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On the chaos synchronization phenomena

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Abstract

Chaos synchronization is an important problem in the nonlinear science. However, several phenomena can be found in the synchronization systems. Here, we discuss several phenomena involved with the chaos synchronization problem. Between the involved phenomena, one can find: Complete, Practical and Partial Synchronization. A feedback controller is used to illustrate such synchronization phenomena. The feedback was recently reported and involves robustness features. Such control actions can induce one more phenomena: the Almost Synchronization (AS). In addition, it is shown that the AS can be found if the master and slave models are strictly different. © 1999 Published by Elsevier Science B.V. All rights reserved.

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1. Introduction

Chaos control is a very interesting topic which has been recently studied. Two basic problems of the chaos control can be identified. Such basic problems are the following: (i) chaos suppression and (ii) chaos synchronization. Chaos suppression mainly consists in the stabilization of the system around regular orbits or equilibrium points. The chaos synchronization problem has the following feature: The trajectories of a slave system must tracks the trajectories of the master system in spite of both master and slave systems being different. There are some uncertainties sources in the chaos control problem. Therefore, robust control schemes are required to achieve suppression or synchronization (for instance, see [1,2]).

The chaos synchronization phenomenon was found early 90s. In [3], the authors presented the first chaos synchronization results regarding the unidirectional synchronization of two identical chaotic systems (i.e., two dynamical systems whose parameters are equal). They reported that as the differences between system increases, the synchronization is lost. Nevertheless, as the synchronization phenomenon has been understood, the researchers have found that synchronization of not identical systems can be achieved (see for instance [2] and references therein).

From the paper by Pecora and Carroll [3] onwards, many papers have been published. Since the synchronization phenomenon is very interesting and important, a lot of effort has been devoted to understand it. For example, Phase Synchronization in bidi-

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rectionally coupled systems was reported in [4]. This is, two nonidentical systems have the same phase but different amplitude. We have found that phase synchronization can be attained is several synchronization phenomena. For instance, the practical synchronization (see below) is a class of synchronization where the systems have same phase but different amplitude. Also, in the partial synchronization [5], the systems can have the same phase and different amplitude. Nevertheless, the relation between the different kinds of the chaos synchronization (and the scenarios to or between them) have not been addressed yet.

This work is focused on the chaos synchronization problem. The main goal is the classification of the phenomena involved with the chaos synchronization problem. The results are restricted to the case where the systems to be synchronized are unidirectionally coupled by feedback. However, general features are discussed. The discussed phenomena are: Complete Synchronization (CS), Partial Synchronization (PS), Practical Synchronization (PrS), Exact Synchronization (ES). In addition, the Almost Synchronization (AS) was found.

In order to present the synchronization phenomena a feedback control scheme was chosen. The controller has a simple (linear) structure. Indeed, it is represented by a Laplace domain equation, which implies that the feedback is linear. The controller contains three parts: (i) A term proportional to the control error, (ii) an integral action which provides stability around an equilibrium point and (iii) a quadratic integral action which yields a dynamic estimated value of the perturbing forces (for example, time-varying reference, internal perturbations or parametric variations). The scheme is so-called PII^2 controller (Proportional-Integral-Quadractic-integral). It has been shown that the chaos control problem can be addressed by means of the PII^2 controller [6].

Since the controller has a simple structure, hence, it is very easy to analyze its effect onto the chaotic system. The results allow to observe the different synchronization phenomena. In addition, the results show that it is possible to find a combination of phenomena in synchronized systems.

In this sense, the aim of the Letter is to discuss the main differences involved in the chaos synchronization. The main contribution consists in the geometrical characterization of the synchronization in such way that important features can be identified. Such phenomena can be found if the PII^2 controller is used. This is, the different synchronization types can be induced by the same class of feedback. The text has been organized as follows. The next section contains a brief presentation of the synchronization scheme. The synchronization phenomena are defined in Section 3. Also, the synchronization phenomena are presented via numerical simulations. The Section 4 contains some concluding remarks.

2. The synchronization scheme

It has been recently established that the synchronization problem can be seem as a chaos stabilization one [2]. This is, from the dynamics of the synchronization error (defined by $x_i = x_{i,\mathrm{M}} - x_{i,\mathrm{S}}$), a feedback control can be constructed in such way that controller is able to lead the trajectories of the synchronization error around the origin in spite of the differences between master and slave model.

Using the design algorithm reported in [6], a Laplace domain controller can be obtained. Such feedback controller is given by

$$u(s) = K_\mathrm{C}\left[1 + \frac{1}{\tau_\mathrm{I} s} + \frac{K_\mathrm{e}}{s(\tau_\mathrm{II} s + 1)}\right]e(s) \qquad (1)$$

where $s = \omega j$, $j = (-1)^{1/2}$, K_C, K_e, τ_I and τ_II are control parameters, which are chosen in such way that the closed-loop system is stable. $e(s) = y(s) - r(s)$ is the control error, $y(s)$ is the system output whereas $r(s)$ is the prescribed signal. In particular, the control goal is to lead the trajectories of the synchronization error to the origin.

The controller (1) contains three parts: (i) A proportional action, which is represented by the constant term K_C. (ii) An integral action, which is given by the term $K_\mathrm{C}/\tau_\mathrm{I}$S, and (iii) A dynamic compensator term, which becomes $K_\mathrm{e}/S(\tau_\mathrm{II} S + 1)$. The feedback (1) does not require prior knowledge about the system to be controlled. In this sense, it can be used to control any chaotic system. In fact, it only requires: (a) Knowledge about the reference signal and (b) On-line measurements of one available state (measurable). *It must be pointed out that the closed-loop scheme (dynamical system under control ac-*

tions) can be interpreted as a dynamical system with an algebraic constraint.

In particular, since the control law (1) was designed to stabilize a dynamical system in spite of modeling errors and perturbing forces, the dynamic equation given by the synchronization error system can be controlled at the origin. In what follows the several phenomena are discussed.

3. The synchronization phenomena

The synchronization phenomena are discussed in this section. It is shown that each synchronization phenomenon can be separately found into the same dynamical system, i.e., since each synchronization phenomenon has different nature; however, it can be displayed by the same dynamical system. In the next section, we show that a combination of the synchronization types can be presented by the same system.

3.1. Exact synchronization

Definition 1. *It is said that two chaotic systems are exactly synchronized if the synchronization error, $x_i = x_{i,\mathrm{M}} - x_{i,\mathrm{S}}$, exponentially converges to the origin. This implies that at a finite time $x_{i,\mathrm{S}} = x_{i,\mathrm{M}}$.*

Remark 2. *Several controllers have been reported in the literature under perfect assumption. This is, the synchronization model is exactly known (see for example [7] and references in [8]). The above definition is general. It involves open-loop as well as closed-loop control schemes. Nevertheless, it has been remarked by Aguirre and Billings [8] that, under control actions, the exact controller has the following practical difficulties: (i) The model describing the system dynamics should be available in order to compute the Lyapunov table and (ii) Even if a model of the system were available, relatively small uncertainties and/or disturbances could provoke chaos in spite of the Lyapunov table indicates that chaos has been suppressed. The latter difficulty can be avoided by means of feedback control schemes. However, the designer of a feedback controller can stumble with the difficulty related with modeling errors [9]. For example, the problem of the synchronization of two strictly different chaotic oscillators requires control in spite of the differences between master and slave system [2].*

Lemma 3. *Let $x_i = x_{i,\mathrm{M}} - x_{i,\mathrm{S}}$ be the synchronization error. Assume that the system $\dot{x} = f(x) + g(x)u$ (where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$ while $f(x)$ and $g(x)$ are known vector fields) represents the chaotic dynamic of the synchronization error. In addition, suppose that the system output (measured state) is $y = Cx$, where C is a vector which can be chosen such that $y \in \mathbb{R}$. Then, the trajectories exponentially converges to zero only if the feedback control force has perfect information of the dynamical system.*

Proof. In seek of clarity and without lost of generality, consider the following feedback control $u = [-f(x) + V(x;K)]/g(x)$, where $V(x;K) = K^{\mathrm{T}}x$ means the desired dynamics to be induced by the controller. Then, the closed-loop system (i.e., the synchronization error system under control actions) becomes $\dot{x} = V(x;K)$. Since $V(x;K) = K^{\mathrm{T}}x$ the control coefficients K are chosen such that the polynomial $s^{\rho} + K_1 s^{\rho-1} + \cdots + K_{\rho-1}s + K_{\rho} = 0$ be Hurwitz. In this sense, such control constant represents the convergence rate. The above controller would asymptotically steer the trajectories of the synchronization error system to zero at finite time t. □

It should be pointed out that the perfect controller given by $u = [-f(x) + V(x;K)]/g(x)$ can be taken into the form (1). To this end, Taylor linearization can be applied to the feedback. After that, Laplace operator is used to get the resulting controller.

Fig. 1 shows dynamics of the synchronization error (all states of the synchronization error system are Complete Exact Synchronized, CES). The master and slave oscillators are represented by the Lorenz equation. Note that the trajectories of the synchronization error system is leaded to zero. The synchronization was carry out by the following feedback: $u = [-f(x) + V(x;K)]/g(x)$. The controller was activated at $t = 50$ (i.e., $u = 0$ for all $t < 50$). Note that the such feedback have prior knowledge about the nonlinear functions $f(x)$ and $g(x)$. In consequence, such controller is so-called *ideal feedback* [1]. The initial conditions of the slave system were chosen as follows: $x_{\mathrm{S}}(0) = (-1.0,0.0,0.5)$. On the

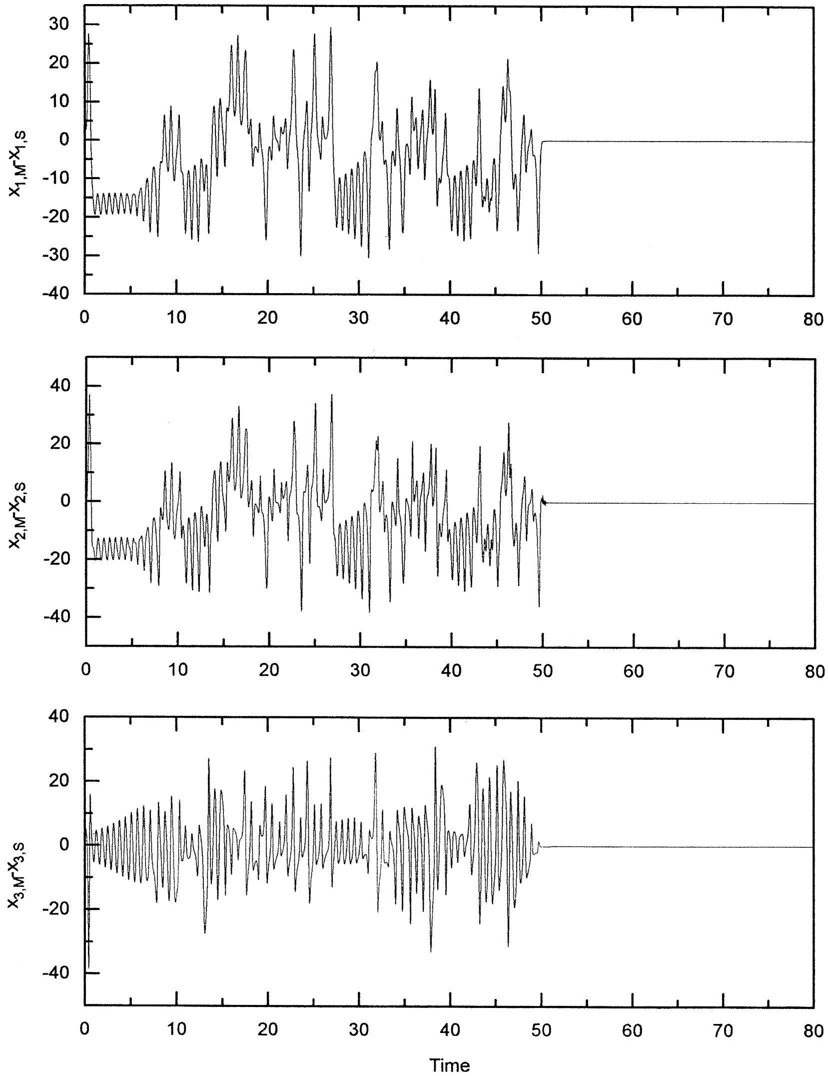

Fig. 1. Complete Exact Synchronization. The dynamics of the synchronization error converges to zero.

other hand, the initial condition of the Master system were chosen as: $x_M(0) = (10.0, -10.0, 10.0)$. In this case the desired dynamics was chosen linear and is given by $V(x;K) = K_1 x_1 + K_2 x_2$, The control constant values $K_1 = K_2 = 250.0$. Note that the trajectories of the synchronization error system converges exponentially to zero. Nevertheless, Note that under the perfect controller (exact synchronization) the feedback requires large parameters values (high-gain feedback control).

3.2. Practical synchronization

Definition 4. *It is said that two chaotic systems are practically synchronized if the trajectories of the synchronization error $x_i = x_{i,M} - x_{i,S}$, converges to a neighborhood around the origin. This implies that for all time $t \leq t^*$ the trajectories of the slave system are close to the master trajectories, i.e., $x_{i,S} \approx x_{i,M}$.*

Lemma 5. *Let $x_i = x_{i,M} - x_{i,S}$ be the synchronization error. Assume that the system $\dot{x} = f(x) +$*

g(x)u (where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$ while f(x) and g(x) are uncertain vector fields) represents the chaotic dynamic of the synchronization error. In addition, suppose that the system output (measured state) is $y = Cx$, where C is a vector which can be chosen such that $y \in \mathbb{R}$. Then, the controller (1) yields the trajectories of the synchronization error system to an arbitrarily small neighborhood, which contains the origin.

Proof. Defining the coordinates exchange $z = \Phi(x)$, the system $\dot{x} = f(x) + g(x)u$ can be rewritten as follows [10]:

$$\dot{z}_i = z_{i+1}; \quad i = 1,2,\ldots,\rho - 1$$
$$\dot{z}_\rho = \alpha(z,\upsilon) + \gamma(z,\upsilon)u$$
$$\dot{\upsilon} = \zeta(z,\upsilon) \tag{2}$$

where $\nu \in \mathbb{R}^{n-\rho}$ represents the unobservable states, $\alpha(z,\nu)$ and $\gamma(z,\nu)$ are unknown nonlinear functions and ρ is the relative degree (i.e., the lowest order time-derivative such that the control command is directly related with the output).

Following the idea reported in [2] (also see for high-order systems), one can construct a feedback which yields chaos control against the unknown functions $\alpha(z,\nu)$ and $\gamma(z,\nu)$. In particular, if $\rho = 1$ the feedback controller is given by the following equations

$$\dot{\hat{z}} = \hat{\eta} + u + L\kappa_1(z - \hat{z})$$
$$\dot{\hat{\eta}} = L^2\kappa_2(z - \hat{z})$$
$$u = [\hat{\eta} + K(\hat{z} - z^*)] \tag{3}$$

where $z = y \in \mathbb{R}$ represents the measured state (system output) of the synchronization error system, is a estimated value of the system output, $\hat{\eta} \in \mathbb{R}$ provides an estimated value of the uncertainties. The unique control parameter is denoted by L (which is positive defined) and K, κ_1 and κ_2 are constant.

Note that the controller (3) is linear, hence the Laplace operator can be used for its transformation to the equivalent form (1). Thus, one has the following equations

$$s\hat{z}(s) = \hat{\eta}(s) + u(s) + L\kappa_1[z(s) - \hat{z}(s)]$$
$$s\hat{\eta}(s) = L^2\kappa_2[z(s) - \hat{z}(s)]$$
$$u(s) = \hat{\eta}(s) + K[z(s) - \hat{z}(s)]$$

Then, by combining the above equations, the controller (1) can be obtained. Finally, since the controller (3) leads the trajectories of a chaotic system around the origin [2] (see also Appendix A in [3]), consequently the equivalent controller (1) leads the chaotic trajectories of synchronization error system around the origin. □

Remark 6. *The controller (1) leads the trajectories of the synchronization error system around zero, this implies that the master and salve are practically synchronized, i.e., $x_{i,\mathrm{S}} \approx x_{i,\mathrm{M}}$. Perhaps several controllers reported in literature can yield practical synchronization. Nevertheless, it has been previously reported that chaos can be controlled by means of the PII^2 feedback control [3]. In fact, the PII^2 feedback stabilizes the unstable periodic orbits (UPO's). In this sense, the PII^2 controller conserves the spirit of the most proposed control schemes. In addition, under the PII^2 controller, an arbitrary reference can be tracked. Notice that the Definition 4 implies the phase synchronization is locking, i.e., if the trajectories of the slave system are close of the master, then their phases are similar (see Fig. 2).*

Definition 7. *It is said that two chaotic system are completely synchronized if and only if all states of both master and slave systems are practically or exactly synchronized.*

Fig. 2 shows the dynamics of the synchronization error for the case of the Complete Practical Synchronization (CPS) of the Lorenz equation. The parameters values of the master system were chosen as follows: $\sigma_\mathrm{M} = 11.0$, $r_\mathrm{M} = 27$ and $b_\mathrm{M} = 2.57$. The slave parameters values were chosen as follows: $\sigma_\mathrm{S} = 10.0$, $r_\mathrm{S} = 28.0$ and $b_\mathrm{S} = 8/3$. The initial conditions were chosen as follows: $x_\mathrm{M}(0) = (1.0,1.0,10.0)$ and $x_\mathrm{S}(0) = (-1.0,10.0 - 1.0)$. The controller was activated at $t = 50$ (this is, for $t \le 50$, $u = 0$). The control parameter values were chosen as $K_\mathrm{C} = K_\mathrm{e} = 5.0$. The controller (1) steers the trajectories of the synchronization error around zero. This means that the synchronization is not exact. The phase of the synchronization error is locking. However, the

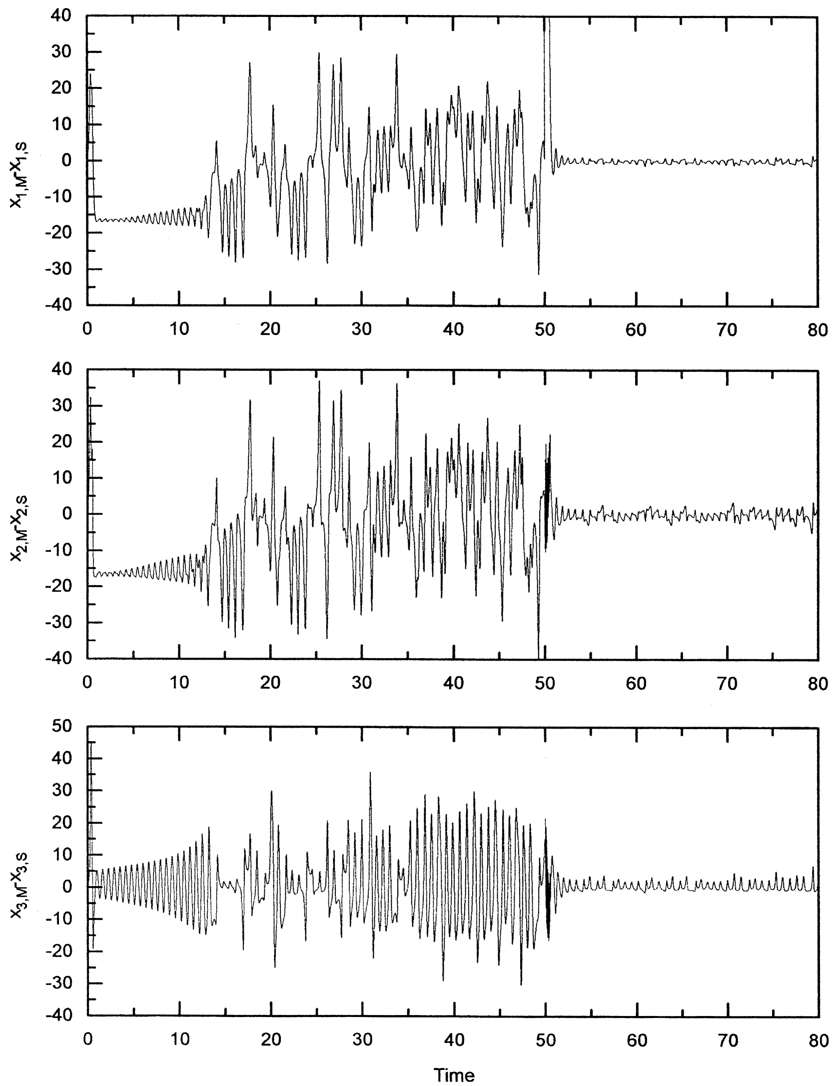

Fig. 2. Complete Practical Synchronization. The dynamics of the synchronization error converges around the origin.

amplitude of the slave system is not the same than the master. In fact, the difference between master and slave does not display a regular behavior. In this sense, the Definition 4 agrees with the previous results reported in [7]. Moreover, such definition provides a geometrical notion of this kind of synchronization (see below).

3.3. Partial synchronization

Definition 8. *It is said that two chaotic systems are partially synchronized if, at least, one of the states of the synchronization error system is either practically or exactly synchronized and only if, at least, one of the states of the synchronization error system is neither practically nor exactly synchronized.*

Remark 9. *Partial synchronization has been found in several systems. In particular, partial synchronization can be found in networks of identical oscillators (even in the absence of noise). Indeed, some results has been published where the open-loop* *partial synchronized has been studied [5], which contains the first evidence that the partial synchronization is related with the bifurcation parameter of the synchronization error system. Nevertheless, the definition of the partial synchronization is assumed.*

Lemma 10. *Let $x_i = x_{i,\mathrm{M}} - x_{i,\mathrm{S}}$ be the synchronization error. Assume that the system $\dot{x} = f(x) + g(x)u$ represents the chaotic dynamic of the synchronization error. Suppose that the master model is strictly different to the slave model. Besides, consider the feedback control (1). Then, there exists a control parameter value such that both master and slave chaotic systems are partially synchronized.*

Proof. The proof follows from the following fact: *As the control parameters increase, the controller (1) leads the trajectories to a neighborhood, which contains the origin. This is, since the controller (1) was obtained from the feedback (3) and such con-*

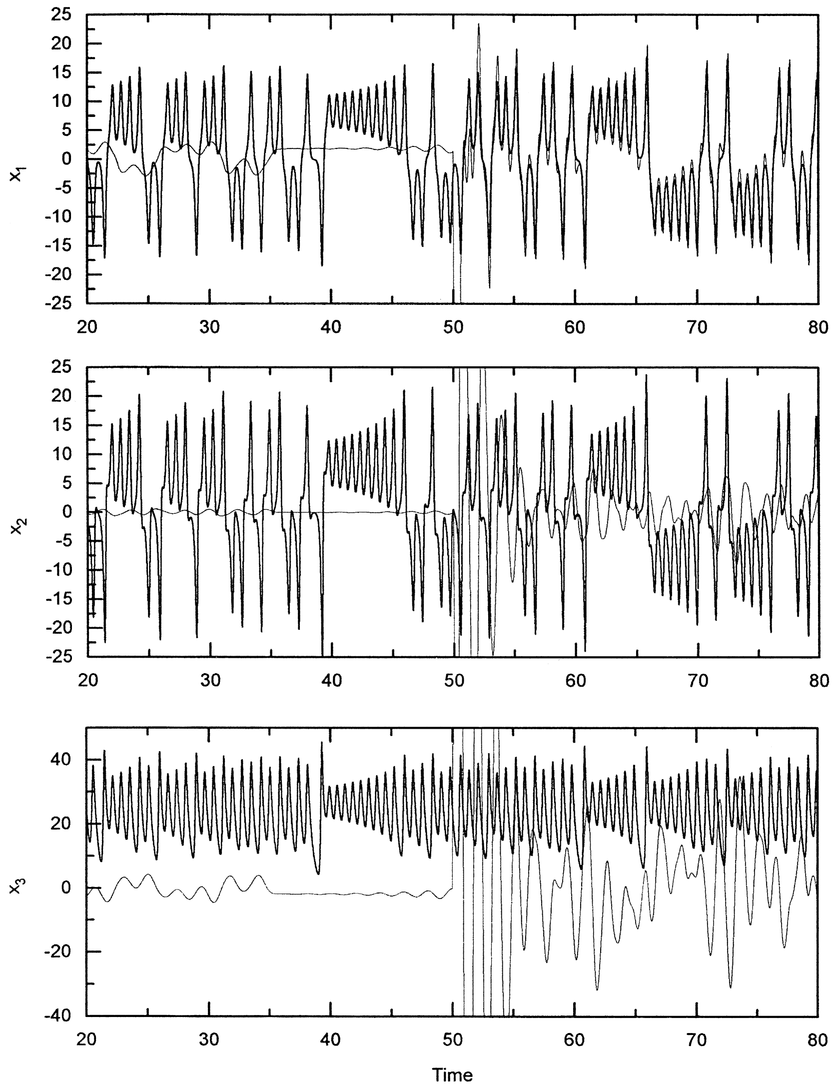

Fig. 3. Partial Practical Synchronization. The dynamics of, at least, one state of the synchronization error system is synchronous while, at least, another one is not synchronous. The thin-line is the dynamics of the slave system.

troller yields asymptotic convergence to a ball with radius $r = O(L^{-1})$ (for more details, see [1]). Hence, as the control constants decrease as the complete synchronization is lost. □

Remark 11. PII^2 *controller has been chosen due to its simple structure. In fact it is a robust approach to the perfect controller used in Lemma 3. Finally, note that the partial synchronization cannot be yielded by the ideal feedback due to it cancels the nonlinearities $f(x)$ and $g(x)$. In this sense, the PS phenomena is induced by the robust features of the feedback.*

Fig. 3 shows the Partial Practical Synchronization (PPS). In this case, the Lorenz system was chosen as the master system whereas the slave system is given by the Chua oscillator (i.e., the synchronization of two strictly different systems). The parameters values of the master system were chosen as follows: $\sigma = 10.0$, $r = 28.0$ and $b = 8/3$. The slave parameters values were chosen as follows: $g_1 = 10.0$ and $g_2 = -14.87$, $\alpha = -1.27$ and $\beta = -0.68$. The initial conditions were arbitrarily chosen as follows: $x_{1,\mathrm{M}} = x_{1,\mathrm{S}} = 0.074$, $x_{2,\mathrm{M}} = x_{2S} = -0.023$ and $x_{1,\mathrm{M}} = x_{1,\mathrm{S}} = -0.063$. The controller was chosen as the feedback (1) and the control parameters values were chosen as follows: $\tau_{\mathrm{I}} = \tau_{\mathrm{II}} = 1.0$, $K_{\mathrm{C}} = K_{\mathrm{e}} = 15.0$. The controller was activated at $t = 50$ (i.e., $u = 0$ for $t \leq 50$). In such case, the first state, $x_1 = x_{1,\mathrm{M}} - x_{1,\mathrm{S}}$, has been synchronized. Note that the dynamics of the second state ($x_2 = x_{2,\mathrm{M}} - x_{2,\mathrm{S}}$) and the third one ($x_3 = x_{3,\mathrm{M}} - x_{3,\mathrm{S}}$) are not synchronous.

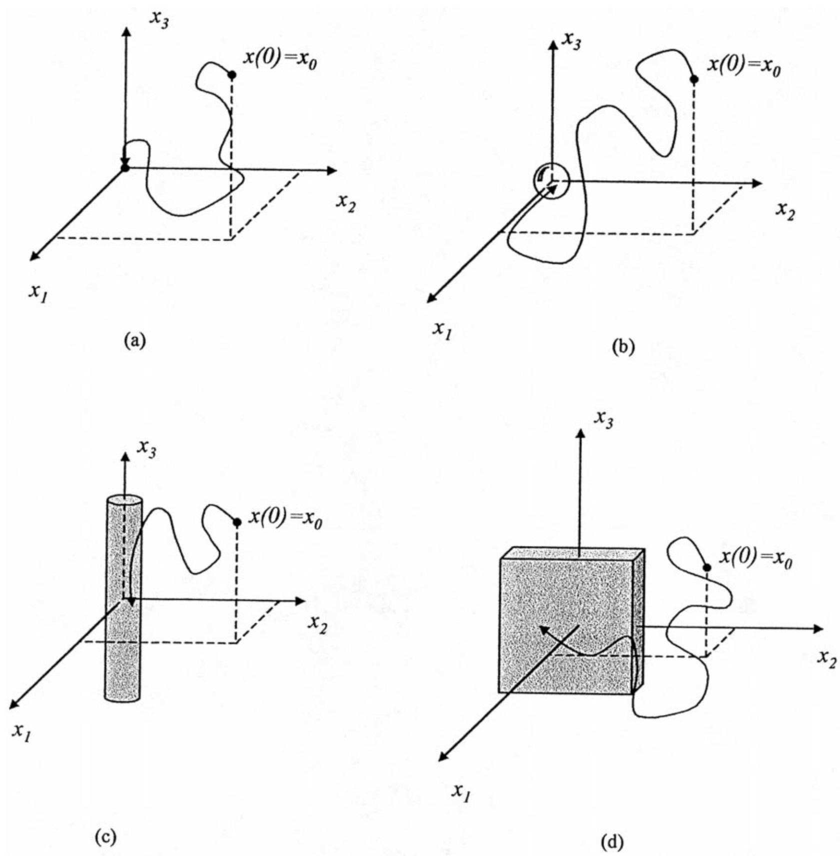

Fig. 4. Geometrical interpretation of the synchronization phenomena. (a) The Complete Exact Synchronization. (b) The Complete Practical Synchronization. (c) Only one state of the synchronization error system does not converge to zero. (d) The synchronization has been lost by two states of the synchronization error system.

A schematic representation of the geometrical interpretation of the Exact Complete Synchronization is illustrated in the Fig. 4.a. In such case the trajectories of the synchronization error converges to the origin. The Fig. 4.b shows the geometrical interpretation of the Practical Complete synchronization, Here, the synchronization error dynamics converges to a ball (represented by a *sphere*), which contains the origin. On the other hand, Fig. 4c shows the schematic interpretation of the Partial Practical Synchronization. In this case, the trajectories are leaded to a ball. The ball has been represented by a *cylinder* (if only one state of the error system is not synchronized) or by a sheet (if two states of the error system are not synchronized, see Fig. 4d). The idea behind the interpretation is as follows: *If the complete synchronization is lost, then the ball is deformed in any direction.* Of course, it could be interesting to study

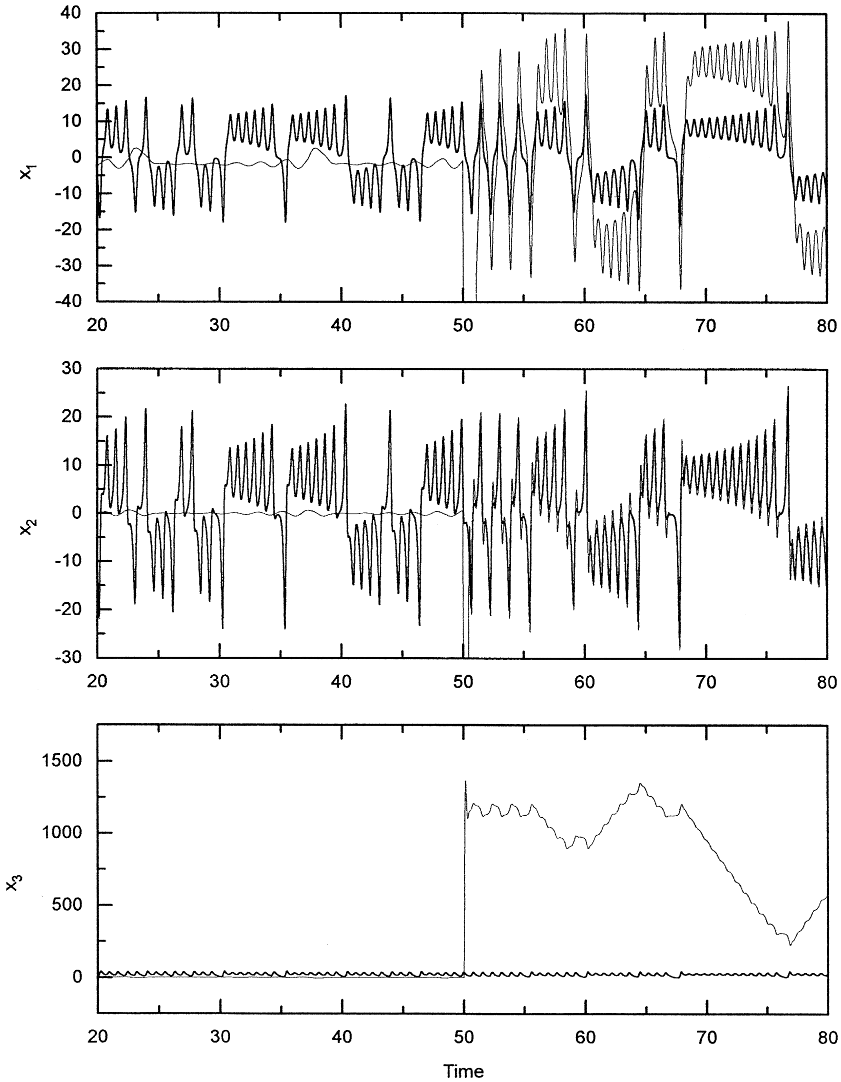

Fig. 5. Almost Practical Synchronization. One state of the synchronization error system is practically synchronized while the second slave state has the same phase than the corresponding master state. Finally, the last one is not synchronous. The thin-line is the dynamics of the slave system.

such deformation; however, such goal is beyond this paper. Results in this direction are under study and will be reported elsewhere.

3.4. Almost synchronization

The controller (1) is able to suppress the chaotic behavior of the synchronization error system. In this way the synchronization of both master and slave system can be achieved. However, the control constants, K_C, K_e, τ_I and τ_{II}, can be interpreted as bifurcation parameters (due to for different values of such parameters one can found different dynamical behavior of the synchronization error system). This is, if the control parameters change the synchronization can be lost into some states of the slave system. Even if the master and slave system are strictly different, the controller (1) is able to carry out the chaos synchronization. This is due to the controller was designed in such way that, in spite of the model errors, the controller can stabilize the synchronization error trajectories around the origin.

Synchronization of nonlinear systems is a very important and interesting problem, which is being widely studied. On the contrary, asynchronous states have received less attention (a few papers can be found in the literature, for instance see [11] and references therein). Nevertheless, such phenomenon is very important and interesting. For example, in a healthy piece of brain tissue, asynchronous states of the neurons (which can be modeled as coupled nonlinear oscillators) can be characteristic of epileptic activity [11,12]. Here, it is shown, via numerical simulations, that nonlinear systems can display synchronous as well as asynchronous states.

Definition 12. *It is said that two chaotic systems are almost synchronized if and only if the both master and slave systems display oscillations with the same phase and different amplitude.*

Remark 13. *Notice that the above definition does not exclude neither almost partial synchronization nor the almost complete synchronization. This is, in principle, it is possible to find the Almost Partial Synchronization (APS). Indeed, the APS can be induced by the controller (1). Fig. 5 shows the synchronization of the Lorenz system and the Chua oscillator. The Lorenz system was chosen as master whereas the Chua oscillator becomes the slave system. The parameter values of the Lorenz system were chosen as follows:* $\sigma = 10.0$, $r = 28.0$ *and* $b = 8/3$. *The parameter values of the Chua oscillator were selected as:* $g_1 = 10.0$, $g_2 = -14.87$, $\alpha = -1.27$ *and* $\beta = -0.68$. *The initial conditions were arbitrarily chosen as follows:* $x_{1,M} = x_{1,S} = 0.074$, $x_{2,M} = x_{2,S} = -0.023$ *and* $x_{3,M} = x_{3,S} = -0.063$. *Note that the state* $x_1 = x_{1,M} - x_{1,S}$ *is practically synchronized while the state* $x_2 = x_{2,M} - x_{2,S}$ *is almost synchronized and the state* $x_3 = x_{3,M} - x_{3,S}$ *is not synchronized.*

4. Concluding remarks

In this Letter, we have analyzed the synchronization phenomena. The synchronization phenomena were characterized in five types: (i) Exact, (ii) Practical, (iii) Complete, (iv) Partial and (v) Almost synchronization. To this end a feedback control structure was used. The feedback structure holds the main features of several chaos control schemes. In addition, in order to illustrate each synchronization type numerical simulations were performed. Each kind of synchronization has been previously reported. For example, the practical synchronization has been reported in [2] and [13]. Or, for instance, it has been proved that partial synchronization can be found even in networks of identical nonlinear oscillators [5].

However, this paper contains evidence of the almost synchronization phenomenon. Moreover, the characteristics of the synchronization phenomena were established. In addition, the main differences between the synchronization types were presented. The Exact, the Practical, the Partial and the Complete synchronization can be found if the master and slave model have the same structure, we propose the following property (which is under study and results in this direction will be reported as soon as possible).

Conjecture 14. *The almost synchronization can be found if and only if: (i) Both master and slave* *systems are strictly different, i.e., both the master and the slave model are different, and (ii) The control structure is based on feedback.*

Finally, the results reported in this letter show that it is possible to find that the combination of the synchronization phenomena can be displayed by the same system. Thus, the main factors are the control parameters.

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