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A Simple Deflection Model for Agricultural Tyres

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A simple geometrical theory is presented for pneumatic tyre deflection under load on a smooth, rigid, plane surface. The model, in terms of dimensionless parameters, is then modified to a dimensional, empirical form suitable for fitting to experimental data relating load, inflation pressure and deflection; and relating maximum permissible load to inflation pressure and tyre geometry. Experimental results for agricultural tyres are used to show that the theoretically-based, empirical equations fit the data well. The model shows promise of being adaptable to study tyre behaviour in soil.

1. Introduction

The pneumatic tyres used on tractors, trailers and other agricultural machinery provide such a convenient compromise between costs, life, adaptability to varying surfaces, traction, manoeuvreability and soil damage that they will remain the most common running gear on this machinery in the foreseeable future. Alternatives such as tracks or cage wheels are useful for specific circumstances, their main disadvantage being lack of adaptability.

Cost, life, adaptability, manoeuvreability and traction have long been important factors, influencing tyre design. Concern about compaction of soil by agricultural equipment has increased recently[1] and has begun to have some effects on the choice and design of tyres.

The purpose of providing the present model of tyre behaviour on a smooth, rigid, plane surface is to prepare the way for further study of tyre behaviour in soil. The shape and position of the tyre/soil interface results from an interaction between tyre and soil properties which are themselves not yet fully understood. The tread (lugs or ribs), the sidewalls and tyre carcase all contribute to supporting the wheel load of a tyre in soil. The various component areas make different contributions to soil compaction, tyre traction and tyre wear.

The tyre model in this paper does not relate tyre behaviour to detailed tyre construction. Rather, the emphasis is on providing a model which accurately represents static tyre behaviour on a hard surface for either radial or crossply tyres, and which shows promise of being adaptable for the study of static tyre behaviour in soil.

2. Dimensional analysis

A relationship is sought between vertical load, L, inflation pressure, p, and vertical deflection, d (*Fig. 1*). Tyre elastic properties will be indicated by k, with dimensions force/length (the "carcase stiffness"). The geometry of the tyre will initially be indicated by a diameter, D, and an equivalent diameter of curvature, C, of the cross-section. A functional relationship can be expressed in terms of dimensionless parameters as

$$\frac{L}{pd^2} = F\left[\frac{k}{pd}, \frac{d}{D}, \frac{d}{C}\right]. \qquad \ldots(1)$$

Dimensional analysis can lead no further than such a result, and the form of Eqn (1) must be decided either by experiment or by theoretical development.

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NOMENCLATURE

a_i	empirical constants
a	tyre contact area
A	an equivalent contact area, Eqn (2)
b_i	empirical regression constants
B	tyre section width
c_i	empirical constants
C	tyre cross section equivalent diameter of curvature
d	tyre deflection (undeflected less deflected diameter)
D	tyre diameter at root of tread on centreline
D	(in section 5) tyre overall diameter
F	an arbitrary function
H	tyre section height
$ijklm$, $i'j'k'l'm'$	points on *Fig. 1*
k	tyre elastic property (force/length)
L	vertical load on tyre
L_m	maximum permissible L
m, n	exponents in Eqn (8)
p	tyre inflation pressure
p'	mean value of contact stress
r	correlation coefficient
T	temporary variable representing either C or D

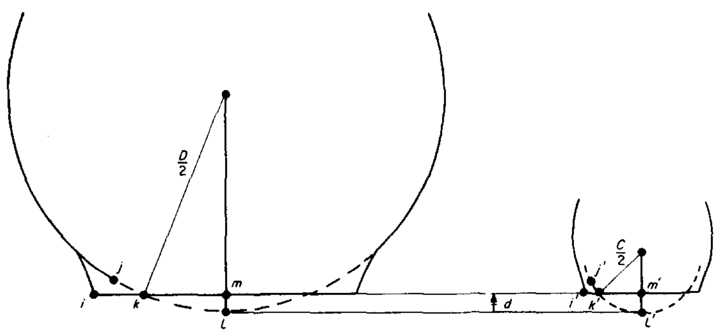

Fig. 1. Side elevation and cross section of an agricultural tyre carcase (tread not shown)

3. A simple geometrical model

Agricultural tyres may have lugs, ribs or tread superimposed on a radial or crossply casing.[2] On a smooth, rigid, plane surface it is these exterior tyre elements which make contact, and the load is supported by the normal stress integrated over the contact area, a. However, tyre inflation pressure is a more suitable parameter than some mean value of the contact normal stress, p', because the latter is not usually known. The inflation pressure does not act directly on the contact area, but it does produce a net vertical force to support that part of the load not supported by the elastic properties of the carcase. It is therefore appropriate to consider an "equivalent contact area",

$$A = \frac{p' a}{p} - \frac{k d}{p}. \qquad \ldots(2)$$

This term was used by Cooper,[3] who also confirmed the implied assumption that the total load, $L = p'a$, can be considered as the sum of an elastic load, kd, and a pneumatic load, pA. If A be replaced by an expression in d, D and C, Eqn (2) becomes a simple model of tyre deflection behaviour.

Fig. 1 shows the side elevation and cross-section of an agricultural tyre carcase (i.e. no lugs, ribs or tread are shown). The undeflected shape of the carcase is shown by broken lines. Three further assumptions are required to develop the model: that the equivalent contact area on which the inflation pressure acts may be represented by the contact area which the carcase alone would make on a smooth, rigid surface when deflected by d; that that contact area would be elliptical; and that an equivalent diameter of curvature of the cross-section can be defined. The equivalent contact area used here is a mathematical convenience, and not necessarily the same as the area obtained by circumscribing the contact pattern of the tread on a hard surface.

The half-length and half-width of the contact area (*Fig. 1*) are given by im and $i'm'$, respectively, where points j, j' originally above the contact plane have moved out and down to i, i'. At least for small deflections, j, j' will be close to k, k' so that im, $i'm'$ can be approximated by kl, $k'l'$. The arc kl has length

$$\frac{D}{2}\cos^{-1}\frac{\left(\frac{D}{2}-d\right)}{\frac{D}{2}} = \frac{D}{2}\cos^{-1}\left(1-2\frac{d}{D}\right),$$

and the arc $k'l'$ likewise has length

$$\frac{C}{2}\cos^{-1}\left(1-2\frac{d}{C}\right),$$

Thus, for the elliptical area,

$$A = \frac{\pi}{4} D\,C\cos^{-1}\left(1-2\frac{d}{D}\right)\cos^{-1}\left(1-2\frac{d}{C}\right). \qquad \ldots(3)$$

(According to Bekker,[4] Rotta[5] similarly described contact geometry in 1949.)

Substituting Eqn (3) into Eqn (2) and rearranging,

$$\frac{L}{pd^2} = \frac{k}{pd} + \frac{\pi}{4}\left(\frac{d}{D}\right)^{-1}\cos^{-1}\left(1-2\frac{d}{D}\right)\left(\frac{d}{C}\right)^{-1}\cos^{-1}\left(1-2\frac{d}{C}\right). \qquad \ldots(4)$$

The simple geometrical model has thus led to a particular form of Eqn (1) which can be expected to be accurate for small values of d/D and d/C.

4. Load related to deflection and inflation pressure

When d/D or d/C are not sufficiently small for Eqn (4) to represent accurately the deflection of real tyres, or when the assumptions are not fully met, the pneumatic last term of Eqn (4) can be generalized using an expression which fits it exactly for small d/D and d/C, and which allows for empirical fitting in a way which is linked to the simple theory. The equality (where T represents either C or D)

$$\left(\frac{d}{T}\right)^{-1}\cos^{-1}\left(1-2\frac{d}{T}\right) = a_1\left(\frac{d}{T}\right)^{-a_2} \qquad \ldots(5)$$

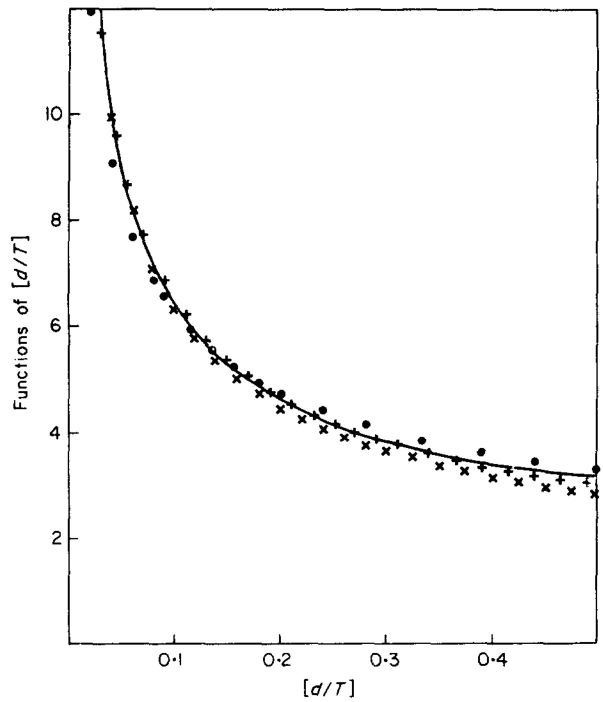

Fig. 2. The relationship between $(d/T)^{-1}\cos^{-1}(1-2\,d/T)$ *(full line) and* $a_1\,(d/T)^{-a_2}$; x $a_1=2$, $a_2=0{\cdot}5$; + $a_1=2{\cdot}15$, $a_2=0{\cdot}48$; ● $a_1=2{\cdot}5$, $a_2=0{\cdot}4$

holds exactly for small d/T when $a_1=1/a_2=2$, because the arc *kl* and the chord *km* (*Fig. 1*) are nearly equal. As d increases towards T, the equality can be approximately maintained by increasing a_1 and decreasing a_2. *Fig. 2* illustrates the goodness of fit when $a_1=2{\cdot}15$, $a_2=0{\cdot}48$; $a_1=2{\cdot}5$, $a_2=0{\cdot}4$ and $a_1=1/a_2=2$, for $0<d/T<0{\cdot}5$.

Values of d/D are likely to be less than 0·1 for many tractor front tyres and implement tyres, and less than 0·05 for many tractor rear tyres. d/C could take larger values, probably less than 0·2 for load/inflation pressure combinations permitted by tyre manufacturers.

A theoretically-based, empirical equation relating load to deflection and inflation pressure is obtained by substituting Eqn (5) into Eqn (4):

$$\frac{L}{pd^2}=\frac{a_0}{pd^2}+\frac{a_5}{pd}+\frac{\pi}{4}a_1\left(\frac{d}{D}\right)^{-a_2}a_3\left(\frac{d}{C}\right)^{-a_4}, \qquad \ldots(6)$$

wherein a_1 and a_3 are constants greater than 2, a_2 and a_4 are constants less than 0·5, by amounts which depend on the ranges of values of d/D and d/C, and a_0 has been introduced to allow for an intercept in empirical fitting. (a_0 could be thought of as a value of load which produces no measurable deflection at zero inflation pressure.) k has been replaced by a_5 to indicate that it might no longer have its usual meaning.

Eqn (6) is still in the form of the dimensionless parameters of Eqn (1) and may be used with any consistent set of units. One use of it will be to interpolate and extrapolate experimental results, when a limited set of L, p, d measurements has been made. The values of a_i and perhaps

C will be unknown. The equation can readily be put in a dimensional form which allows data fitting by multiple regression:

$$L = a_0 + a_5 d + \frac{\pi}{4} p d^2 \, a_1 \, a_3 \, D^{a_2} \, C^{a_4} \, d^{-a_2} \, d^{-a_4}.$$

For a particular tyre (D, C constant)

$$L = a_0 + a_5 \, d + a_6 \, p d^{2-a_2-a_4}. \qquad \text{...(7)}$$

4.1. *Testing the deflection model*

Data from 6 agricultural tyres (Table I) of different constructions and sizes and one tyre carcase from which the tread had been removed[3] were used to test the model. Data for 2 tractor driving wheel tyres (one radial, one crossply), a trailer tyre and 2 implement tyres were supplied by B. D. Tipper.[6] The data for one tractor driving wheel tyre were measured at the SIAE.

TABLE I

Fit of tyre data to Eqn (7)*

Nominal size	*Tread*	*Best interaction*	%*XP*†	a_0 (*N*)	a_5 (*N mm*$^{-1}$)	a_6 (*mm*$^{a_2+a_4}$)	$a_1 a_3$‡	a_2+a_4§	*Ref.*
7·50–16	lug	$pd^{1\cdot3}$	99·4	36·5	32·9	333·5	6·2	0·7	6
7·50–16	rib	$pd^{1\cdot3}$	99·8	−57·1	27·5	292·0	5·5	0·7	6
12·00–18	rib	$pd^{1\cdot5}$	99·7	335·5	80·5	142·7	8·0	0·5	6
9·00–20	none	$pd^{1\cdot2}$	99·9	−999·8	170·3	553·1	5·1	0·8	3
13·6–38	lug	$pd^{1\cdot3}$	99·0	61·0	16·0	448·0	5·6	0·7	
16·9–30	lug	$pd^{1\cdot3}$	98·7	506·7	50·3	647·3	7·7	0·7	6
16·9R–30	lug	$pd^{1\cdot3}$	99·8	259·0	80·6	465·5	5·6	0·7	6

* L, newtons; d, mm; p, N mm^{-2}
† The sum of squares L attributable to the regression as a percentage of the total sum of squares of L, i.e. 100 r^2
‡ Expected value, greater than 4
§ Expected value, less than 1

A generalized form of Eqn (7) and other regression models were fitted to the data using a standard multiple regression computer program. The forms of equations used were

$$L = b_0 \, d^{b_1} \, p^{b_2} \quad (\ln L = \ln b_0 + b_1 \ln d + b_2 \ln p)$$

and ...(8)

$$L = b_0 + b_1 \, d + b_2 \, p^m \, d^n$$

where $0 < m < 2$; $0 < n < 2$. For each tyre, the best fit was obtained using Eqn (8) with $m = 1$ [thus equivalent to Eqn (7)] and $1 < n < 1\cdot5$. The results for all 7 tyres are summarized in Table I. *Fig. 3* is a lattice diagram[2] illustrating the results for the 9·00–20 tyre with no tread. Data for this tyre had to be read from a published graph[3] and it is possible that some "smoothing" took place.

Fig. 4 shows the data measured at the SIAE, for a 13·6–38 rear tractor tyre on 2 different occasions. The weighbridge determination of L was to $\pm 0\cdot5$ kN accuracy, and tyre inflation pressure to about $\pm 0\cdot02$ bar. Measurement of d is least accurate; it is the difference between a laden and an unladen radius and each of these depends on the fitting of the tyre to the rim, the uniformity of the tyre itself, and whether the radius measured lies along a lug, or in between

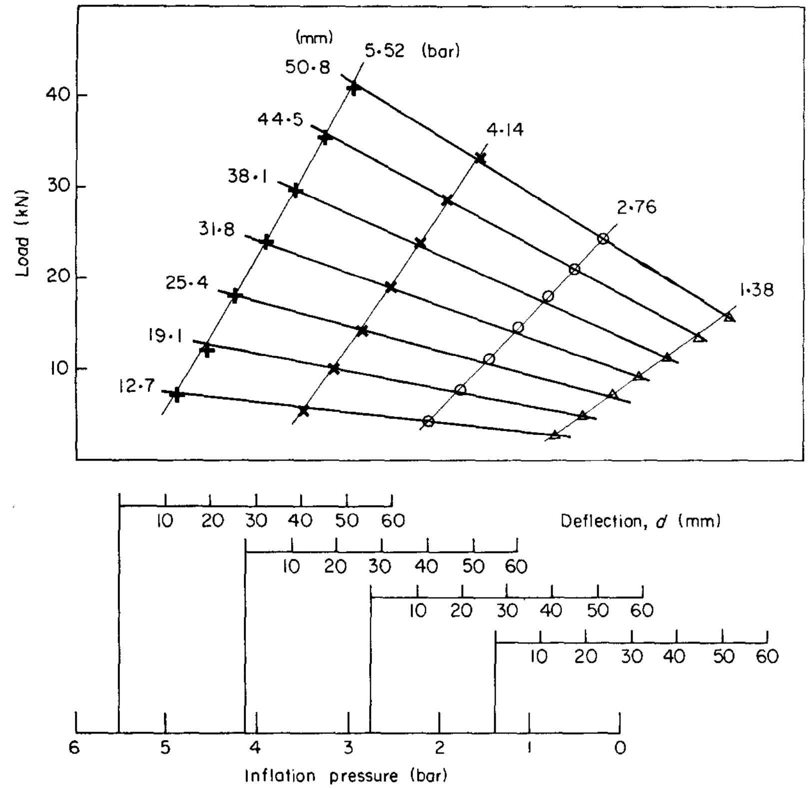

Fig. 3. Lattice diagram relating tyre load to deflection and inflation pressure for a 9·00–20 tyre without tread. Full lines from Eqn (7). Symbols represent experimental data from Cooper[3]

2 lugs. d was measured to an accuracy of perhaps ± 10 mm. The possible errors in d alone are great enough to explain all the observed differences in *Fig. 4* between predicted and measured values. Most of the differences are much less than could be expected from the comparatively simple measuring methods used.

5. Maximum load related to tyre pressure and geometry

To protect tyres from carcase damage manufacturers specify maximum permissible loads, L_m, which are functions of tyre geometry, inflation pressure, maximum wheel speed and nature of the loading. Although such "load capacities" in tyre catalogues will often be decided as a compromise between commercial advantage and service complaints, their more rational basis lies in restricting deflections to some fraction of section height, H. H is not always given in catalogues, but can be derived from the overall tyre diameter and the rim diameter. The catalogue overall diameter is greater than the D of *Fig. 1* by twice the tread height, but this difference will usually be less than a few percent. The equivalent diameter of curvature, C in *Fig. 1*, has not been precisely defined but it is of order B, the section width, and will be taken as equal to B multiplied by a constant.

It has been suggested[2] that d/H should be restricted to about 0·2. Putting $d/H = c_1$, Eqn (6) can be converted to the dimensional form

$$L_m = a_0 + a_5\, c_1\, H + \frac{\pi}{4} p H^2\, c_1^{\,2-a_2-a_4}\, a_1 a_3\, D^{a_2}\, C^{a_4}\, H^{-(a_2+a_4)}$$

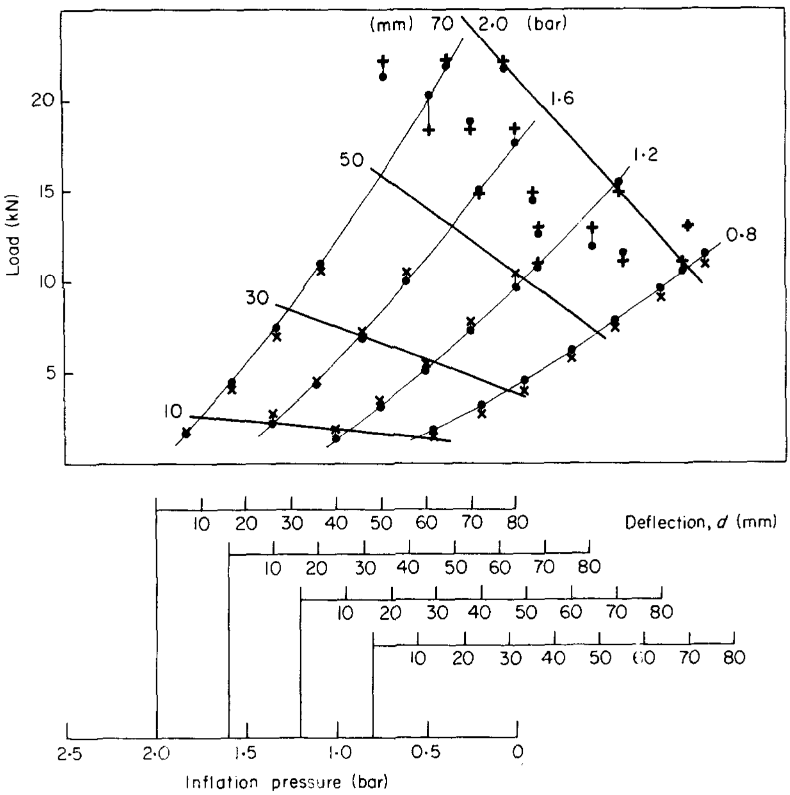

Fig. 4. Lattice diagram relating tyre load to deflection and inflation pressure for a 13·6/12–38 6-ply rear tractor tyre. Full lines from Eqn (7); + and × measured values on 2 occasions; difference between measured and predicted load at the same p and d

$$= a_0 + c_2 \left(\frac{H}{B}\right) B + c_3\, p \left(\frac{H}{B}\right)^{2-(a_2+a_4)} B^{2-a_2} D^{a_2} \qquad \ldots(9)$$

(H/B is the aspect ratio).

Alternatively it could be suggested that the permissible deflections should be less at the higher inflation pressures (corresponding to higher maximum permissible loads), because of the greater risks from higher dynamic loads.

A simple assumption is

$$\frac{d}{H} = c_4\, p^{-c_5},$$

wherein c_4, c_5 are positive constants. In this case Eqn (6) can be converted to the dimensional form

$$L_m = a_0 + c_6 \left(\frac{H}{B}\right) B p^{-c_5} + c_7 p^{\,1-2c_5+c_5(a_2+a_4)} \left(\frac{H}{B}\right)^{2-(a_2+a_4)} B^{2-a_2} D^{a_2}. \qquad \ldots(10)$$

Either of Eqns (9) or (10) could be used to examine the relationship between L_m and p, B, D, H/B when the data have been established according to some rule on deflections like the assumptions for d/H which led to them.

5.1. *Testing the maximum load model*

Values of L_m, p, B, D and H/B taken from an agricultural tyre manufacturer's engineering data book[7] were used to test Eqns (9) and (10). The values, which are based on data sheets of the European Tyre and Rim Technical Organisation, also conform to the relevant British Standard, and so a similar result could probably have been obtained using values from the British Standard.[8]

Values of a_2 and a_4 were estimated for tractor driving wheel tyres from the data for the last 3 tyres in Table I. $a_2 = a_4 = 0{\cdot}35$ were selected. One hundred and twenty-six sets of values of L_m and p were taken from the data book for 15 tractor driving wheel tyres ranging from 23·1–26 to 11·2–28. The regression equation

$$L_m = b_0 + b_1 B + b_2\, p\, B^{1{\cdot}65}\, D^{0{\cdot}35} \qquad \text{...(11)}$$

was fitted to the data. Eqn (11) is derived from Eqn (9), but H/B has been omitted as it had a restricted range in the data ($0{\cdot}80 < H/B < 0{\cdot}87$), with two-thirds of the values between 0·86 and 0·87. For units N, mm, N mm^{-2},

$$L_m = -4620 + 31{\cdot}6B + 0{\cdot}00268\, p\, B^{1{\cdot}65}\, D^{0{\cdot}35} \qquad \text{...(12)}$$

fitted the data well, with a correlation coefficient of 0·989 (97·8% XP, see notes to Table 1).

Relaxing the constraints of the present model so that the permissible load data could be fitted more closely was not deemed as important for present purposes as fitting the model to the load–pressure–deflection behaviour described in section 4, and so an exhaustive, unrestricted multiple regression analysis was not pursued. However, it was established that

$$L_m = -4679 + 31{\cdot}2\, B + 0{\cdot}0000156\, p\, B^{1{\cdot}3}\, D^{1{\cdot}35} \qquad \text{...(13)}$$

fitted the data better than equations with other powers of B and D, with a correlation coefficient of 0·998 (99·7% XP). This was a slightly better fit than could be obtained using a term in $p^{0{\cdot}585}$, akin to the Hoover and Steiner and Söhne[9] formulae (see Discussion, section 7.2). An equation

$$L_m = 104 + 0{\cdot}0096\, p^{0{\cdot}585}\, B^{1{\cdot}4}\, D^{0{\cdot}8} \qquad \text{...(14)}$$

gave a correlation coefficient of 0·998 (99·6% XP). The fit of Eqns (13) and (14) to the manufacturer's data is illustrated in *Fig. 5*.

6. Contact areas and stresses

Lug, rib or tread contact area on a hard surface, a, is relatively simple to measure and tyre "footprints" were measured 30 years ago[10] if not earlier. If load is measured, the mean value of the contact normal stress, p', can be calculated. Eqn (3) should represent quite accurately the area obtained by circumscribing the contact area of a shallow-ribbed implement tyre on a hard surface. It is unlikely that values derived from

$$A = \frac{\pi}{4} a_1\, a_3\, D^{a_2}\, C^{a_2}\, d^{2-(a_2+a_4)}$$

which is obtained from Eqn (6), would represent very well the areas obtained by circumscribing the contact area of other agricultural tyres, especially once the side walls begin to bulge to appreciably greater width than the contact area width (compare the assumptions in section 3).

However, the problem of more interest to many who use and study agricultural tyres is their interaction with surfaces which are not smooth, plane or rigid; soil surfaces in particular. Use of the present model should allow values of relevant empirical constants (e.g. a_0 to a_5) to be derived from measurements on a smooth, rigid, plane surface, as Cooper[3] has already suggested, although he used a different approach in estimating them.

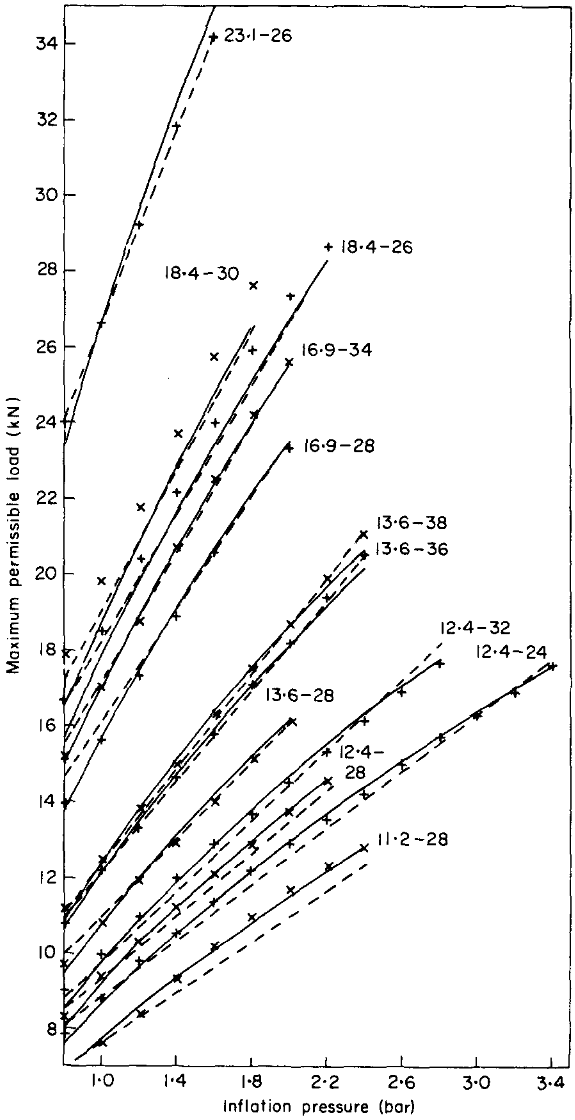

Fig. 5. The fit of Eqn (13) (broken lines) and Eqn (14) (full lines) to a manufacturer's data[7] (×, +). *(3 tyres omitted for clarity of presentation)*

What remains to be determined is whether an equation of form [cf. Eqn (2)]

$$L = pA + kd$$

still holds for a tyre in soil, and if so, how A and k change compared with their values on a hard surface. Also of considerable interest, for example in soil compaction studies, is how the load is distributed between the carcase base, the sidewalls and the lugs, tread or ribs when the tyre is operating in a soil rut. Although some direct measurements of tyre/soil interface stresses have been made,[11] they have been limited to either smooth tyres, or to measurements on particular tyre elements such as lugs.

7. Discussion

7.1. *Load, pressure and deflection*

The theoretically-based, empirical equation (6) was tested by fitting it in dimensional form [Eqn (7)] to data from 7 quite different tyres. The results summarized in Table I show that it fitted the data very well, and gave values of the empirical constants which were in agreement with the simple theory on which it was based. Comparison with the theory allows values of the product a_1a_3 and the sum (a_2+a_4) to be estimated. If it is assumed that $a_1=a_3$, $a_2=a_4$ in Eqn (6), then Table I shows that $2{\cdot}3<(a_1 \text{ or } a_3)<2{\cdot}8$ and $0{\cdot}25<(a_2 \text{ or } a_4)<0{\cdot}4$; values greater than 2 and less than 0·5, respectively, were expected from the theory.

The values of a_0 (the intercept terms) are nearly always small by comparison with the values of L to which they contribute. Even the largest value, for the 9·00–20 tyre, is less than 1 kN (absolute value) and other values range from 0·04 to 0·51 kN, compared with likely wheel loads from a few kN to a few tens of kN.

Considered merely as the coefficient of the second term in Eqn (7), a_5 has a magnitude which ensures that the contribution of the third (pneumatic) term to the total load is always much greater than that of the second (carcase) term, for all practical values of p and d. Although a_5 has units N mm^{-1}, and is analogous to k, the carcase stiffness, it cannot be claimed that it maintains the same physical meaning when Eqn (7) has been empirically fitted to data. Cooper[3] gave a value of $k=137$ N mm^{-1} for the 9·00–20 tyre, compared with 170·3 in Table I. Experimental curves were autographically drawn (using a hydraulically-operated testing machine) at zero inflation pressure for 5 of the tyres[6] in Table I. Although these give load–deflection curves somewhat sigmoid in shape, the gradients are, in all cases, between 16 and 108 N mm^{-1}, with the measured "k" value being less than the corresponding value of a_5 in Table I. In contrast, the absolute values of a_5 are all less than values for k given by Laib.[12] For 5 tractor driving wheel tyres he obtained values of about 150–250 N mm^{-1}. The reason for these differences might be of some interest to those involved in relating tyre performance to construction, but is of less importance for the present purposes.

The present model can be used for purposes other than those specifically mentioned so far in this paper. If the coefficients in Eqn (7) are known for a particular tyre, then the load being supported by that tyre can be estimated using a measuring tape, inflation pressure gauge and pocket calculator, by measuring vertical deflection on a hard surface. (But note the difficulties mentioned in section 4.1, as well as temperature effects.) This could be, for many tractor operators, a more acceptable method of working out wheel loading than calculations based on tractor and implement weights and dimensions, water ballast and wheel weights.[13,14] Use of such a method would also promote an awareness among tractor operators of tyre deflections and possibly reduce the occurrence of tyre damage due to excessive deflection. Representatives of some tyre manufacturers already advocate the use of static laden radius, given in manufacturers' data books, to check on whether tyre loading is acceptable at the appropriate inflation pressure.[15] An ADAS Mechanisation leaflet advocates and explains the use of deflection.[16] Lattice diagrams such as *Figs 3* and *4* could be used in much the same way if they were available, but many people unused to this way of presenting information would find them unwieldy to use.

7.2. *Maximum loads*

Eqn (11) was derived from Eqn (9), which in turn came from the theoretically-based empirical Eqn (6). Eqn (11) has been shown to fit the data from a manufacturer's catalogue[7] well, and would probably do likewise if tested against the British Standard.[8] Steiner and Söhne[9] have recently published empirical relationships for "load capacity" (L_m) in terms of p, B, D, H/B derived from data in the German standards for tractor rear and front tyres (DIN 7807, DIN 7808). Their fully empirical relationship for rear tractor tyres is

$$L_m = 110+1372\,p^{0\cdot593}\left(\frac{H}{B}\right)^{0\cdot756} B^{1\cdot437}\,D^{0\cdot820},$$

and for tractor front tyres is

$$L_m = -400+0{\cdot}59\,(p+1{\cdot}5)\,B^{0\cdot8}\,D^{1\cdot3}.$$

The load is in newtons, p in bars, B and D in cm. Other similar formulae are those attributed[6, 15] to C. G. Hoover in the 1930's whose influence has made its way via the U.S.A. Tire and Rim Association into manufacturers' load tables. One such formula had the form

$$L_m = C_8\,p^{0\cdot585}\,B^{1\cdot39}\,D$$

(D therein was equal to the sum of rim diameter and tyre width).

It is immediately obvious that Hoover's deliberations and multiple regression techniques using transformed variables to seek best fit have led to equations containing terms similar in form to the third terms in the theoretically-based empirical relationships, Eqns (9), (10). The first term is always a small contribution to L_m for Steiner and Söhne's data, as is the contribution of the first and second terms in Eqns (12) or (13). *Fig. 5* indicates that for practical purposes the theoretically-based empirical equations fit manufacturers' data as well as the fully empirical equations.

Better fit to experimental data can probably be obtained by unrestricted multiple regression such as Steiner and Söhne[9] have employed, and resulting equations like Eqn (15) will be useful for many purposes, including soil compaction studies. An advantage of a theoretically-based, empirical relationship such as Eqn (6), is that the derived relationships are physically meaningful and do not change in form or dimensions for widely differing tyres. They are thus more likely to be adaptable to study tyre behaviour in soil, when the model can be suitably modified to take into account the interaction between soil and tyre properties.

7.3. *Temperature and wear effects*

In this initial study, the dependence of the relationships obtained on the tyre temperature has not been ascertained. The magnitude of deflections occurring in use and their frequency cause temperature increases in the tyre which would result in tyre damage if not limited, as intended by manufacturers' load, inflation pressure and speed restrictions.

The tyres tested were all new, or nearly new. The major change expected with older tyres would be the changes in dimensions due to tyre growth and tread wear.

8. Conclusions

Eqn (6) is a theoretically-based, empirical equation which provides an accurate model of agricultural tyre load–pressure–deflection behaviour on a plane, rigid surface. It has been adapted to predict maximum permissible loads from tyre geometry and inflation pressure, given a rule relating maximum deflection to tyre geometry and inflation pressure. Wheel loads can also be estimated from vertical deflection and inflation pressure. The model has been constructed so as to provide a basis for extension to tyre/soil interfaces.

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