical solution of (9) requires the evaluation of a set of eigenvalues that satisfy the relationship

$$([K] - \beta_n [C])\{T\} = \{0\} \tag{10}$$

In this case, n is finite and corresponds to the number of nodes at which the temperature is not known.

An excellent discussion of the solution of (10) is given by Bathe and Wilson (1976). Equation 10 can be solved for all of the eigenvalues using one of several methods. Jacobi's method and Householder's method are two of these. Alternatively, (10) can be solved for the lowest eigenvalue or the largest eigenvalue using the inverse iteration method or forward iteration method, respectively. The most important concept, however, is that the lowest eigenvalue for (10) approaches the value of the lowest eigenvalue for the analytical solution of (1).

The evaluation of the lowest eigenvalue, β, for the cooling problems presented in this paper used the finite element method to generate [C] and [K] in (9) and the inverse iteration method discussed by Bathe and Wilson (1976) to obtain β. The lowest eigenvalue for any problem has a fixed value and the value calculated from a finite element or finite difference grid will converge as the grid is refined. The amount of the grid refinement may affect the rate of convergence but should not influence its final value.

A complicating factor when using the finite element method to formulate (9) is that there are two ways to define the capacitance matrix [C]: The lumped formulation and the consistent formulation, Segerlind (1984). The lumped formulation produces a diagonal [C] and is similar to the finite difference formulation. The consistent formulation produces [C] with nonzero off-diagonal values. Calculations reveal that the lumped formulation produces eigenvalues that converge to β from below while the consistent formulation produces values that converge to β from above, Hughes (1987). Hughes shows that an average of the values calculated using the two formulations gives an excellent estimate for the lowest eigenvalue when solving one-dimensional problems. Preliminary calculations done for this study indicated that the same property existed for axisymmetric problems. The average of the lowest eigenvalue calculated using the lumped and consistent formulations is very close to the analytical value for a finite cylinder with the temperature specified on the boundary. Preliminary calculations also showed that the grid should have a minimum of 16 nodes that are not a part of the boundary conditions in order to obtain an accurate value for β.

EVALUATION OF THE COEFFICIENT A

The simplified form of the cooling curve in (8) has a multiplying coefficient A. The value of A for the step change problem in (3) is $A = 4/\pi = 1.27$. The value changes with the boundary conditions and with the geometry of the problem. The cooling curves for a particular type of problem are not defined until A has been evaluated. The following procedure was used to evaluate A.

(1) Define the geometry for the shape and generate a finite element grid.
(2) Specify the boundary conditions which consist of known temperatures on the boundary or the convection heat loss condition. When convection heat loss occurs, the derivative boundary condition contains the ratio h/k where h is the convection coefficient, $W/m^2 \cdot °C$ and k is the thermal conductivity of the material, $W/m \cdot °C$.
(3) Evaluate the lowest eigenvalue using the inverse iteration method.
(4) Solve the transient heat transfer problem in time using the finite element method to generate the matrices in (9). A lumped formulation should be used for the capacitance matrix. Equation 9 is then solved using a central difference method in time because this method is second order accurate, Gear (1971). The advantages of the lumped formulation over the consistent formulation when developing (9) are discussed by Segerlind (1984). The selection of the time step required for an accurate solution using the central difference method is discussed by Mohtar (1994).
(5) Determine the time required for the dimensionless time ratio to decrease from one to a specified value. A value of 0.125 was used in this study. Equation (8) can be solved for the coefficient A from the time required to reach the specified dimensionless time ratio and the value of β.

The procedure outlined above must be performed for the shape of interest. The thermal diffusivity, a, and thermal conductivity, k, must be known for the material being cooled.

DETERMINATION OF THE COOLING EQUATIONS FOR A ROME APPLE AND A BARTLETT PEAR

Cooling curves were determined for two agricultural products, a Rome apple and a Bartlett pear. Each product was assumed to be axisymmetric. The cross section and the finite element grid for each are presented in Fig. 1. The Rome apple had a maximum horizontal diameter of 0.091m and a height of 0.067m. The Bartlett pear had a maximum horizontal diameter of 0.079m and a height of 0.107m. The thermal properties for an apple are given by Singh and Heldman (1993). The thermal properties for a pear are given by DeBaerdemaeker *et al.* (1977). A thermal diffusivity of a = 1.39E-07 m^2/sec and a thermal conductivity of k = 0.4 W/m·°C were used for the apple. A

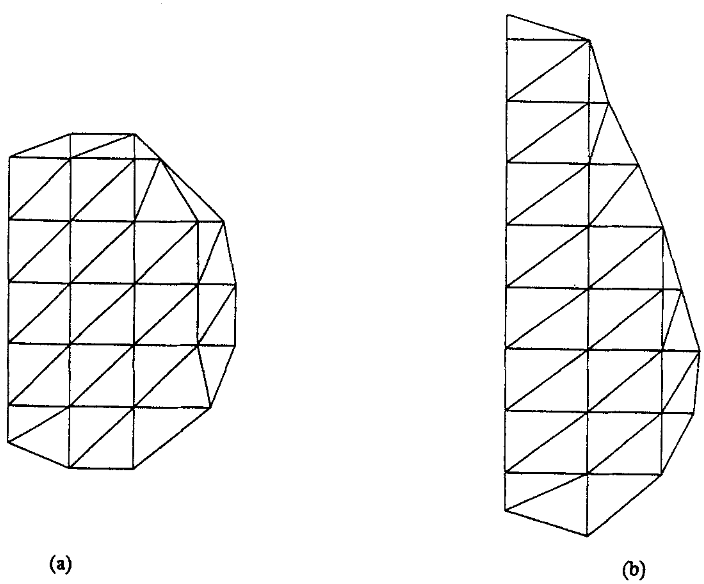

FIG. 1. THE FINITE ELEMENT GRIDS FOR (a) ROME APPLE AND (b) BARTLETT PEAR

thermal diffusivity of a = 1.65E-07 m^2/sec and a thermal conductivity of k = 0.6 W/m·°C were used for the pear.

The lowest eigenvalue and the coefficient A were evaluated for nine different values of h/k for each product. The lowest value of 10 is in the range for cooling by natural convection. Using the values given by Fraser and Otten (1992) and Leinhard (1981), h for natural convection is in the range of 5 or 6 W/m^2·°C. Using h = 5, h/k = 5/0.4 = 12.5 m^{-1} for the apple and h/k = 8.33 m^{-1} for the pear. The largest value of h/k = ∞ occurs when the temperature on the boundary is known.

The calculated values for the lowest eigenvalue and the A coefficient are summarized in Tables 1 and 2 for the apple and pear, respectively. The values in this table can be used in (8) to provide an explicit equation for the cooling process. The values can also be used to generate product specific cooling curves that allow a graphical solution. The variation of the lowest eigenvalue with h/k is presented in Fig. 2. The cooling curves for the apple and pear are given in Fig. 3 and 4, respectively.

The temperature response curves (Singh and Heldman 1993) are presented using a Biot number as one of the dimensionless parameters. The Biot number is Bi = hD/k where D is a characteristic length. The results in Table 1 and 2 are presented in terms of h/k and a Biot number calculated using the largest radius value for each product as D. The primary reason for presenting the Biot values is to allow researchers familiar with the Biot number a basis on which to evaluate the results. The calculations presented in the next section use the h/k values when interpolating to determine β and A. The definition of a characteristic length for an irregular shaped food product could be useful and should be studied.

TABLE 1.
COOLING PARAMETERS FOR A ROME APPLE

$\frac{h}{k}, \frac{1}{m}$	Biot Number	β	A
10	0.455	0.212E-04	1.01
25	1.14	0.508E-04	1.05
50	2.28	0.952E-04	1.09
125	5.69	2.030E-04	1.23
250	11.4	3.280E-04	1.43
625	28.4	5.140E-04	1.76
1250	56.9	6.500E-04	1.94
∞	∞	7.550E-04	2.06

TABLE 2.
COOLING PARAMETERS FOR A BARTLETT PEAR

$\frac{h}{k}, \frac{1}{m}$	Biot Number	β	A
10	0.395	0.457E-04	1.05
25	0.988	1.061E-04	1.10
50	1.98	1.918E-04	1.19
125	4.94	3.790E-04	1.37
250	9.88	5.600E-04	1.57
625	24.7	7.650E-04	1.77
1250	49.4	8.580E-04	1.83
∞	∞	9.630E-04	1.87

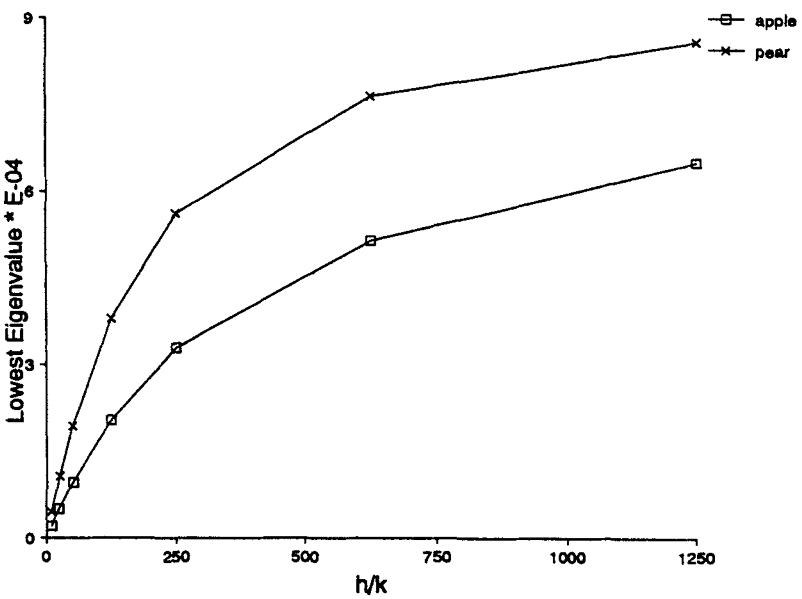

FIG. 2. THE LOWEST EIGENVALUE FOR A ROME APPLE
AND A BARTLETT PEAR

EXAMPLE PROBLEM

A Bartlett pear of the size in this study (0.107 m height, 0.079 m in diameter) is being cooled from 20C to 2C using forced air with h = 25 $W/m^2 \cdot °C$. The thermal conductivity of the pear is k = 0.6 $W/m \cdot °C$. How long will it take for the pears to cool to 3°C. The dimensionless temperature ratio is

$$DTR = (3\text{-}2)/(20\text{-}2) = 0.056$$

The convection/conduction ratio is h/k = 25/0.6 = 41.67 m^{-1}. Linear interpolation of the values in Table 2 gives β = 1.63E-04 and A = 1.17. The equation for the cooling of the pears is

$$DTR = Ae^{-\beta t} = 1.17e^{-1.63E-04)t}$$

Since DTR = 0.056,

$$t = -\frac{\ln (DTR/A)}{\beta}$$

$$t = -\frac{\ln (0.056/1.17)}{1.63E-04}$$

$$t = 18650s = 5.18h$$

The cooling curve in Fig. 4 could have been used to solve this problem. The answer would have been subject to the visual interpolation errors that go with a graphical solution.

COMPARISON WITH STANDARD SHAPES

Given a method for defining the cooling equation for an irregular shaped food product, it is of interest to compare the cooling time using this equation with the time calculated using a standard shape such as the infinite slab, infinite cylinder or a sphere. A comparison of the cooling time for the Rome apple when modeled as a sphere and the cooling time for the Bartlett pear when modeled as an infinite cylinder is discussed here.

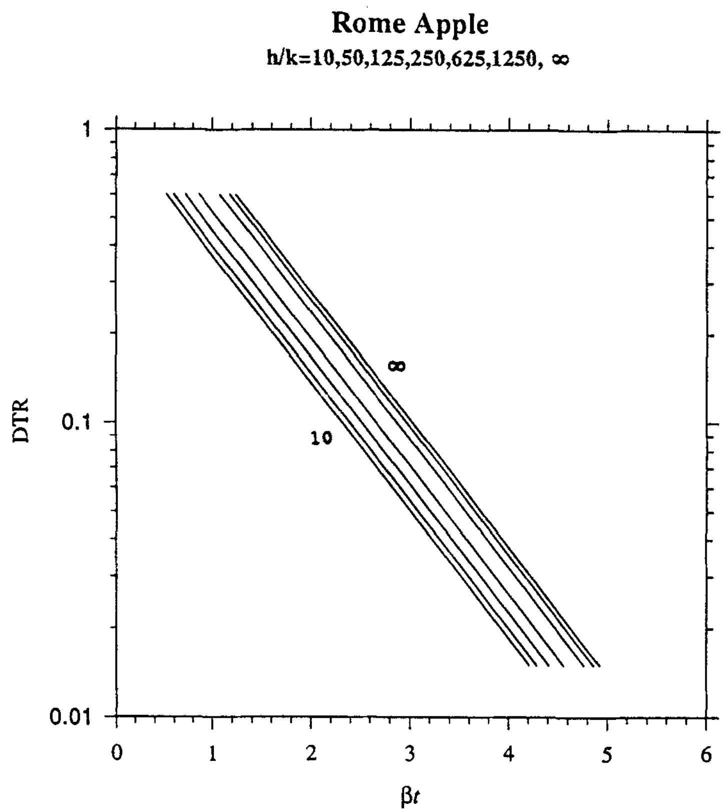

FIG. 3. COOLING CURVES FOR A ROME APPLE

The volume of the Rome apple and the Bartlett pear were calculated during the finite element formulation of (9). The volumes were 3.463E-04 m^3 and 3.043E-04 m^3 for the apple and pear, respectively. A sphere with the same volume as the apple has a radius of 0.0436m. A finite cylinder of height 0.107m with the same volume as the pear has a radius of 0.0301m.

The temperature response curves given by Singh and Heldman (1993), and other books, require a significant amount of visual interpolation unless values are selected on the basis of readability. Easily readable values were selected for this part of the study. The curves are given in terms of the dimensionless temperature ratio, DTR, the inverse of the Biot number and the dimensionless parameter, at/D^2 where D is the outside radius. A dimensionless temperature ratio of 0.100 was used. Inverse Biot values of 0, 0.5 and 1.0 were used for the apple and 0, 0.4 and 1.0 were used for the pear because the curves for these values intercepted the DTR value of 0.100 at locations which reduced the amount of visual interpolation required.

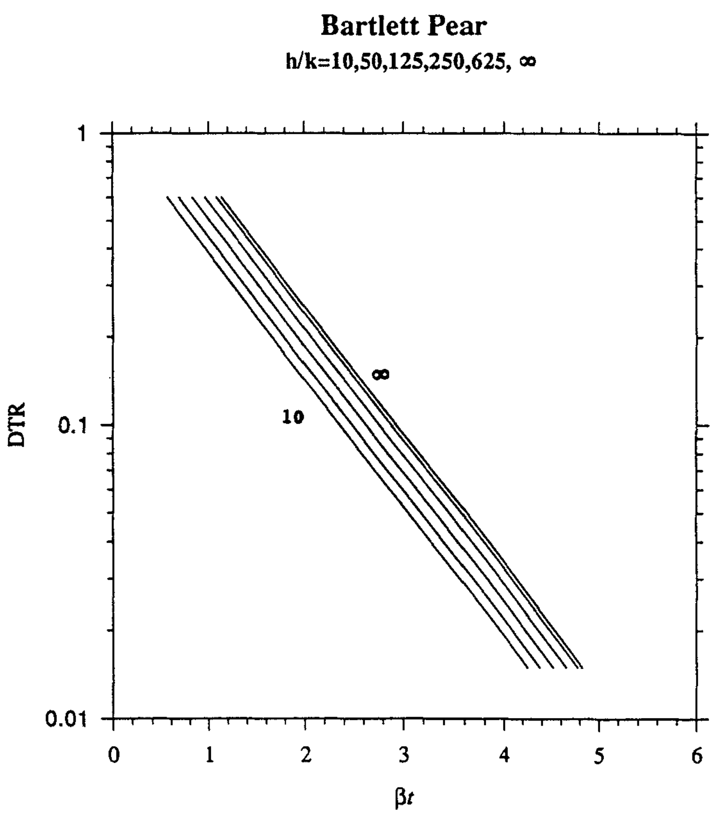

FIG. 4. COOLING CURVES FOR A BARTLETT PEAR

The time to cool each product to DTR = 0.100 was calculated using the finite element solution of (9) as described in step 4 to evaluate the coefficient A. The cooling time was also calculated using the numerical value of at/D^2 obtained from the temperature response curves. The results of each set of calculations are summarized in Tables 3 and 4.

TABLE 3.
COOLING TIME TO DTR = 0.100
USING TWO DIFFERENT METHODS, ROME APPLE

Type of Calculation	Bi^{-1} 0	0.5	1.0
Temperature Response Curve	4790	9436	14360
Numerical Analysis	3220	27000	50000

Calculated values are in seconds

TABLE 4.
COOLING TIME TO DTR = 0.100
USING TWO DIFFERENT METHODS, BARTLETT PEAR

Type of Calculation	Bi^{-1} 0	0.4	1.0
Temperature Response Curve	2730	4360	8720
Numerical Analysis	3060	8770	17700

Calculate values are in seconds

The conclusion of this comparison is that there is no relationship between the two methods of calculating the cooling values. In fact, the disparities between the calculated cooling times are so great, the finite element code was rechecked for programming errors and rechecked using a finite cylindrical shape. No errors were found in the software. It appears from these calculations that an apple should not be modeled by a sphere nor should a pear be modeled by an infinite cylinder when trying to estimate the time to cool to a specified temperature. These results may not be as surprising as they seem because the cooling of an apple or pear is a special two-dimensional problem, axisymmetric, and the infinite slab, the infinite cylinder and a sphere are one-dimensional problems.

COOLING TIME ESTIMATE

Equation (8) and the A values for the apple and pear can be used to generate a simple equation to estimate the cooling time. The equation contains a numerical value divided by the lowest eigenvalue. The equation has the simple form

$$t = \frac{C}{\beta} \tag{11}$$

The cooling time to a specified DTR value is

$$t = -\frac{\ln (DTR/A)}{\beta} \tag{12}$$

The numerator in (12) has similar values for apples and pears for the same h/k values. Suppose h/k = 125 m^{-1} and DTR = 0.02. The corresponding A values are 1.23 and 1.37 for the apple and pear, respectively. The numerator values are 1.789 and 1.836 with an average of 1.81. The time required to cool the apple or pear is

$$t = \frac{1.81}{\beta} \tag{13}$$

Using the eigenvalue for each fruit when h/k = 125 m^{-1} in (12), the estimated time to cool each fruit 98% is 8920 s for the apple and 4780 s for the pear. These values compare with 8810 s, apple, and 4840 s, pear, calculated using (8). The cooling time for each fruit has a 1.2% error. The largest time difference is less than 2 min.

Table 5 contains the C values required for (11) for six different DTR values and convection coefficients corresponding to natural convection, h = 5 $W/m^2{}^\circ C$, forced convection, h = 25 $W/m^2{}^\circ C$, and hydrocooling, h = 70 $W/m^2{}^\circ C$. The convection coefficients are given by Fraser and Otten (1992). The C coefficients were calculated using the A values given in Table 5. Each A value was determined using linear interpolation in Table 2 or Table 3 and is the average for the apple and the pear.

The advantage of the cooling time estimate given by (13) is that the lowest eigenvalue is much easier to evaluate than a time dependent finite difference or finite element solution of the fruit. No decisions have to be made about the solution procedure or the time step. A good estimate of the lowest eigenvalue, β, is obtained from a relatively coarse grid of the shape. The best approach is to average the lowest eigenvalue for the lumped and consistent formulations of the time problem. The C values may also be relatively independent of the axisymmetric shape. Further study is needed on this aspect.

TABLE 5.
THE COEFFICIENT C USED TO ESTIMATE THE TIME REQUIRED TO COOL A ROME APPLE AND A BARTLETT PEAR

Type of Cooling	A	DTR 0.125	0.100	0.075	0.050	0.025	0.010
Natural	1.03	0.916	1.01	1.14	1.30	1.61	2.04
Forced	1.14	0.960	1.06	1.18	1.36	1.66	2.06
Hydro	1.33	1.03	1.12	1.25	1.42	1.72	2.12

SUMMARY

A procedure for developing a cooling equation for an irregular shaped food product has been presented. The cooling equation includes two parameters that are a function of the product shape, material properties and boundary conditions. The procedure utilizes the finite element method to calculate the lowest eigenvalue for the transient heat transfer problem. The second parameter is obtained by determining the time required for the dimensionless temperature ratio to reach a specified value. The method was applied to the cooling of an apple and a pear. A table of the lowest eigenvalues and the multiplying coefficient A as a function of h/k is given for the Rome apple and the Bartlett pear, respectively. The method gives reasonable cooling time values for the two products analyzed. A comparison with temperature response curves indicates that the calculations for an apple modeled using a sphere with the same volume are not comparable. The same noncomparble result was found for the pear when it was modeled using a finite cylinder with the same volume.

The method of analysis presented here could be the basis for a computer code that is capable of determining the cooling time for axisymmetric shapes because the cooling time can be represented by a single numerical coefficient divided by the lowest eigenvalue for the product being cooled. All of the thermal properties of the cooling problem are incorporated in the lowest eigenvalue. The lowest eigenvalue is a more desirable parameter than the Biot number because the Biot number is not clearly defined for irregular geometries. The computer software would ask for information that defines the shape and thermal properties, calculate the lowest eigenvalue for the product and give the time to cool the product to a specified dimensionless temperature ratio.

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