Effect of Shallow Penetration and Streambed Sediments on Aquifer Response to Stream Stage Fluctuations (Analytical Model)

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Abstract

An analytical model of stream-aquifer interaction is proposed that considers the effects from a small degree of aquifer penetration and low-permeability sediments on the head response to an arbitrary stream-stage hydrograph. Aquifer sections under the stream and beyond are considered in a single model. The model of ground water flow in the aquifer is based on the Dupuit assumptions corrected for leakage from the stream. The model can use stream-stage hydrographs in both analytical and tabular forms. The nondimensional linear boundary value problem is solved for hydraulic head in the aquifer using numerical Laplace transforms and a convolution algorithm. The proposed solution is used to assess the impact of shallow penetration and low-permeability streambed sediments on head responses by comparison with available solutions which neglect these factors.

Introduction

The evaluation of pumping-induced stream depletion is a critical step in the design of watershed-scale management plans (Bouwer and Maddock 1997).

The early work on depletion estimation focused on analytical solutions. Theis (1941) was the first to propose a transient method for evaluation of the impact of ground water pumping on a nearby stream. This approach, later generalized by Glover and Balmer (1954), is based on a series of idealistic assumptions that include a fully penetrating stream and perfect hydraulic connection between stream and aquifer. Hantush (1965) extended this approach to consider an imperfect hydraulic connection produced by clogging (siltation) at the stream-aquifer interface. Although based on a more realistic depiction of conditions at the stream-aquifer interface, the Hantush model has seen little use in practical applications. Instead, Jenkins' (1968) implementation of the Theis/Glover-Balmer solutions has become the standard tool for use in water-management design and water-rights adjudication in the United States and many other countries. Note that this method is based on the assumption that all of the pumped water comes from the stream at large times, which, as Larkin and Sharp (1992) have shown, is often not the case. In addition, analytical models have been developed for estimating changes in bank storage and ground water contribution to streamflow (Ferris et al. 1962; Cooper and Rorabaugh 1963; Moore and Jenkins 1966; Moench et al. 1974).

Hydrogeologists have long recognized that the approaches of Theis (1941) and Jenkins (1968) have serious limitations for many regions of the United States. The methods are based on an idealized flow system that bears little resemblance to stream-aquifer systems (e.g., stream channels in the Great Plains only partially penetrate through and are imperfectly connected to the surrounding aquifer). The impact of an imperfect hydraulic connection and/or partial penetration was demonstrated in a number of field studies in the 1960s and 1970s (e.g., Moore and Jenkins 1966; Moench et al. 1974). Despite the findings of these studies, the method of Jenkins is still the most commonly used tool for water-rights adjudication in the United States primarily because of its simplicity and the lack of convenient-to-use alternatives.

In the last decade, much new light has been shed on the concept of stream depletion (Wilson 1993; Winter 1995). Carefully performed field studies have revealed the heterogeneous nature of the stream-aquifer interface and re-emphasized the impact of partial penetration. For example, Sophocleous et al. (1988) found that pumping near a partially penetrating stream induces drawdown on the opposite side of the stream and that stream depletion estimates calculated from stream-flow measurements never reached the value predicted by Jenkins' method. The impact of partial penetration and an imperfect hydraulic connection on ground water flow near a ditch was demonstrated by Chambers and Bahr (1992) and Meigs and Bahr (1995).

These field studies have been supplemented by a number of numerical modeling investigations of the role of the stream-aquifer interaction. Pinder and Sauer (1971), Spalding and Khaleel (1991), Sophocleous et al. (1995), and Conrad and Beljin (1996) have explored the impact of a number of factors on stream depletion estimates. They found that neglect of partial penetration and imperfect hydraulic connection can result in significant overestimation of stream depletion. Note that Govindaraju and Koellicker (1994) performed a stochastic analysis of stream-aquifer interactions that found that variability in aquifer parameters has a minor impact on stream-depletion estimates for the case of a perfect hydraulic connection and a fully penetrating stream.

It is of interest to note that in Europe, and particularly in the former Soviet Union, analytical solutions that incorporate a simplified representation of imperfect hydraulic connections and partial penetration (Grigoryev 1957; Shestakov 1965; Bochever 1966; Minkin 1973; Vasilyev et al. 1975; Zlotnik et al. 1985) have been used routinely for the design of wellfields in alluvial aquifers. The primary objectives of this study are as follows: (1) to present a model

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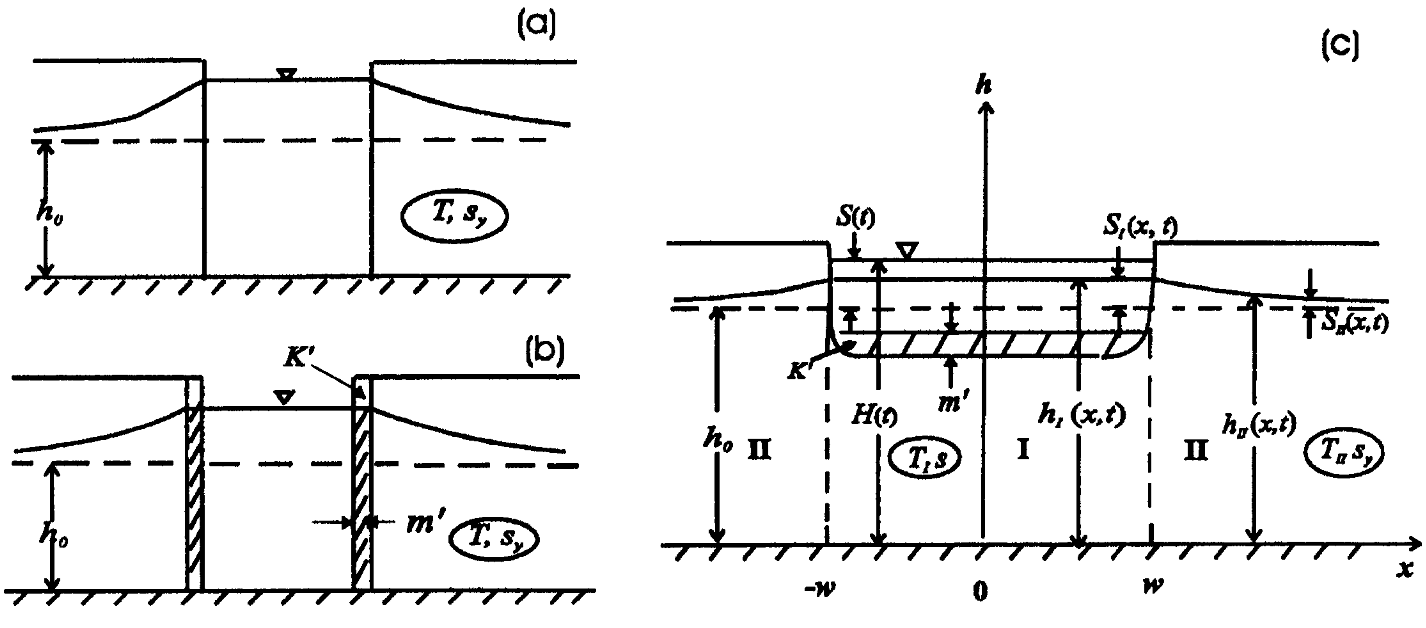

Figure 1. Different hydrogeological conditions near the stream-aquifer interface: (a) fully penetrating stream without streambed clogging by Theis (1941), Glover and Balmer (1954), and Jenkins (1968); (b) fully penetrating stream with streambed clogging by Hantush (1965); and (c) partially penetrating stream with streambed clogging by Grigoryev (1957) and Bochever (1966).

of stream-aquifer interaction using realistic streambed description; (2) to apply this model to obtain an analytical solution describing bank storage effects; (3) to explore sensitivity of bank storage effects with respect to various hydrogeological parameters; and (4) to demonstrate the conceptual improvements achieved in comparison with Theis (1941), Glover and Balmer (1954), and Hantush (1965) models based on simplified streambed description.

Statement of the Problem

The major emphasis will be on the development of models of stream-aquifer interactions in conditions typical of the Great Plains. Figures 1a and 1b depict the conditions assumed in commonly used models of stream depletion. Theis (1941), Glover and Balmer (1954), and Jenkins (1968) considered streams which fully penetrate the aquifers without streambed clogging, and Hantush (1965) investigated the additional effect of streambed clogging for fully penetrating streams. Since these conditions are often not representative of alluvial valleys in the Great Plains, where shallow stream penetration and large stream width-to-depth ratios are the norm, we will investigate a model that incorporates shallow stream penetration and large width-to-depth ratios (Figure 1c). This approach was proposed for description of the sub-stream zone by Grigoryev (1957) for the analysis of well hydraulics in alluvial aquifers. The proposed modeling approach involves dividing the aquifer into two zones. Zone I, the central strip, includes both the stream-aquifer interface and the portion of the aquifer under the stream, while Zone II includes the remainder of the aquifer which is symmetrical with respect to the stream axis. This model can be generalized to include a leaky aquitard that is often a common feature of alluvial valleys (e.g., Larkin and Sharp 1992).

The following assumptions are made in the model (Figure 1c) in order to obtain an analytical solution: (1) the stream is infinitely long in the horizontal plane and has low sinuosity; (2) the aquifer is homogeneous, isotropic, and semi-infinite in lateral extent; (3) the stream and the aquifer are initially at hydraulic equilibrium, and the water table is initially horizontal at some level h_0; (4) the streambed partially penetrates the aquifer with a hydraulic conductivity much less than the aquifer (shallow stream); (5) drawdown in the aquifer is small compared with the saturated aquifer thickness so that the Dupuit approximation is applicable; (6) leakage across the streambed (to or from the stream) is vertical and occurs only through the bottom sediments of the stream; (7) the ground water flow under the stream in Zone I is confined while ground water flow in Zone II is unconfined.

Consider hydraulic head h(x, t) in the aquifer with the aquifer base as a reference level, and a stream stage H(t) with the same reference level. Drawdown in the aquifer is $S_i(x, t) = h_0 - h_i(x, t)$, where i = I for Zone I and i = II for Zone II, and stream-stage change is $S(t) = h_0 - H(t)$. Confined ground water flow in Zone I can be described by the following equation:

$$s\frac{\partial S_I}{\partial t} = T_I\frac{\partial^2 S_I}{\partial x^2} + \frac{K'}{m'}(S - S_I) \quad 0 < x < w, t > 0 \qquad (1)$$

Here, K′ is the hydraulic conductivity of the semi-pervious "clogging" layer of the streambed, m′ is thickness of the semi-pervious layer of the streambed, w is half-width of the stream channel, T_I is transmissivity of Zone I, s is storativity of Zone I, x is the distance from the observation well to the stream axis, and t is time.

The boundary condition at the stream axis x = 0 indicates flow symmetry:

$$\frac{\partial S_I}{\partial x} = 0 \qquad (2)$$

Unconfined ground water flow in Zone II can be described by the following equation:

$$s_y\frac{\partial S_{II}}{\partial t} = T_{II}\frac{\partial^2 S_{II}}{\partial x^2} \quad x > w, t > 0 \qquad (3)$$

where s_y and T_{II} are specific yield and transmissivity, respectively, in Zone II, and head is unperturbed at large distances from the stream:

$$S_{II} = 0 \qquad x = \infty, t > 0 \tag{4}$$

Continuity of the hydraulic head and flux requires the following conditions along the vertical interface between Zones I and II:

$$S_I = S_{II},\ T_I \frac{\partial S_I}{\partial x} = T_{II} \frac{\partial S_{II}}{\partial x} \qquad x = w, t > 0 \tag{5}$$

Initial conditions for the stream stage and the head in the aquifer are as follows:

$$S_I(x, 0) = S_{II}(x, 0) = S(0) = 0 \tag{6}$$

Unlike the Glover and Balmer (1954) or Hantush (1965) models, this model considers partial (shallow) penetration, streambed hydraulic conductivity, and stream width.

Solution

The introduction of dimensionless variables

$$\bar{x} = \frac{x}{w},\ \bar{t} = \frac{T_{II}t}{s_y w^2} \text{ and } \sigma = \frac{s}{s_y},\ \beta = \frac{T_I}{T_{II}},\ \gamma = \frac{K'w^2}{m'T_{II}} \tag{7}$$

allows the problem be rewritten in a dimensionless format. Parameter γ was used by Grigoryev (1957) for characterization of the stream-aquifer interface in hydraulic analysis of alluvial aquifers, and parameter σ was proposed by Neuman (1972). The solution of the boundary value problem (Equations 1 through 5) with the initial condition (Equation 6) can be obtained using Laplace transforms.

Due to the relatively small size of Zone I, one naturally can explore the two particular cases. First case ($\sigma > 0$) considers compressibility in both Zones I and II explicitly while the second one ($\sigma = 0$) corresponds to an assumption of the incompressibility of Zone I.

General Case: Finite Storativity and Specific Yield

In this case, $s \neq 0$ or $\sigma \neq 0$. Derivation of closed form of the solution is tedious. However, one can easily obtain the Laplace transform solutions $\overline{S_I}(\bar{x},p)$ and $\overline{S_{II}}(\bar{x},p)$ in both Zones I and II, respectively, as shown in Appendix A:

$$\begin{cases} \overline{S_I}(\bar{x},p) = \dfrac{\gamma}{\beta\omega^2}\left(1 - \dfrac{\sqrt{p}\cosh(\omega\bar{x})}{\beta\omega\sinh\omega + \sqrt{p}\cosh\omega}\right)\bar{S}(p) \\ \overline{S_{II}}(\bar{x},p) = \dfrac{\gamma e^{-\sqrt{p}(\bar{x}-1)}\tanh\omega}{\omega(\beta\omega\tanh\omega + \sqrt{p})}\bar{S}(p) \end{cases} \tag{8}$$

where $\omega^2 = (\gamma + \sigma p)/\beta$, p is the Laplace transform parameter, and

$$\bar{S}(p) = \int_0^\infty e^{-p\tau} S(\tau)d\tau,\ \bar{S}_i(\bar{x}, p) = \int_0^\infty e^{-p\tau} S_i(\bar{x},\tau)d\tau,\ i = \text{I and II} \tag{9}$$

$S_I(\bar{x},\bar{t})$ and $S_{II}(\bar{x},\bar{t})$ can be obtained from $\overline{S_I}(\bar{x},p)$ and $\overline{S_{II}}(\bar{x},p)$ by using the Stehfest (1970) algorithm for the numerical Laplace transform inversion with parameter N = 16.

Special Case: Neglecting the Storativity of the Confined Aquifer under the Streambed

In this case, setting the parameter $\sigma = 0$, $\omega^2 = \gamma/\beta$ and using Laplace transform, one obtains the following solutions as shown in Appendix B:

$$\begin{cases} S_I(\bar{x},\bar{t}) = \dfrac{\cosh\omega\bar{x}}{\cosh\omega}[S_{II}(1,\bar{t}) - S(\bar{t})] + S(\bar{t}) \\ S_{II}(\bar{x},\bar{t}) = \displaystyle\int_0^{\bar{t}} G(\bar{x},\tau)\, S(\bar{t} - \tau)d\tau \end{cases} \tag{10}$$

where kernel G in the integrand is expressed through the parameter ξ:

$$G(\bar{x},\tau) = \xi\left[\left(\frac{1}{\pi\tau}\right)^{\frac{1}{2}} e^{-(\bar{x}-1)^2/4\tau} - \xi e^{\xi(\bar{x}-1)+\tau\xi^2}\operatorname{erfc}\left(\frac{\bar{x}-1}{2\sqrt{\tau}} + \xi\sqrt{\tau}\right)\right] \tag{11}$$

$$\xi = \omega\tanh\omega \tag{12}$$

The solutions S_I and S_{II} can be obtained for any arbitrary S (t).

For the unit step function,

$$S(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases} \tag{13}$$

the drawdown from Equation 10 in Zone II can be found as follows:

$$S_{II}(\bar{x},\bar{t}) = \operatorname{erfc}\left(\frac{\bar{x}-1}{2\sqrt{\bar{t}}}\right) - \exp[\xi(\bar{x}-1) + \bar{t}x^2]\operatorname{erfc}\left(\frac{\bar{x}-1}{2\sqrt{\bar{t}}} + \xi\sqrt{\bar{t}}\right) \tag{14}$$

The above solution is analogous to the following one (Hall and Moench 1972; Moench et al. 1974):

$$S(x,t) = \operatorname{erfc}\left(\frac{x}{2(\alpha t)^{1/2}}\right) - \exp\left(\frac{x}{a} + \frac{\alpha t}{a^2}\right)\operatorname{erfc}\left(\frac{x}{2(\alpha t)^{1/2}} + \frac{(\alpha t)^{1/2}}{a}\right) \tag{15}$$

where $\alpha = T_{II}/s_y$ is the notation of aquifer diffusivity. "Retardation" parameter $a = K_{II}m'/K'$ (see notation on Figure 1b) was introduced by Hantush (1965) to accommodate the realistic features of streambed. Moench et al. (1974) used retardation as a fitting parameter. (Note, that in the latter formula, coordinate x has a stream bank location as a reference point instead of a stream axis). In our interpretation, parameter ξ has a more direct hydrogeological meaning which explicitly takes into account streambed parameters w, K′, and m′.

Effects of the Hydrogeological Parameters on Stream-Aquifer Interaction

Analysis of Equations 10 and 14 was made using an example of a stream with typical characteristics for many alluvial aquifers in the Great Plains. Consider the stream which does not penetrate the unconfined aquifer with parameters $T_I = T_{II} = 200$ m²/day, s = 0.001, $s_y = 0.2$, w = 5 m, K′ = 0.1 m/day, m′ = 0.1m. Observations are made in the well at the distance of x = 4w from the stream axis. Using the typical ranges of these parameters, one can evaluate their importance and the sensitivity of the proposed model. The developed methodology is not limited by any specific

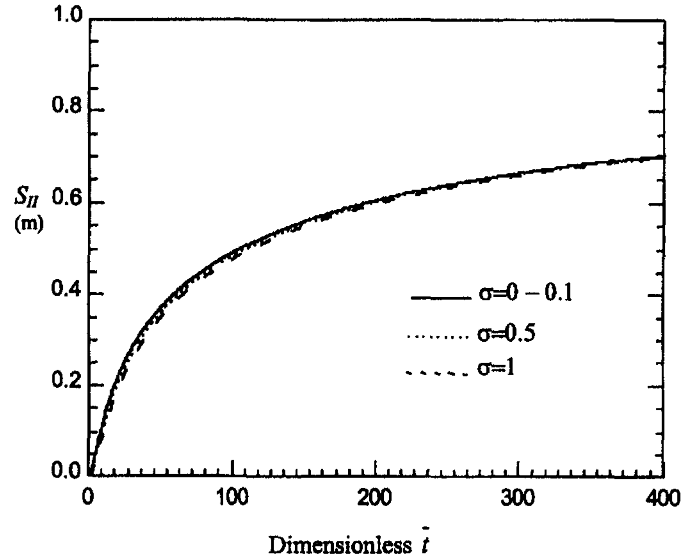

Figure 2. Effect of compressibility in Zone I on head changes at the distance 4 w between stream axis and well located in Zone II.

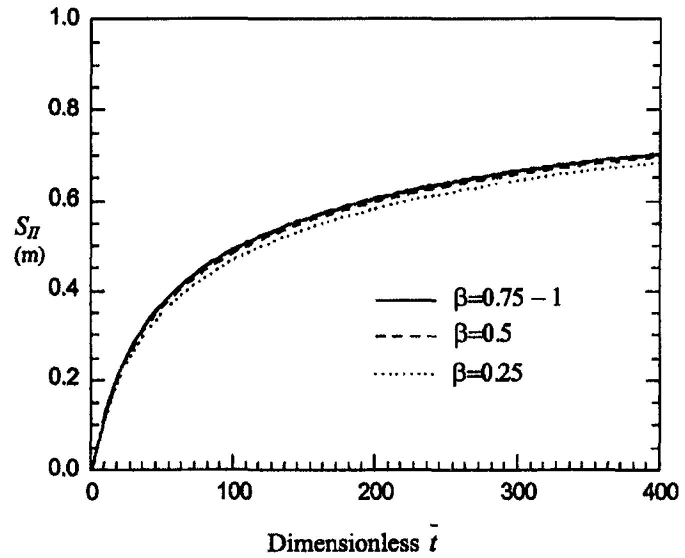

Figure 3. Effect of degree of penetration on the head changes in the well at the distance 4 w between stream axis and well located located in Zone II.

form of the hydrograph, and we will use several different hydrograph types.

Accuracy of the Simplified Solution (Effect of σ)

The simplified Equations 10 and 14 were obtained by neglecting the compressibility in Zone I ($\sigma = 0$). The accuracy of these solutions can be demonstrated for a unit-step stream-stage function (Equation 13) with a corresponding analytical solution (Equation 14). The results are compared with the "exact" solution (Equations 8 and 9) using the Stehfest (1970) algorithm for the well located at the distance of 4w in Zone II. The same response was calculated from Equation 14 using the range of σ values from 0.005 to 1 by changing storativity s from 0.001 to 0.2 and setting s_y unchanged. The results, shown in Figure 2, indicate that the approximation of incompressible Zone I yields high accuracy (relative error is better than 0.5%). Therefore, the storativity of the confined aquifer in Zone I can be neglected for practical purposes, and parameter σ can be set to zero.

Effect of Penetration (Parameter (β)

Parameter β describes the effects of the degree of aquifer penetration by the stream if one assumes the uniformity of hydraulic conductivity in Zone I and Zone II. In this case, $\beta < 1$ means a reduction of the saturated thickness of the aquifer in Zone I compared to Zone II according to the β definition based on transmissivity values of both zones. The sensitivity of this solution can be demonstrated for a unit step stream stage function (Equation 13) with corresponding analytical solution (Equation 14) which assumes $\sigma = 0$ according to the previous result. By changing T_I from 50 m^2/day to 200 m^2/day, one finds that the head response in the well located at the distance of 4w is virtually unchanged (Figure 3). Note that this analysis assumes shallow penetration ($\beta \approx 1$). Therefore, the model with $\sigma = 0$ and $\beta = 1$ can be a proper idealization of the general case.

Effect of the Stream Width (Parameter w)

Unlike the previously developed models of bank storage effect, the proposed model of stream-aquifer interactions accounts explicitly for stream width. For example, in the case of full stream pen-

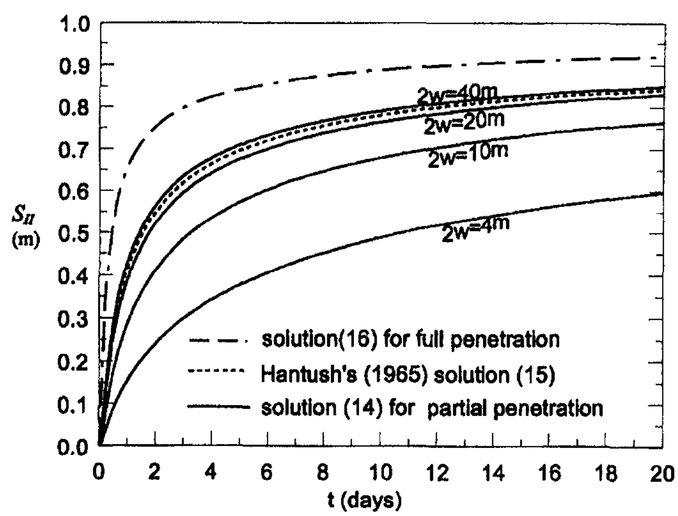

Figure 4. Effect of stream width on head changes in the well located at the distance of 20 m between stream bank and well located in Zone II.

etration, the response in the well to the unit step hydrograph is as follows (Ferris et al. 1962; Hall and Moench 1972):

$$S_{II} = S_0 \operatorname{erfc}\left(\frac{\bar{x} - 1}{\sqrt{4\bar{t}}}\right), S_0 = 1 \qquad (16)$$

and the Hantush (1965) formula (Equation 15) includes hydraulic conductivity and the thickness of streambed sediments only.

In order to assess the impact of stream width and to compare with the previous models, the same hypothetical stream is considered (see aforementioned dimensional parameters). Figure 4 illustrates water level changes in the observation well located at a distance 20 m from the stream bank. These changes were computed from Equation 14 for the unit step hydrograph. (Note, these curves were generated for wells located at the different distances from the stream axis.) Results demonstrate that the observed water level

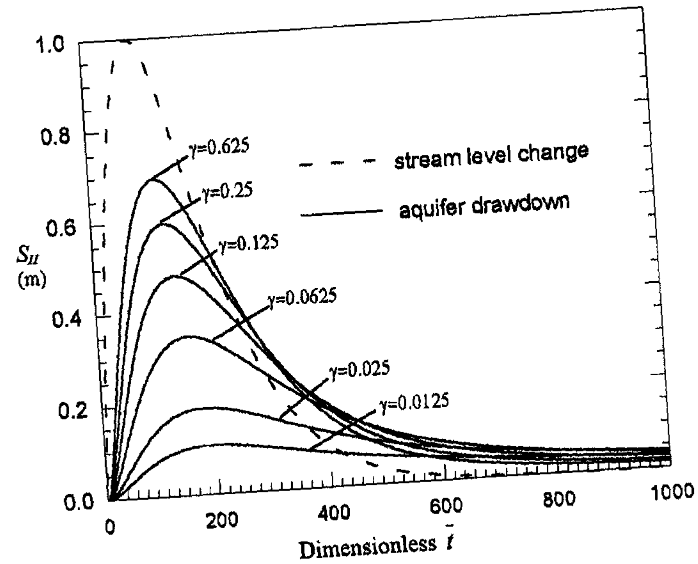

Figure 5. Effect of parameter γ on well response to the Gylybov (1977) hydrograph.

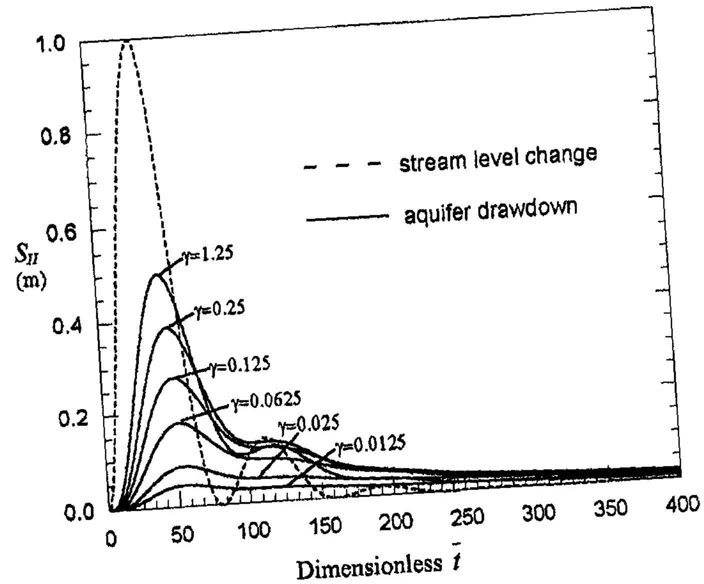

Figure 6. Effect of parameter γ on well response to the modified Cooper and Rorabaugh (1963) hydrograph.

changes are dependent on stream width, while water level changes from the Hantush (1965) formula do not depend on this parameter.

Effect of Stream-Aquifer Interface Parameter on Bank Storage Effect (Parameter γ)

The magnitude of bank storage effect strongly depends on the value of the Grigoryev parameter γ. It can be demonstrated using two different river stage hydrographs. One analytical form of the hydrograph was proposed by Gylybov (1977):

$$H(t) = H_0 c \left(\frac{t}{\tau}\right)^n \exp\left(-\frac{t}{\tau}\right)$$

$$n = 1, H_0 = 1, c = e = 2.7182..., \tau = 2 \text{ days} \qquad (17)$$

and the other form is a generalization of the Cooper and Rorabaugh (1963) hydrograph:

$$H(t) = H_0 \, c \exp\left(-\frac{t}{\tau_1}\right)\left[1 - \cos\left(2\pi\frac{t}{\tau_2}\right)\right] \qquad (18)$$

$$H_0 = 1, \tau_1 = 1 \text{ day}, \tau_2 = 2 \text{ days}$$

The results were computed using Equations 10 and 11, which neglect the compressibility of Zone I. Figures 5 and 6 indicate that an increase in the parameter γ enhances aquifer sensitivity to stream stage changes. The magnitude of the well response becomes comparable to a river stage if γ values are on the order of one. Note, the decrease of this parameter significantly smoothes out the well response to the stream stage changes.

Example

Let us consider a practical application of the methodology. We will determine the streambed conductance per unit length (2wK′)/m′, where all parameters are explained in Figure 1c. This parameter is an important factor in numerical modeling of stream depletion (Anderson and Woessner 1992).

Moench et al. (1974) used the Hantush model (Equation 15) for the North Canadian River in central Oklahoma. Retardation parameter a = 46 m was obtained from the convolution technique for the stream section between streamflow stations near Canton and El Reno. Aquifer transmissivity T_{II} = 330 m²/day was determined by a pumping test. The stream has a shallow penetration into the aquifer: $T_I = T_{II}$, thus β = 1 (Figure 1c). Using Equation 14, one can derive the value of conductance of the stream-aquifer interface if stream width is known.

Indeed, from comparison of Equations 14 and 15 it follows that $a/w = \xi = \omega \tanh \omega$, where for $\omega^2 = \gamma = (K'w^2)/m'T_{II}$ for β = 1. Using the aforementioned values of parameters a and T_{II}, and stream width w, one can solve the equations for parameters ξ and ω to yield values for stream conductance (2wK′)/m′.

Unfortunately, stream width is not a well-defined parameter during flood events at this particular stream section. Therefore, we assumed three values 2w = 32, 64, and 128 m, which are generally within range of observed data. Corresponding values of the stream conductance are 16.2, 18.4, and 24.1 m/day, respectively. These data on conductance can be used as a first approximation in numerical ground water flow models.

Conclusions

Commonly used analytical models of Theis (1941), Glover and Balmer (1954), Ferris et al. (1962), Cooper and Rorabaugh (1963), Hantush (1965), and Jenkins (1968) disregard the effects of the partial penetration of the stream and/or streambed clogging on aquifer response to stream-stage fluctuations. A new analytical solution has been developed to explicitly consider these effects. This solution uses the Dupuit assumptions to incorporate the effects of stream width, the hydraulic conductivity and thickness of streambed sediments, and the degree of penetration of the stream. When combined with a convolution algorithm, the solution can be used with any type of stream-stage hydrograph.

Evaluation of the solution indicates that stream-aquifer interactions are insensitive to the storage properties of the sub-stream aquifer zone, thereby significantly simplifying the required calculations. However, results of the evaluation also indicate that the dimensionless parameter of stream-aquifer interface γ, which incorporates the effects of stream width and the hydraulic conductivity and thickness of the streambed sediments, plays an important role in stream-aquifer interactions. Large values of this parameter ($\gamma >> 1$) correspond to a better hydraulic connection between the stream and the aquifer. These results indicate that wide streams will have a more pronounced effect on head changes in the aquifer, approaching the effect of a fully penetrating stream. Although it has not been recognized by Hantush (1965), the stream width is an important characteristic that should be incorporated into analyses of stream-aquifer interactions.

Appendix A: Derivation of Equation 8 for Finite Storativity and Specific Yield

For general case ($\sigma \neq 0$), Equations 1 through 6 in dimensionless variables (Equation 7) becomes

$$\sigma \frac{\partial S_I}{\partial \bar{t}} = \beta \frac{\partial^2 S_I}{\partial \bar{x}^2} + \gamma (S - S_I) \quad 0 < \bar{x} < 1,\ \bar{t} > 0 \qquad \text{(A1)}$$

$$\frac{\partial S_I}{\partial \bar{x}} = 0 \qquad \bar{x} = 0 \qquad \text{(A2)}$$

$$\frac{\partial S_{II}}{\partial \bar{t}} = \frac{\partial^2 S_{II}}{\partial \bar{x}^2} \qquad \bar{x} > 1, \bar{t} > 0 \qquad \text{(A3)}$$

$$S_{II} = 0 \qquad \bar{x} = \infty, \bar{t} > 0 \qquad \text{(A4)}$$

$$S_I = S_{II}, T_I \frac{\partial S_I}{\partial \bar{x}} = T_{II} \frac{\partial S_{II}}{\partial \bar{x}} \qquad \bar{x} = 1, t > 0 \qquad \text{(A5)}$$

$$S_I(\bar{x}, 0) = S_{II}(\bar{x}, 0) = S(0) = 0 \qquad \text{(A6)}$$

Applying Laplace transform and using Equation A6, one obtains

$$\frac{\partial^2 \bar{S}_I}{\partial \bar{x}^2} - \omega^2 \bar{S}_I = -\frac{\gamma}{\beta} \bar{S} \qquad 0 < \bar{x} < 1 \qquad \text{(A7)}$$

$$\frac{\partial \bar{S}_I}{\partial \bar{x}} = 0 \qquad \bar{x} = 0 \qquad \text{(A8)}$$

$$\frac{\partial^2 \bar{S}_{II}}{\partial \bar{x}^2} - p\bar{S}_{II} = 0 \qquad \bar{x} > 1 \qquad \text{(A9)}$$

$$\bar{S}_{II} = 0 \qquad \bar{x} = \infty \qquad \text{(A10)}$$

$$\bar{S}_I = \bar{S}_{II}, T_I \frac{\partial \bar{S}_I}{\partial \bar{x}} = T_{II} \frac{\partial \bar{S}_{II}}{\partial \bar{x}} \qquad \bar{x} = 1 \qquad \text{(A11)}$$

Using Equation A10, the solutions $\bar{S}_I(\bar{x},p)$ and $\bar{S}_{II}(\bar{x},p)$ can be represented as follows:

$$\begin{cases} \bar{S}_I(\bar{x},p) = Ae^{\omega(\bar{x}-1)} + Be^{-\omega(\bar{x}-1)} + (\gamma/(\beta\omega^2))\,\bar{S}(p) \\ \bar{S}_{II}(\bar{x},p) = Ce^{-\sqrt{p}(\bar{x}-1)}\,\bar{S}(p) \end{cases} \qquad \text{(A12)}$$

where unknown coefficients A, B, and C can be obtained from a system of linear equations using Equations A8 and A11. The solution of this system yields

$$\begin{cases} A = \dfrac{-e^{\omega}\gamma\sqrt{p}}{2\beta\omega^2(\beta\omega \sinh \omega + \sqrt{p}\cosh \omega)}\bar{S} \\ B = \dfrac{-e^{-\omega}\gamma\sqrt{p}}{2\beta\omega^2(\beta\omega \sinh \omega + \sqrt{p}\cosh \omega)}\bar{S} \\ C = \dfrac{\gamma \sinh \omega}{\omega(\beta\omega \sinh \omega + \sqrt{p}\cosh \omega)}\bar{S} \end{cases} \qquad \text{(A13)}$$

Equation A12 with coefficients (Equation A13) results in Equation 8.

Appendix B: Derivation of Equation10 for Negligible Storativity

In this case, $\sigma = 0$ and $\omega^2 = \gamma/\beta$ in the system of Equations A1 through A6 in Appendix A. Equation A1 becomes

$$\frac{\partial^2 S_I}{\partial \bar{x}^2} - \frac{\gamma}{\beta} S_I = \frac{\gamma}{\beta} S \qquad 0 < \bar{x} < 1, \bar{t} > 0 \qquad \text{(B1)}$$

Using Equations A2 and A5, one obtains

$$S_I(\bar{x},\bar{t}) = \frac{\cosh \omega\bar{x}}{\cosh \omega}\left[S_{II}(1,\bar{t}) - S(\bar{t})\right] + S(\bar{t}) \qquad \text{(B2)}$$

and

$$\frac{\partial S_{II}}{\partial \bar{x}}\bigg|_{\bar{x}=1} = \frac{\partial S_I}{\partial \bar{x}}\bigg|_{\bar{x}=1} = \xi(S_{II}\big|_{\bar{x}=1} - S), \xi = \omega \tanh \omega \qquad \text{(B3)}$$

where the function $S_{II}(1,\bar{t})$ has to be determined. This equation can be used as the boundary condition for the aquifer zone II. Applying Laplace transform to Equation B3, one obtains

$$\frac{\partial \bar{S}_{II}}{\partial \bar{x}} = \xi(\bar{S}_{II} - \bar{S}) \qquad \bar{x} = 1 \qquad \text{(B4)}$$

Applying Laplace transform to Equation A3, and boundary condition (Equation A4) and using Equation B4, one obtains

$$\bar{S}_{II}(\bar{x},p) = \frac{\xi}{\xi + \sqrt{p}} e^{-\sqrt{p}(\bar{x}-1)}\bar{S} \qquad \text{(B5)}$$

Equations 10 through 12 follow from Laplace transform tables (Carslaw and Jaeger 1959).

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References

Anderson, M.P., and W.W. Woessner. 1992. *Applied Groundwater Modeling*. San Diego, California: Academic Press.

Bochever, F.M. 1966, Evaluation of well-field yield in alluvial aquifers: The impact of a partially penetrating stream. In *Proceedings of VODGEO* (Hydrogeology) Moscow, no. 13, 84–115 (in Russian).

Bouwer, H., and T. Maddock III. 1997. Making sense of the interactions between groundwater and streamflow: Lessons for water masters and adjudicators. *Rivers* 6, no. 1: 19–31.

Carslaw, H.S., and J.C. Jaeger. 1959. *Conduction of Heat in Solids*. London: Oxford University Press.

Chambers, L.C., and J.M. Bahr. 1992. Tracer test evaluation of drainage ditch capture zone. *Ground Water* 30, no. 5: 667–677.

Conrad, L.P., and M.S. Beljin. 1996. Evaluation of an induced infiltration model as applied to glacial aquifer systems. *Water Resour. Bull.* 32, no. 6: 1209–1220.

Cooper, H.H., and M.J. Rorabaugh. 1963. Groundwater movements and bank storage due to flood stages in surface streams. U.S. Geol. Surv. Water Supply Paper 1536-J, 343–366.

Ferris, J.F., D.B. Knowles, R.H. Brown, and R.W. Stallman, 1962. Theory of aquifer tests. U.S. Geol. Surv. Water Supply Paper 1536–E, 126–131.

Glover, R.E., and G.G. Balmer. 1954. River depletion resulting from pumping a well near a river. *Trans. Am. Geophys. Union* 35, no. 3: 468–470.

Govindaraju, R.S., and J.K. Koelliker. 1994. Applicability of linearized Boussinesq equation for modeling bank storage under uncertain aquifer parameters. *J. Hydrology* 157, 349–366.

Grigoryev, V. M. 1957. The effect of streambed siltation on well-field yield in alluvial aquifers. *Water Supply and Sanitation* (Moscow), no. 6, 110-118 (in Russian).

Gylybov, M. 1977. Bank-storage dynamics due to rivers. *J. Hydrological Sciences* 4, no. 3-4: 206–215.

Hall, F.R., and A.F. Moench. 1972. Application of the convolution equation to stream-aquifer relationship. *Water Resour. Res.* 8, no. 2: 487–493.

Hantush, M.S. 1965. Wells near streams with semi-pervious beds. *J. Geophys. Res.* 70, no. 12: 2829–2838.

Jenkins, C. T. 1968, Techniques for computing rate and volume of stream depletion by the wells. *Ground Water* 6, no. 2: 37–46.

Larkin, R.G., and J.M. Sharp Jr. 1992. On the relationship between river-basin geomorphology, aquifer hydraulics, and ground-water flow direction in alluvial aquifers. *Geological Soc. America Bull.* 104, 1608–1620.

Meigs, L.C., and J.M. Bahr. 1995. Three-dimensional groundwater flow near narrow surface water bodies. *Water Resour. Res.* 31, no. 12: 3299–3307.

Minkin, E.L. 1973. *Groundwater-Surface Water Interactions and Their Role in Solutions of Problems of Applied Hydrogeology*. Moscow: Stroyizdat Press (in Russian).

Moench, A.F., V.B. Sauer, and M.E. Jennings. 1974. Modification of routed streamflow by channel loss and base flow. *Water Resour. Res.* 10, no. 5: 963–968.

Moore, I.E., and C.T. Jenkins. 1966. An evaluation of the effect of groundwater pumpage on the infiltration rate of a semipervious streambed. *Water Resour. Res.* 2, no. 4: 691–696.

Neuman, S.P. 1972. Theory of flow in unconfined aquifers considering delayed response of the water table. *Water Resour. Res.* 8, no. 4:, 1031–1045.

Pinder, G.F., and S.P. Sauer. 1971. Numerical simulation of flood wave modification due to bank storage effects. *Water Resour. Res.* 7, no. 1: 63–70.

Shestakov, V. M. 1965. *Theoretical Foundations of Evaluation of Afflux and Drainage*. Moscow, USSR: Moscow State University Press (in Russian).

Sophocleous, M.A., A. Koussis, J.L. Martin, and S.P. Perkins. 1995. Evaluation of simplified stream-aquifer depletion models for water rights administration. *Ground Water* 33, no. 4: 579–588.

Sophocleous, M.A., M.A. Townsend, L.D. Vogler, T.J. McClain, E.T. Marks, and G.R. Coble. 1988. Experimental studies in stream-aquifer interaction along the Arkansas River in central Kansas—Field testing and analysis. *J. Hydrology* 98, 249–273.

Spalding, C.P., and R. Khaleel. 1991. An evaluation of analytical solutions to estimate drawdowns and stream depletions by wells. *Water Resour. Res.* 27, no. 4: 597–610.

Stehfest, H. 1970. Numerical inversion of Laplace transforms. *Commun. ACM* 13, no.1: 47–49.

Theis, C.V. 1941. The effect of a well on the flow of a nearby stream. *Trans. Am. Geophys. Union* 22, no. 3: 734–738.

Vasilyev, S.V., N.N. Verigin, G.A. Razumov, and B.S. Sherzhukov. 1975. *Losses from Surface Water Reservoirs*. Moscow: Kolos Press (in Russian).

Wilson, J.L. 1993. Induced infiltration in aquifers with ambient flow. *Water Resour. Res.* 29, no. 10: 3503–3512.

Winter, T. 1995. Recent advances in understanding the interaction of groundwater and surface water. *Reviews of Geophysics, Supplement, July. U.S. National Report to International Union of Geodesy and Geophysics 1991–1994*, 985–994. Washington, D.C.: AGU.

Zlotnik, V.A., M. Kalinin, V. Usenko, and M. Cherepansky. 1985. *Forecasting Groundwater Use Influence on the Hydrogeological Environment*. Minsk, USSR: Nauka i Technika Press (in Russian).