

Intertemporal and Intergenerational Pareto Efficiency¹

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Efficiency conditions are derived for both private and public goods which provide benefits over time. In deriving these conditions, the paper extends the notion of efficiency to an intertemporal Pareto-optimal concept requiring the maximization of the i th individual's utility at a point of time subject to the constancy of his utility in all future periods and that of all other individuals during the relevant time span. By permitting births and deaths, a generalization of the basic model recasts the analysis into an intergenerational setting. Additional extensions involve learning by doing and perpetual public goods. The paper concludes that several of the conventional practices in public expenditure analysis do not conform to our definition of intertemporal Pareto efficiency.

1. INTRODUCTION

The questions associated with intergenerational resource allocation and equity have increasingly occupied the attention of economists.² John Rawls' widely read volume, "A Theory of Justice," is, in part, responsible for this concern. Additional stimulus to the research can be found in the efforts to amend the theory of economic growth, moving it away from a utilitarian perspective.³ In both cases, the focus of attention has been on those issues associated with the distribution of economic goods. While efficiency questions have not been completely ignored, they have remained in a secondary role.

This paper demonstrates that the generalization of the concept of Pareto efficiency to intertemporal and intergenerational perspectives has important implications for a number of problems, including both the criteria for public expenditure analysis and

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² Examples of this interest can be found in the symposium on Rawls in the *Quarterly Journal of Economics* [1, 5, 13, 19]. References [6, 16, 17] are also further examples.

³ Solow [24] makes this point in his recent paper. He notes that "The theory of optimal economic growth, in the form given it by Frank Ramsey and developed by many others, is thoroughly utilitarian in conception. It is utilitarian in the broad sense that social states are valued as a function of the utilities of individuals. . . . It is also utilitarian in the narrow sense that social welfare is (usually) defined as a sum of the utilities of different individuals or generations" (p. 29).

the modeling of optimal local government decision making.⁴ Both private (x) and public (y) goods are considered in our generalization.

Section 2 begins with a review of the conventional Pareto efficiency criteria and ends with a discussion of our revised objective function. Section 3 considers each of five specifications of the resource allocation problem. In Section 4, the implications and conclusions are presented.

2. CONVENTIONAL PARETO EFFICIENCY AND A GENERALIZATION

A Pareto optimum is attained when it is not possible, through alterations in the resource allocation, to improve the utility of any member of the defined *reference group* without loss of utility by some other member (or members). Optimality, of course, assumes that the production and resource availability constraints are satisfied. Our analysis will proceed by progressively expanding the *reference group* under consideration. As a consequence we shall adopt a notational format that appears cumbersome and unnecessary for the most limited definition of the *reference group*, but is *necessary* for the fully generalized specification. Table I defines the symbols we shall use throughout the paper. All utility functions are assumed differentiable and concave, and the transformation function is assumed to be differentiable and convex in order to satisfy sufficiency conditions. Equation (1) specifies the conventional Lagrangian for a Pareto optimum.

$$L = u^{1i}(x_{11}, \dots, x_{1T}; r_{1i}; y_1, \dots, y_T) + \sum_{i=2}^S \lambda^{ii}(u^{ii}(\cdot) - k^{ii}) \\ + \sigma F(R_i; X_1, \dots, X_T; y_1, \dots, y_T) + \sum_{j=1}^T \phi^j(\sum_{i=1}^S x_{ij} - X_j) + \gamma^i(\sum_{i=1}^S r_{ii} - R_i) \quad (1)$$

where $u^{ij}(\cdot) = u^{ij}(x_{i1}, \dots, x_{iT}; r_{ij}; y_1, \dots, y_T)$.

In converting our general notation to this problem, we have held the time period (index j) constant at \bar{l} . Equation (1) states that we maximize the utility of individual 1 in a given period while holding all others in that period at a constant utility and satisfying the technological production and distribution constraints. x_{ij} can be treated as individual i 's share of good j or, as we have defined for the general model, the consumption of x (by i) in each of j periods. The y_j 's can be treated as different public goods or the same public good in each time period. The good r_{ii} serves as a numeraire in the \bar{l} period.

The Pareto-efficient conditions call for equality of the marginal rate of substitution (MRS) between each private good (or each period's consumption of the given private good) and the numeraire (r_{ii}) for all individuals. Moreover, this MRS must equal the marginal rate of transformation between these goods (MRT). For the public goods,

⁴ Efficiency conditions for public goods with interregional spillovers are derived in [3, 18, 20, 22, 25]. The analysis of this paper can be used to extend the Pareto efficiency conditions derived in these contributions by summing each of the conditions of Table II over all relevant regions. Recasting local government provision of public goods within an intertemporal analysis makes for a much richer analysis due to an increase in the number of independent adjustment rules that fail to adjust for all relevant spillovers. Pareto inefficiency may result from a failure to account for interregional, intertemporal or intergenerational spillovers.

TABLE I
Definition of Symbols

$u^i(\cdot)$	= i th individual's t th period utility function.
$F(\cdot) = 0$	= intertemporal transformation function.
r_{ij}	= i th individual's consumption of numeraire good in the j th period.
x_{ij}	= i th individual's consumption of the private good in j th period.
y_j	= quantity of public good in the j th period.
X_j	= quantity of private good available in j th period.
R_j	= quantity of numeraire good available in j th period.
$R = \sum_{j=1}^T R_j$	= quantity of numeraire good for all periods.
$MRS_{x_p r}^{ij}$	= $(\partial u^i / \partial x_{ip}) / (\partial u^i / \partial r_{ij})$.
$MRS_{y_p r}^{ij}$	= $(\partial u^i / \partial y_p) / (\partial u^i / \partial r_{ij})$.
$MRT_{x_p R}$	= $(\partial F / \partial X_p) / (\partial F / \partial R)$.
$MRT_{y_p R}$	= $(\partial F / \partial y_p) / (\partial F / \partial R)$.
S, S_j	= number of individuals in period j .
T	= total time horizon.
T_{i0}	= initial period of time horizon for i th individual.
T_{if}	= final period of time horizon for i th individual.
k^{ij}	= constant utility level for the i th individual in j th period.
p, j	= subscripts for time periods.
i	= subscript for individuals.
ϕ^i, σ, γ^j	= nonpositive unspecified Lagrangian multipliers.
λ^{ij}	= nonnegative unspecified Lagrangian multipliers.

the sum of the MRSs across individuals for each public good (or each period's consumption) must equal the corresponding MRT. Equations (2) and (3) provide these conditions more formally.

$$MRS_{x_p r}^{1i} = MRS_{x_p r}^{2i} = \dots = MRS_{x_p r}^{Si} = MRT_{x_p R} \quad \text{for } p = 1, \dots, T \quad (2)$$

$$\sum_{i=1}^S MRS_{y_p r}^{ii} = MRT_{y_p R} \quad \text{for } p = 1, \dots, T. \quad (3)$$

These conditions for private and public goods are, of course, well known. Before generalizing them we should note that our specification of different utility functions over the T periods (for each individual) generalizes the conventional statement of the multiperiod utility function (see [9, p. 298]) in which a given utility function is specified for all consumption bundles over time. This specification can be deduced from our treatment which allows a more detailed account to be given to particular time patterns of consumption.⁵

The intertemporal generalization of the concept of Pareto efficiency requires expansion of the *reference group* noted in our definition. That is, we consider each individual in each time period as a unique entity and maximize the utility of one individual at one point in time subject to the constancy of his utility in all future periods and that of all other individuals in the initial and all future periods. Equation (4) specifies the amended version of Eq. (1).

⁵ See Section 3 for examples of these differing consumption patterns.

$$\begin{aligned}
L = & u^{11}(x_{11}, \dots, x_{1T}; r_{11}; y_1, \dots, y_T) \\
& + \sum_{j=1}^T \sum_{i=2}^S \lambda^{ij}(u^{ij}(\cdot) - k^{ij}) + \sum_{j=2}^T \lambda^{1j}(u^{1j}(\cdot) - k^{1j}) + \sigma F(R; X_1, \dots, X_T; y_1, \dots, y_T) \\
& + \sum_{j=1}^T \phi^j(\sum_{i=1}^S x_{ij} - X_j) + \sum_{j=1}^T \gamma^j(\sum_{i=1}^S r_{ij} - R_j) \quad (4)
\end{aligned}$$

where

$$R = \sum_{j=1}^T R_j.$$

A comparison of (1) and (4) suggests that we are taking full account of the implications of any action in terms of both the initial period and future periods.

3. RESOURCE ALLOCATION: INTERTEMPORAL AND INTERGENERATIONAL EFFICIENCY

In what follows we shall examine five specifications of the intertemporal efficiency definition. Each specification is consistent with the logic outlined in the previous section. That is, the definition of the reference group is a fundamentally important concept for the relevance of Pareto efficiency. The cases serve to illustrate the implications of the overall generalization as we change the character of the goods and/or individuals considered.

(a) *Base Case: Intertemporal Efficiency with Static Constituency*⁶

In the base case, defined in Eq. (4), each individual's utility is affected by the full set of intertemporal choices for the private and public goods. Thus, an increment of the private good to each individual generates utility in every period. Although we do not assume a pattern for this stream of benefits, there are many examples that can be cited to parallel these effects. For example, education has been considered as affecting the consumer's efficiency in nonmarket production (see [12]). This observation can be adjusted to our model by substituting the household production functions into the utility function that is expressed in basic commodities [2] or produced service flows. In so doing, we have education affecting both present and future utility through its influence on consumption. Health expenditures can be treated in much the same way (see [8]).

The public goods' intertemporal effect also parallels observed behavior. This effect can be used as one means of taking account of the "learning-by-doing" phenomena found to be important in many resource-based recreational activities (see [4, 7]). Davidson, Adams, and Seneca note for water-based activities that:

If water recreational facilities are neither available nor easily accessible, people tend not to engage in these activities. Should they participate, however, their realized enjoyment often exceeds their expectations, and as a result they will tend to increase their demand for facilities. Moreover, skill is often essential for the enjoyment of these activities. When facilities are not readily available, skills will not be developed and, consequently, there may be little desire to participate in these activities [7, p. 186].

Pareto efficiency in this case replaces condition (2) with equality of the sum over the periods for each individual's MRSs. These sums must be equal to the relevant MRT.

⁶ In what follows constituency and reference groups will be treated as synonyms.

There are T sets of conditions of this form, one for each period's consumption of x . The public good's influence is across all individuals and their utility in all periods so that (3) must be replaced with a double sum of the relevant MRSs across individuals and time periods for a given period's quantity of the public good. As before, since each y_p gives rise to an efficiency condition, there are T sets of conditions.

(b) *Intergenerational Efficiency: A Changing Constituency with Time Period*

Here we allow the individuals present in each time period to change. This specification allows a natural sequencing of generations. The notation necessary to describe this case becomes somewhat cumbersome, but the logic underlying the notational changes is clear-cut. For the private good, we want to count only the utilities generated while each individual exists in the reference group, (thus accounting for migration and death) when summing each individual's MRSs over time. In equating these sums across individuals, we consider only those members of the constituency in each period. Similarly, for the public good, our sum of the MRSs between public good and numeraire must take account of only the individuals in existence and the time period of their enjoyment of the good. Hence, public good efficiency requires equality of the double summation of the MRSs and the MTR; however, the MRSs are summed over the cardinality of the set of all individuals present during any part of the time span of T periods. The T_{i_0} and T_{i_f} limits on the period sums (see Table II) are indicative of the initial and final time periods, respectively, for the i th individual (i.e., $T_{i_f} - T_{i_0}$ is his time span in the reference group). In this formulation, residency in the reference group is assumed continuous.

Examples for this case are akin to those in the base case with the important refinement that we can allow for migration or death.⁷

(c) *Unidirectional Consumption Effects*

In this case, we alter the specification of each individual's utility function to permit only past and present consumption to affect utility. In the base case, the causality worked in both directions. Here future consumption choices in either the private or public good do not affect present utility. This case is likely to be more in keeping with the examples discussed with out base case scenario. The alterations to conditions (2) and (3) are once again straightforward. Increments to satisfaction from the private good are counted from the j th time period they are realized (i.e., $j = p$ for y_p and x_p) to the end of the time horizon in constructing the relevant sums of each individual's MRSs. These sums are, in turn, equated across individuals and to the MRT. Analogous alterations hold for the public good conditions. If one accepts this preference structure (as several researchers have for the case of a variety of environmental resources), then conventional Pareto efficiency conditions will *not* be appropriate guides to efficient resource allocation. The modification of benefit-cost techniques implicitly called for in Krutilla's seminal paper on conservation problems assumes the kind of Pareto conditions implied in this case.⁸

⁷ Migration is considered important here because our concept of a reference group requires that we clearly identify the frame of reference for efficient resource allocation. If it is regional, then we consider migration as movement out of the region; if national, then out of the nation; if global, the concept is not meaningful, at present.

⁸ Krutilla and Fisher [11] discuss in some detail the public characteristics of the recreational or amenity services provided by natural environments.

(d) *Perpetual Public Goods and Efficiency*

Consider a public good which is public across individuals and time during a given time horizon. Once provided, this class of public goods remains in existence, at the same level, for all ensuing time periods. Flood protection might be an example of this type of public good. The public aspects of a unique natural environment remain as long as it is preserved. The decision to reserve it under statutory protection (i.e., National Wilderness System, National Wildlife Refuges, National Park System, etc.), is similar to the allocation of a perpetual public good. For this case, we need to alter the efficiency conditions so as to sum the MRSs for the public good and numeraire over the time span in which the good is enjoyed by an individual, the individuals who enjoy the good, and the time periods during which it is available. Hence, a triple summation is required.

(e) *Asset Utilization and Public Goods*

Consider the problems associated with reserving a natural environment in some protected status. Our previous discussion assumes that the same level of the public

TABLE II
Pareto Optimality Conditions for Intertemporal Efficiency

Case	Assumptions	Optimality conditions
1 (base case)	$u^{ij} = u^{ij}(x_{i1}, \dots, x_{iT}; r_{ij}; y_1, \dots, y_T)$ production constraint $F(X_1, \dots, X_T; R; y_1, \dots, y_T) = 0$ $R = \sum_{j=1}^T R_j$ numeraire assumption S individuals in each of T periods; same individuals	$\text{Public good } \sum_{i=1}^S \sum_{j=1}^T \text{MRS}_{y_p r^{ij}} = \text{MRT}_{y_p R}$ for all $p = 1, \dots, T$ $\text{Private good } \sum_{j=1}^T \text{MRS}_{x_p r^{1j}} = \dots = \sum_{j=1}^T \text{MRS}_{x_p r^{Sj}}$ $= \text{MRT}_{x_p R}$ for all $p = 1, \dots, T$
2	Take account of generational differences in composition of community. S_j denotes the set of individuals in j th period. $\bigcup_{j=1}^T S_j = \Omega$ (cardinality of the set)	$\text{Public good } \sum_{i=1}^{\Omega} \sum_{j=T_{i0}}^{T_{if}} \text{MRS}_{y_p r^{ij}} = \text{MRT}_{y_p R}$ for all $p = 1, \dots, T$ $\text{Private good } \sum_{i=T_{10}}^{T_{1f}} \text{MRS}_{x_p r^{1i}} = \dots = \sum_{i=T_{\Omega 0}}^{T_{\Omega f}} \text{MRS}_{x_p r^{\Omega i}}$ $= \text{MRT}_{x_p R}$ for all $p = 1, \dots, T$
3	Learning by doing, past and present consumption provides utility into the future, but future consumption does not enter present utility function. Similar in other respects to case 1.	$\text{Public good } \sum_{i=1}^S \sum_{j=p}^T \text{MRS}_{y_p r^{ij}} = \text{MRT}_{y_p R}$ for all $p = 1, \dots, T$ $\text{Private good } \sum_{j=p}^T \text{MRS}_{x_p r^{1j}} = \dots = \sum_{j=p}^T \text{MRS}_{x_p r^{Sj}}$ $= \text{MRT}_{x_p R}$ for all $p = 1, \dots, T$

TABLE II -continued

Case	Assumptions	Optimality conditions
4	<p>Public good is public across individuals and time periods</p> $y_1 = y_2 = \dots = y_T = y$ <p>Transformation function given by:</p> $F(X_1, \dots, X_T; R; y)$	<p><i>Public good</i></p> $\sum_{p=1}^T \sum_{i=1}^S \sum_{j=1}^T \text{MRS}_{y_p, x_i^j} = \text{MRT}_{x_p, R}$ <p><i>Private good</i></p> $\sum_{j=1}^T \text{MRS}_{x_p, y^j} = \dots = \sum_{j=1}^T \text{MRS}_{x_p, y^{Sj}}$ $= \text{MRT}_{x_p, R}$ <p>for all $p = 1, \dots, T$</p>
5	<p>Public good provides resource utilization which determines the level of public good in any period. Production constraint given as</p> $F(X_1, \dots, X_T; R; y)$ $y_j = y_j(y) \text{ for } j = 1, \dots, T$	<p><i>Public good</i></p> $\sum_{i=1}^S \sum_{j=1}^T \text{MRS}_{y_1, x_i^j} \frac{dy_1}{dy} + \dots$ $+ \sum_{i=1}^S \sum_{j=1}^T \text{MRS}_{y_T, x_i^j} \frac{dy_T}{dy} = \text{MRT}_{y, R}$ <p><i>Private good</i></p> $\sum_{j=1}^T \text{MRS}_{x_p, y^j} = \dots = \sum_{j=1}^T \text{MRS}_{x_p, y^{Sj}}$ $= \text{MRT}_{x_p, R}$ <p>for all $p = 1, \dots, T$</p>

good will be consumed each period after the decision. For a practical perspective this is unlikely. Rather, it would seem more reasonable to assume that the decision makes an asset available, and that in each period a choice will be made on how much is to be consumed. Of course, this choice is constrained by the initial decision.

In considering the consequences of a decision on the size of the asset reserved, we must amend our expression for the MRS of the public good to reflect the asset's ability to produce certain consumption levels in each period. Otherwise, the conditions defined in (3) remain the same. Least this description give the impression the amendment is of trivial importance, we shall discuss one example of this relationship between the asset and the public good. It is this problem that Krutilla [10] clearly had in mind in discussing the asymmetry in technological change and our need to preserve a sufficient set of unique natural environments for the amenities they offer to future generations. He noted:

. . .while the supply of fabricated goods and commercial services may be capable of continuous expansion from a given resource base by reason of scientific discovery and mastery of technique, the supply of natural phenomena is virtually inelastic. That is, we may preserve the natural environment which remains to provide amenities of this sort for the future, but there are significant limitations on reproducing it in the future should be fail to preserve it [10. p. 783].

Table II summarizes, in detail, the explicit conditions for each of these cases. Each of these amendments to conventionally defined Pareto efficiency conditions can be readily related to the concept of a social welfare function generalized over individuals and time periods which, in principle, resolves the distributional issues.

4. IMPLICATIONS

This paper has proposed an extension to the definition of Pareto efficiency in resource allocation. By introducing the concept of a *reference group* we extend the static definition of efficiency to intertemporal and intergenerational generalizations.⁹ There is a growing body of research (see [11]) that indicates that the traditional concepts of efficiency in resource allocation must be generalized for a large class of decisions involving the allocation of environmental resources. To illustrate this point, we consider the contention that discounting is Pareto inefficient over time (see [11, pp. 67–69; 17]). For those cases where this result has been asserted, intertemporal Pareto efficiency was not defined. Our analysis allows a direct proof of the conditions under which these conclusions will be correct. We shall confine our attention to public goods. Pareto-efficient resource allocation over time requires that we treat each person's incremental benefits from the public good in question *equally* regardless of the time they receive the benefits. This observation is clear from all the cases in Table II. Discounting would replace the unitary weights given either future periods or individuals with declining weights. Thus, instead of Eq. (5) we would have (6) for case 1.¹⁰

$$\sum_{i=1}^S \sum_{j=1}^T \text{MRS}_{y_p r}^{ij} = \text{MRT}_{y_p R} \quad p = 1, \dots, T \quad (5)$$

$$\sum_{i=1}^S \sum_{j=1}^T \beta^j \text{MRS}_{y_p r}^{ij} = \text{MRT}_{y_p R} \quad (6)$$

where $0 < \beta^j < 1$. Unless we are willing to assume a rather special relationship between each $u^{ij}(\cdot)$ as $j = 1, \dots, T$ such that (a) the marginal utilities of y_p decline as β^j would suggest, and (b) the pattern of decline is identical across individuals, then we are forced to conclude discounting at a constant rate for all individuals and time periods in Pareto inefficient over time. The same finding can be derived from each of the cases we have studied with, in some instances, more stringent conditions necessary to permit discounting to be intertemporally efficient. This conclusion is particularly clear for cases 4 and 5 (perhaps those cases that are most relevant for environmental problems).

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⁹ The present analysis has focused on pure public goods, the findings can be easily extended to cases which may be more germane to "real world" problems. Introducing quasi-public goods (i.e., $y = A_{ij}y_j$) simply introduces a set of A_{ij} 's into the necessary conditions, scaling the MRS for each individual and period. Efficiency rules stated in Table II can be easily modified to include these A_{ij} 's. Congestion effects also may be introduced and these will serve to change the definition of MRS in our Pareto conditions, but not our overall findings. See [15, 20, 21, 23].

¹⁰ If the decision on y_p for $p = 1, \dots, T$ is made in the current period, then further discounting must be applied to reflect the future stream of benefits.

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