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Trajectory stabilization of a model car via fuzzy control

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Abstract

This paper deals with trajectory stabilization of a computer simulated model car via fuzzy control. Stability conditions of fuzzy systems are given in accordance with the definition of stability in the sense of Lyapunov. First, we approximate a computer simulated model car, whose dynamics is nonlinear, by T–S (Takagi and Sugeno) fuzzy model. Fuzzy control rules, which guarantee stability of the control system under a condition, are derived from the approximated fuzzy model. The simulation results show that the fuzzy control rules effectively realize trajectory stabilization of the model car along a given reference trajectory from all initial positions under a condition and the dynamics of the approximated fuzzy model agrees well with that of the model car.

Keywords: Fuzzy control; Trajectory stabilization; Model car; Takagi and Sugeno's fuzzy model

1. Introduction

Steering control for automobiles such as parallel parking is suitable for fuzzy control, because human control steering angle only by using his experiential knowledge without considering a mathematical model for automobile. Actually, Zadeh explained the idea of fuzzy algorithm [15] by showing an example of parking a car. Sugeno [7–9] first demonstrated that a real model car can be effectively controlled by fuzzy control rules derived from human experiential knowledge. Some papers [1, 6, 4] on fuzzy control of automobiles have been reported since Sugeno's papers were published.

Fuzzy control can be widely applied to more complicated, sensitive and dangerous objects such as nuclear reactor plants if we show that a designed fuzzy controller always works well in all situation. One of the possible ways for showing it is to guarantee stability of control system. In real industrial applications, in particular, in control of complicated, sensitive and dangerous objects, stability is required at least when designing control systems. Automobile robots have to be flexibly controlled because of restrictions such as real time obstacle avoidance and determination of moving path according to control situations. Trajectory stabilization of automobile robots is important, because out of trajectory control causes crashes. Stability

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analysis of fuzzy control systems has been difficult because fuzzy systems are essentially nonlinear systems. Recently, some useful stability techniques [5, 3, 14, 2, 13], which are based on nonlinear stability theory, have been reported. One of the authors derived stability conditions in accordance with the definition of stability in the sense of Lyapunov.

This paper deals with design of a fuzzy controller which guarantees stability of control system for a computer simulated model car, i.e., which effectively realize trajectory stabilization. As pointed out above, automobile robots control is actually difficult because of restrictions such as real time obstacle avoidance and determination of moving path according to control situations. In order to succeed trajectory control of automobile robots to the desired position, they often have to attempt backing, going forward, backing again, going forward again, etc. Thus, the forward and backward movements help to position mobile robots for successful trajectory control to the desired position. A more difficult trajectory control would only allow forward movements, with no backward movements permitted. The specific problem in this simulation is to control a computer simulated model car along the desired trajectory from an arbitrary initial position by manipulating the steering angle. Of course, only forward movements are allowed. Some papers [1, 4, 6–9, 15] have reported fuzzy control of automobile robots. However, as far as we know, these studies have never given guarantee of stability of the control system, i.e., trajectory stabilization. Our goal in this simulation is to design a fuzzy controller such that the control system is asymptotically stable in the large, i.e., such that the trajectory control can be perfectly achieved from all initial positions.

2. Model of a car

Fig. 1 shows a model car and its coordinate system used in this simulation. The model can be described as

$$x_0(k+1) = x_0(k) + vt/l \tan[u(k)], \tag{1}$$

$$x_1(k+1) = x_1(k) + vt \sin[x_0(k)], \tag{2}$$

$$x_2(k+1) = x_2(k) + vt \cos[x_0(k)], \tag{3}$$

where $x_0(k)$ is the angle of the car, $x_1(k)$ is the vertical position of the rear end of the car, $x_2(k)$ is the horizontal position of the rear end of the car, $u(k)$ is the steering angle, l is the length of truck, t is the sampling time, and v is the constant speed. In this paper, $l = 2.8$ (m), $v = 1.0$ (m/s), and $t = 1.0$ (s).

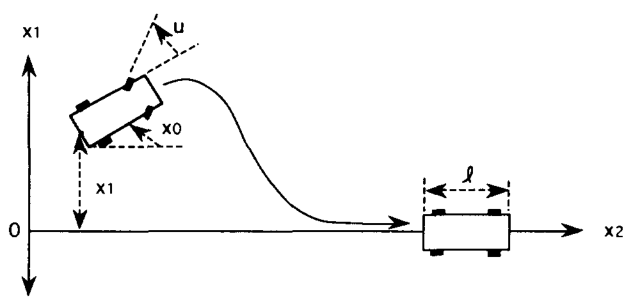

Fig. 1. A model car and its coordinate system.

3. Fuzzy controller design

3.1. Takagi and Sugeno's fuzzy model and stability analysis

The fuzzy system, proposed by Takagi and Sugeno [12], is described by fuzzy IF–THEN rules which locally represent linear input–output relations of a system. The ith rule of this fuzzy system is of the following form:

Rule i: IF $x_1(k)$ is A_{1i} and $\cdots$ and $x_n(k)$ is A_{ni}
THEN $\boldsymbol{x}_i(k+1) = \boldsymbol{A}_i\boldsymbol{x}(k) + \boldsymbol{B}_i\boldsymbol{u}(k), \quad i = 1, 2, \ldots, r,$

where

$$\boldsymbol{x}^{\mathrm{T}}(k) = [x_1(k), x_2(k), \ldots, x_n(k)],$$

$$\boldsymbol{u}^{\mathrm{T}}(k) = [u_1(k), u_2(k), \ldots, u_n(k)],$$

r is the number of IF–THEN rules. $\boldsymbol{x}_i(k+1)$ is the output from the ith IF–THEN rule, and A_{ij} is the fuzzy set. Given a pair of $(\boldsymbol{x}(k), \boldsymbol{u}(k))$, the final output of the fuzzy system is inferred as follows:

$$\boldsymbol{x}(k+1) = \frac{\sum_{i=1}^{r} w_i(k)\{\boldsymbol{A}_i\boldsymbol{x}(k) + \boldsymbol{B}_i\boldsymbol{u}(k)\}}{\sum_{i=1}^{r} w_i(k)}, \tag{4}$$

where

$$w_i(k) = \prod_{j=1}^{n} A_{ij}(x_j(k)),$$

$A_{ij}(x_j(k))$ is the grade of membership of $x_j(k)$ in A_{ij}.

The free system of (4) is defined as

$$\boldsymbol{x}(k+1) = \frac{\sum_{i=1}^{r} w_i(k)\boldsymbol{A}_i\boldsymbol{x}(k)}{\sum_{i=1}^{r} w_i(k)}. \tag{5}$$

Let us assume in this paper that

$$\begin{aligned} &\sum_{i=1}^{r} w_i(k) > 0, \\ &w_i(k) \geqslant 0, \quad i = 1, 2, \ldots, r \end{aligned} \tag{6}$$

for all k. Each linear consequent equation represented by $\boldsymbol{A}_i\boldsymbol{x}(k)$ is called "subsystem".

A stability condition, proposed by Tanaka and Sugeno, for ensuring stability of (5) is given as follows.

Theorem 1 [14]. *The equilibrium of a fuzzy system described by* (5) *is asymptotically stable in the large if there exists a common positive-definite matrix* $\boldsymbol{P}$ *such that*

$$\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A}_i - \boldsymbol{P} < \boldsymbol{0} \tag{7}$$

for $i = 1, 2, \ldots, r$, *i.e., for all the subsystems.*

We should notice that (7) depends only on $\boldsymbol{A}_i$. In other words, it does not depend on $w_i(k)$. This theorem is reduced to the Lyapunov stability theorem for linear discrete systems when $r = 1$. Theorem 1 gives, of course, a sufficient condition for ensuring stability of (5). We may intuitively guess that a fuzzy system is asymptotically stable in the large if all its subsystems are stable, i.e. if all its $\boldsymbol{A}_i$'s are stable matrices. However, this is not the case in general. We pointed it out in [14].

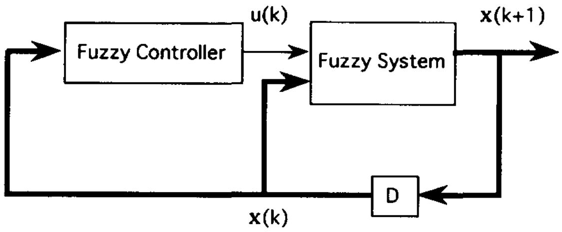

Fig. 2. Fuzzy control system.

Let us consider the fuzzy control system shown in Fig. 2. We use the following fuzzy controller in order to stabilize the fuzzy system (4).

Control Rule i: IF $x_1(k)$ is A_{1i} and $\cdots$ and $x_n(k)$ is A_{ni}
THEN $\boldsymbol{u}_i(k) = \boldsymbol{F}_i\boldsymbol{x}(k)$, $i = 1, 2, \ldots, r$.

The final output of this fuzzy controller is calculated by

$$\boldsymbol{u}(k) = \frac{\sum_{i=1}^{r} w_i(k)\boldsymbol{F}_i\boldsymbol{x}(k)}{\sum_{i=1}^{r} w_i(k)}, \tag{8}$$

where we use the same weight, $w_i(k)$, as the weight of ith rule of the fuzzy controller. Of course, the consequent matrices $\boldsymbol{F}_i$ are feedback gains in each control rule. By substituting (8) into (4), we obtain

$$\boldsymbol{x}(k+1) = \frac{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)w_j(k)\{\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{F}_j\}\boldsymbol{x}(k)}{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)w_j(k)}. \tag{9}$$

From (9),

$$\begin{aligned}\boldsymbol{x}(k+1) &= \frac{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)w_j(k)\{\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{F}_j\}\boldsymbol{x}(k)}{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)w_j(k)} \\ &= \frac{1}{R}\left[\sum_{i=1}^{r} w_i(k)w_i(k)\boldsymbol{G}_{ii}\boldsymbol{x}(k) + 2\sum_{i<j} w_i(k)w_j(k)\frac{\boldsymbol{G}_{ij} + \boldsymbol{G}_{ji}}{2}\boldsymbol{x}(k)\right],\end{aligned} \tag{10}$$

where

$$\boldsymbol{G}_{ij} = \boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{F}_j,$$

$$R = \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(k)w_j(k).$$

Without loss of generality, (10) can be rewritten as follows:

$$\boldsymbol{x}(k+1) = \frac{\sum_{i=1}^{r(r+1)/2} v_i(k)\boldsymbol{H}_i\boldsymbol{x}(k)}{\sum_{i=1}^{r(r+1)/2} v_i(k)}, \tag{11}$$

where

$$\boldsymbol{H}_{\sum_{t=1}^{j}(t-1)+i} = \boldsymbol{G}_{ij}, \quad i = j,$$
$$\boldsymbol{H}_{\sum_{t=1}^{j}(t-1)+i} = (\boldsymbol{G}_{ij} + \boldsymbol{G}_{ji})/2, \quad i < j,$$
$$v_{\sum_{t=1}^{j}(t-1)+i}(k) = w_i(k)w_j(k), \quad i = j,$$
$$v_{\sum_{t=1}^{j}(t-1)+i}(k) = 2w_i(k)w_j(k), \quad i < j.$$

By applying Theorem 1 to (11), we can derive a stability condition for the fuzzy control system (11).

Theorem 2. *The equilibrium of a fuzzy control system described by* (11) *is asymptotically stable in the large if there exists a common positive-definite matrix* $\boldsymbol{P}$ *such that*

$$\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{H}_i - \boldsymbol{P} < \boldsymbol{0} \tag{12}$$

for $i = 1, 2, \ldots, r(r+1)/2$.

Proof. It follows directly from Theorem 1. □

The design problem for Theorem 2 is to select $\boldsymbol{F}_j$ $(j = 1, 2, \ldots, r)$ which satisfies the condition of (12) for a common positive-definite matrix $\boldsymbol{P}$ when $\boldsymbol{A}_j$ and $\boldsymbol{B}_j$ are given.

3.2. Fuzzy model of the car

Let us simplify the model car (original model) described by (1)–(3) before approximating it by a fuzzy model. If $u(k)$ is always a small value, the model of the car can be simplified as follows:

$$x_0(k+1) = x_0(k) + vt/lu(k), \tag{13}$$

$$x_1(k+1) = x_1(k) + vt\sin[x_0(k)], \tag{14}$$

$$x_2(k+1) = x_2(k) + vt\cos[x_0(k)]. \tag{15}$$

Of course, the dynamics of this simplified model may not perfectly agree with that of the original model when the value of $u(k)$ is a large value. We will consider influence of the model error in the simulation.

In the case of trajectory control, the controlled variable $x_2(k)$ is not necessary, because the purpose of this simulation is to control the car along a desired trajectory (the straight line of $x_1(k) = 0$, i.e. to regulate $x_0(k)$ and $x_1(k)$ by manipulating the steering angle $u(k)$.

Next, let us approximate the simplified model by a fuzzy model. We proposed a method [10] for approximating a nonlinear system by a fuzzy model. Notice that sine function has the property,

$$0 \leqslant \sin(x_0(k)) \leqslant x_0(k), \qquad -180^\circ \leqslant x_0(k) \leqslant 180^\circ, \tag{16}$$

where

$$\sin(x_0(k)) = x_0(k),$$

when $x_0(k) = 0^\circ$, and

$$\sin(x_0(k)) \to 0,$$

when $x_0(k) \to -180^\circ$ or $x_0(k) \to 180^\circ$. Therefore, when

$$x_0(k) \cong 0 \text{ (deg.)},$$

the state equation of the simplified mode, (13) and (14), is approximated by

$$\begin{bmatrix} x_0(k+1) \\ x_1(k+1) \end{bmatrix} = \begin{bmatrix} 1.0 & 0 \\ vt & 1.0 \end{bmatrix} \begin{bmatrix} x_0(k) \\ x_1(k) \end{bmatrix} + \begin{bmatrix} \dfrac{vt}{l} \\ 0 \end{bmatrix} u(k). \tag{17}$$

On the other hand, when

$$x_0(k) = 180^\circ \text{ or } -180^\circ \ (\pi\text{(rad.) or } -\pi\text{(rad.)}),$$

the state equation of the simplified model is approximated by

$$\begin{bmatrix} x_0(k+1) \\ x_1(k+1) \end{bmatrix} = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix} \begin{bmatrix} x_0(k) \\ x_1(k) \end{bmatrix} + \begin{bmatrix} \frac{vt}{l} \\ 0 \end{bmatrix} u(k). \tag{18}$$

However, (18) is uncontrollable. Therefore, a control law for (18) cannot be uniquely determined. To avoid it, we approximate the simplified model for

$$x_0(k) = 179.997 \text{ (deg.) or } -179.997 \text{ (deg.)} \tag{19}$$

instead of

$$x_0(k) = 180 \text{ (deg.) or } -180 \text{ (deg.)}.$$

Then, it can be described by

$$\begin{bmatrix} x_0(k+1) \\ x_1(k+1) \end{bmatrix} = \begin{bmatrix} 1.0 & 0 \\ gvt & 1.0 \end{bmatrix} \begin{bmatrix} x_0(k) \\ x_1(k) \end{bmatrix} + \begin{bmatrix} \frac{vt}{l} \\ 0 \end{bmatrix} u(k), \tag{20}$$

where $g = 10^{-2}/\pi$. Of course, this system is theoretically controllable.

From (17) and (20), a fuzzy model, which approximately represents the dynamics of the model car, can be derived as follows.
Rule 1: If $x_0(k)$ is "about 0 (rad.)", then $\boldsymbol{x}(k+1) = \boldsymbol{A}_1\boldsymbol{x}(k) + \boldsymbol{b}_1 u(k)$,
Rule 2: If $x_0(k)$ is "about π (rad.) or $-\pi$ (rad.)", then $\boldsymbol{x}(k+1) = \boldsymbol{A}_2\boldsymbol{x}(k) + \boldsymbol{b}_2 u(k)$,
where

$$\boldsymbol{x}(k)^{\mathrm{T}} = [x_0(k), x_1(k)],$$

$$\boldsymbol{A}_1 = \begin{bmatrix} 1.0 & 0 \\ vt & 1.0 \end{bmatrix}, \qquad \boldsymbol{b}_1 = \begin{bmatrix} vt/l \\ 0 \end{bmatrix},$$

$$\boldsymbol{A}_2 = \begin{bmatrix} 1.0 & 0 \\ gvt & 1.0 \end{bmatrix}, \qquad \boldsymbol{b}_2 = \begin{bmatrix} vt/l \\ 0 \end{bmatrix}.$$

The consequent equations of Rules 1 and 2 correspond to (17) and (20), respectively. After all, the dynamics of the approximated fuzzy model is represented by

$$\boldsymbol{x}(k+1) = \frac{\sum_{i=1}^{2} w_i(k)\{\boldsymbol{A}_i\boldsymbol{x}(k) + \boldsymbol{b}_i u(k)\}}{\sum_{i=1}^{2} w_i(k)}, \tag{21}$$

where $w_i(k)$ is membership value of the fuzzy set in Rule i. Fig. 3 shows the fuzzy sets of "about 0 (rad.)" and "about π(rad.) or $-\pi$(rad.)". We define the fuzzy sets as simple triangles. The influence of model error between the original model and the fuzzy model, (21), will be considered in the simulation.

3.3. Controller design and stability analysis

We design a fuzzy controller for the car modeled by (21). The main idea of the controller design is to derive each control rule so as to compensate each rule of the fuzzy model (21). From Rules 1 and 2 of the

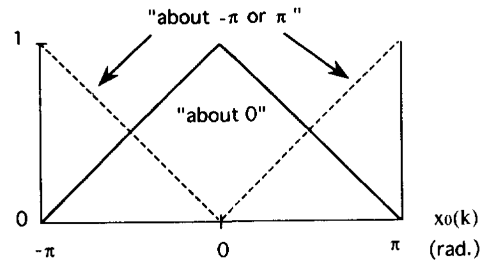

Fig. 3. Fuzzy sets.

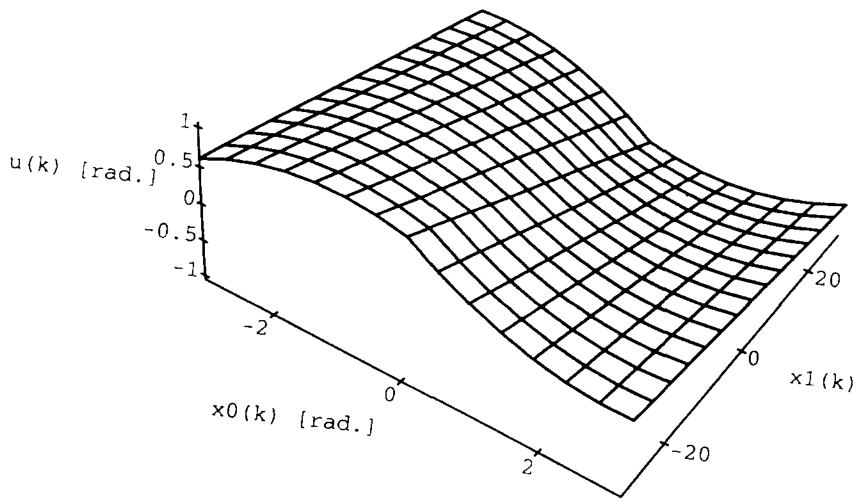

Fig. 4. Control surface.

approximated fuzzy model, we derive Control Rule 1 and Control Rule 2 of fuzzy controller, respectively:

Control Rule 1: If $x_0(k)$ is "about 0 (rad.)", then $u(k) = f_1 x(k)$,

Control Rule 2: If $x_0(k)$ is "about π(rad.) or $-\pi$(rad.)", then $u(k) = f_2 x(k)$,

where f_1 and f_2 are feedback gains. We use the exact same fuzzy sets in the premise part of the fuzzy controller. The purpose of controller design is to determine feedback gains of f_1 and f_2. The following feedback gains are used in the simulation:

$$f_{11} = -0.4212, \qquad f_{12} = -0.02933,$$

$$f_{21} = -0.0991, \qquad f_{22} = -0.00967.$$

Ricati equation for linear discrete systems was used to determine these feedback gains because each consequent part is represented by a linear state equation. Fig. 4 shows the control surface of the controller. It is found from Fig. 4 that the input–output relation of the fuzzy controller has a high nonlinearity.

Next, we consider stability of the control system. From A_1, A_2, b_1, and b_2 of the approximated fuzzy model and f_1, and f_2 of the fuzzy controller, we obtain

$$H_1 = A_1 + b_1 \cdot f_1 = \begin{bmatrix} 0.850 & -0.0105 \\ 1.0 & 1.0 \end{bmatrix},$$

$$H_2 = \frac{\{A_1 + b_1 \cdot f_2\} + \{A_2 + b_2 \cdot f_1\}}{2} = \begin{bmatrix} 0.907 & -0.00696 \\ 0.502 & 1.0 \end{bmatrix},$$

$$H_3 = A_2 + b_2 \cdot f_2 = \begin{bmatrix} 0.965 & -0.00345 \\ 0.00318 & 1.0 \end{bmatrix}.$$

If we select,

$$P = \begin{bmatrix} 989.0 & 75.25 \\ 75.25 & 26.29 \end{bmatrix}$$

as a common positive-definite matrix P, then the stability condition (12) of Theorem 2 is satisfied, i.e.

$$H_1^T P H_1 - P = \begin{bmatrix} -120.4 & 6.008 \\ 6.008 & -1.468 \end{bmatrix} < 0,$$

$$H_2^T P H_2 - P = \begin{bmatrix} -68.31 & -5.873 \\ -5.873 & -0.5079 \end{bmatrix} < 0,$$

$$H_3^T P H_3 - P = \begin{bmatrix} -100.0 & -0.000114 \\ -0.000114 & -1.000 \end{bmatrix} < 0.$$

Therefore, the fuzzy controller guarantees stability of the control system. In other words, it perfectly realizes trajectory control without no steady error under the condition of (19) if we can show that the dynamics of the original model agrees very well with that of the fuzzy model. We will show in the simulation that there is no influence of model error between dynamics of the original model and that of the fuzzy model. The matrix P was found by the construction procedure of the literature [11].

4. Simulation results

In this simulation, we use three kinds of the controlled objects: the original model, the simplified model and the approximated fuzzy model. Table 1 shows 24 cases of initial positions of the model car used in this simulation. Cases 13–18 require to turn the model car in order to realize a perfect trajectory control. Figs. 5–8 show simulation results for the fuzzy model. Figs. 9–12 show simulation results for the simplified model. Figs. 13–16 show simulation results for the original model.

The following points can be pointed out from the simulation results.

1. The designed fuzzy controller can effectively achieve trajectory control of the model car even from difficult initial positions such as cases 3, 9, 15 and 21.
2. The dynamics of the approximated fuzzy model agrees well with those of the original model and the simplified model.

Nobody can deny the first point, because it is shown that the control system is asymptotically stable in the large under the condition of (19). The second point shows that a fairly good approximation is realized.

Table 1
Initial portions of model car (24 cases)

	$x_0(0)$ (deg)	$x_1(0)$ (m)
Case 1	0	30
Case 2	0	20
Case 3	0	10
Case 4	0	− 10
Case 5	0	− 20
Case 6	0	− 30
Case 7	90	30
Case 8	90	20
Case 9	90	10
Case 10	90	− 10
Case 11	90	− 20
Case 12	90	− 30
Case 13	180	30
Case 14	180	20
Case 15	180	10
Case 16	180	− 10
Case 17	180	− 20
Case 18	180	− 30
Case 19	− 90	30
Case 20	− 90	20
Case 21	− 90	10
Case 22	− 90	− 10
Case 23	− 90	− 20
Case 24	− 90	− 30

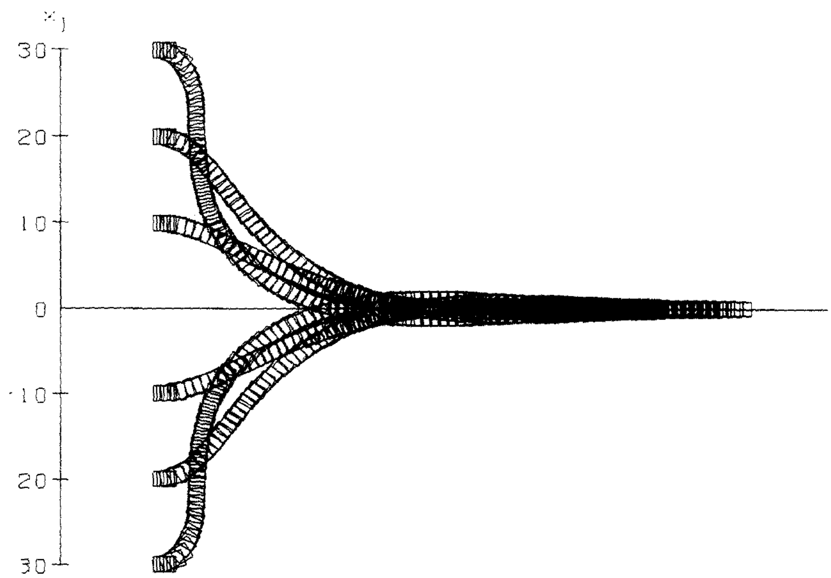

Fig. 5. Control result for cases 1–6 (fuzzy model).

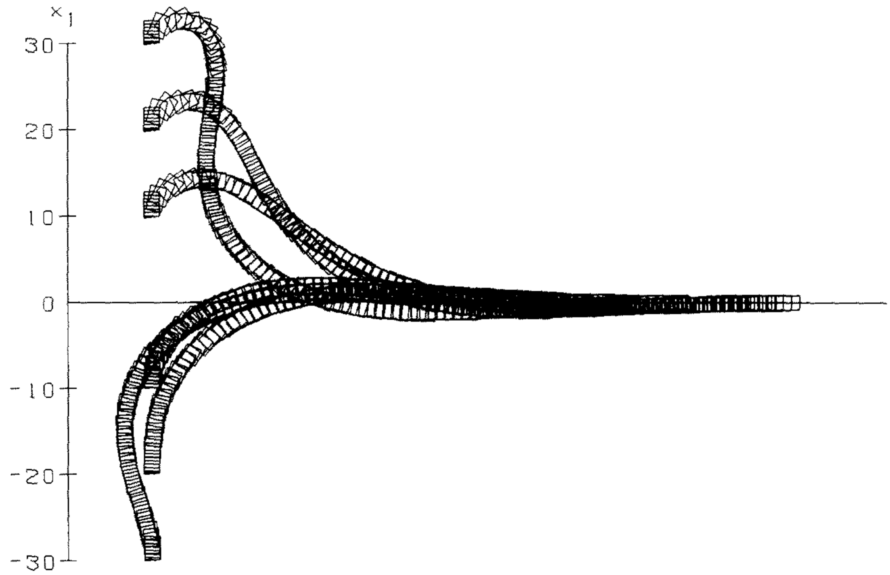

Fig. 6. Control result for cases 7–12 (fuzzy model).

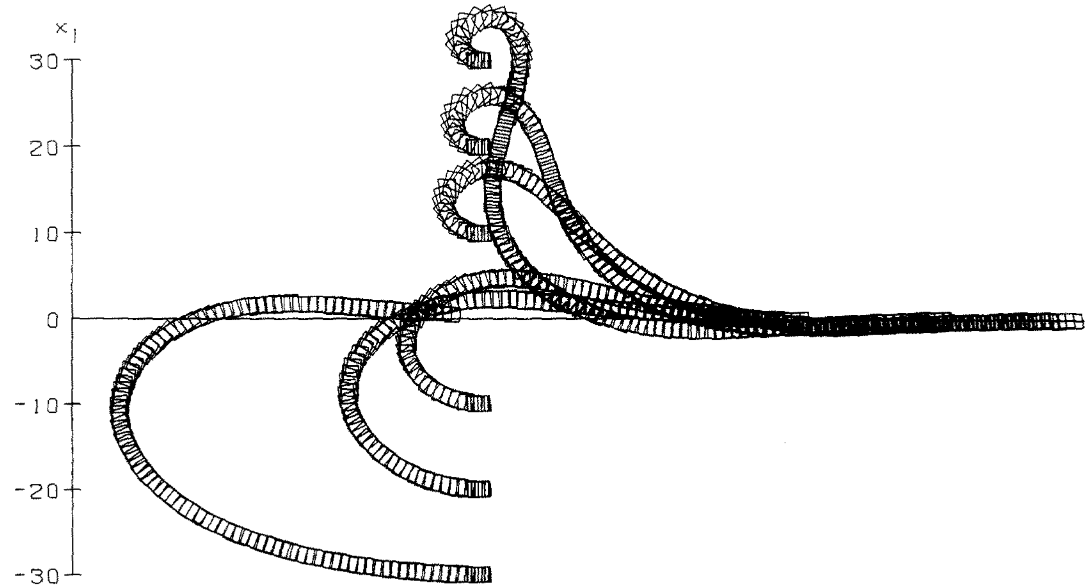

Fig. 7. Control result for cases 13–18 (fuzzy model).

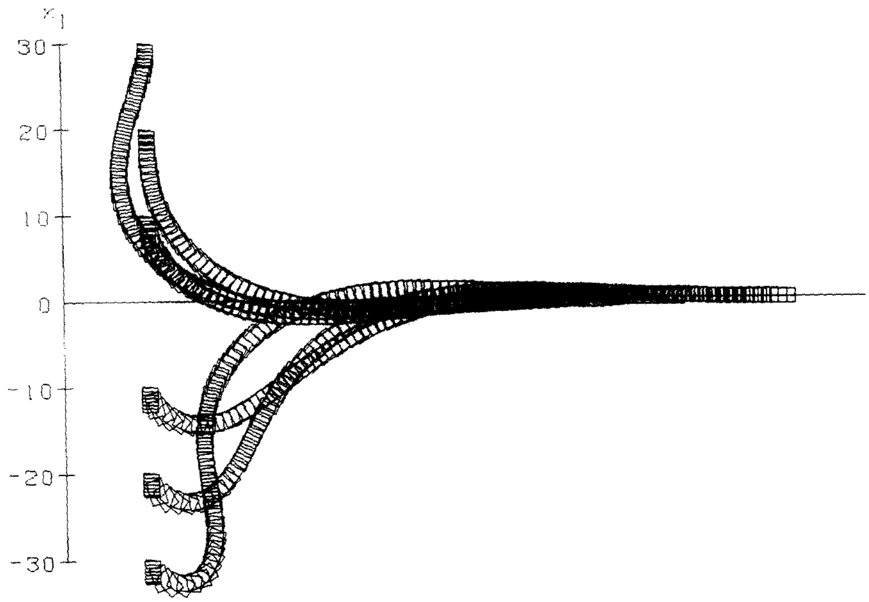

Fig. 8. Control result for cases 19–24 (fuzzy model).

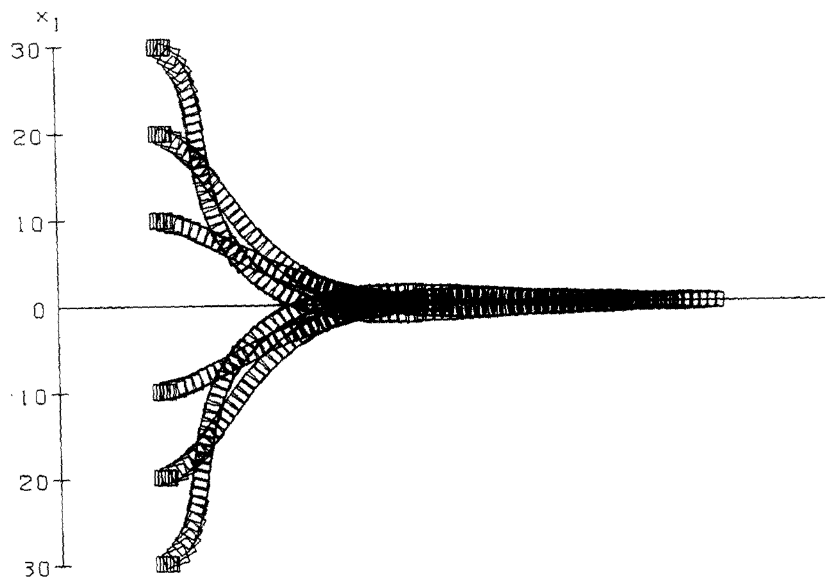

Fig. 9. Control result for cases 1–6 (simplified model).

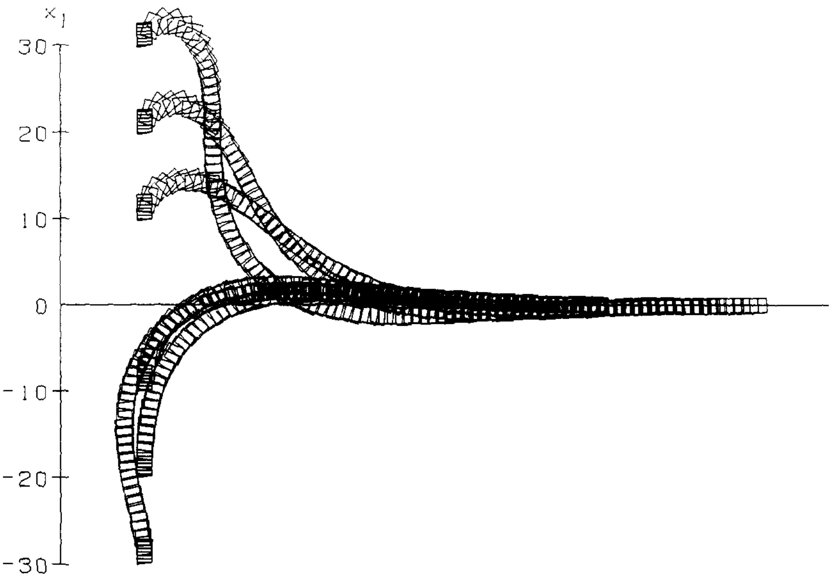

Fig. 10. Control result for cases 7–12 (simplified model).

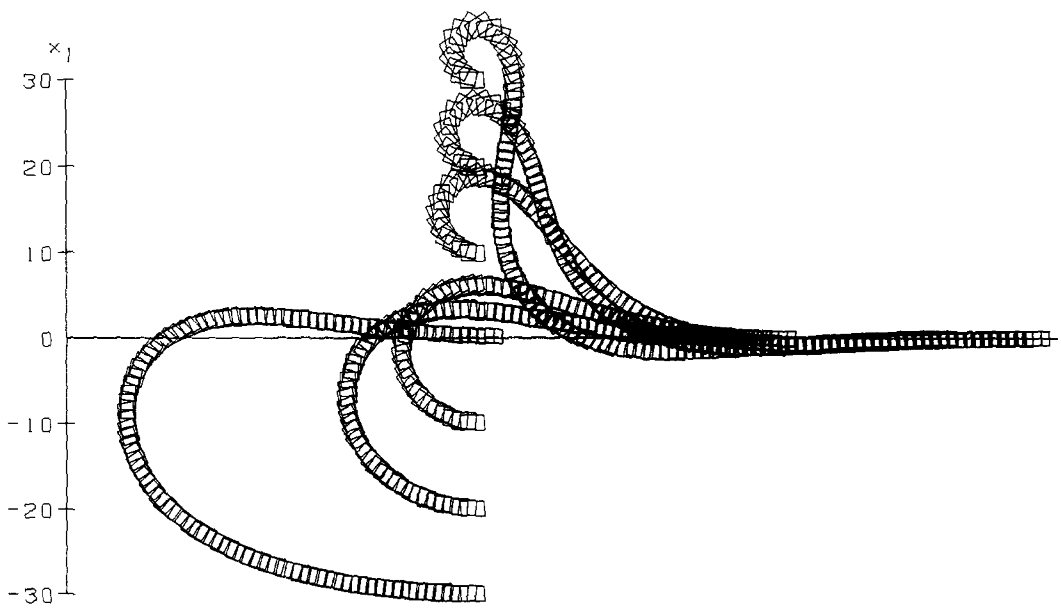

Fig. 11. Control result for cases 13–18 (simplified model).

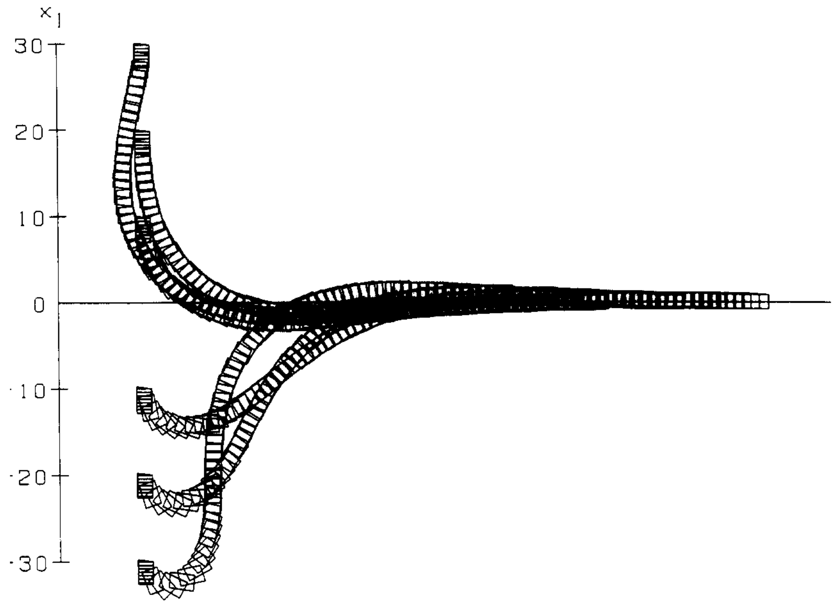

Fig. 12. Control result for cases 19–24 (simplified model).

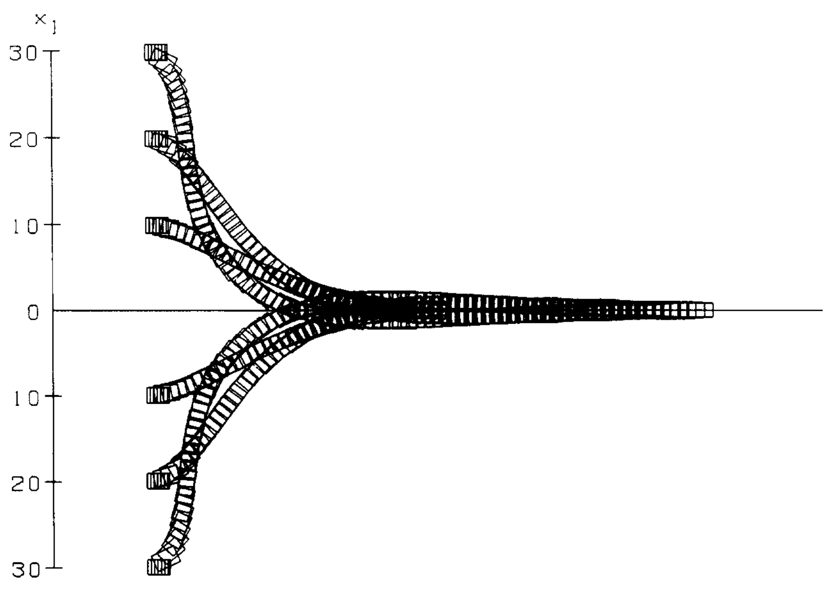

Fig. 13. Control result for cases 1–6 (original model).

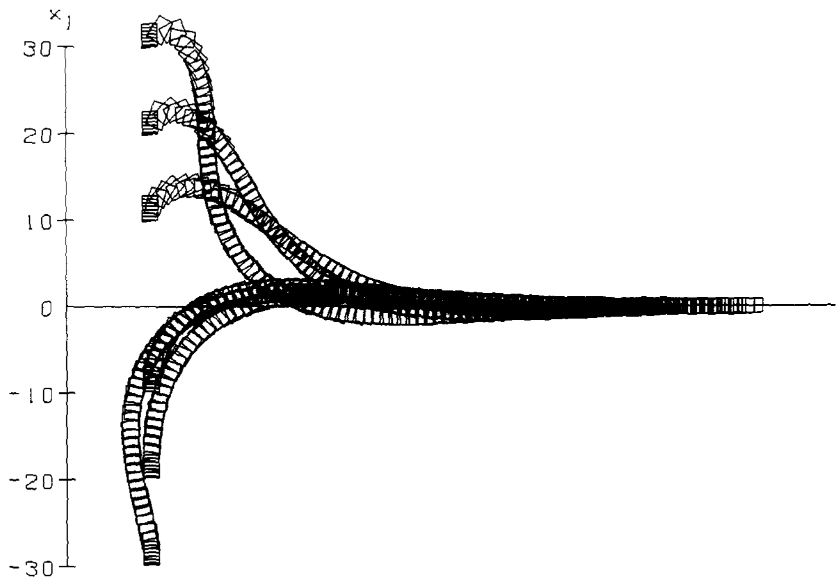

Fig. 14. Control result for cases 7–12 (original model).

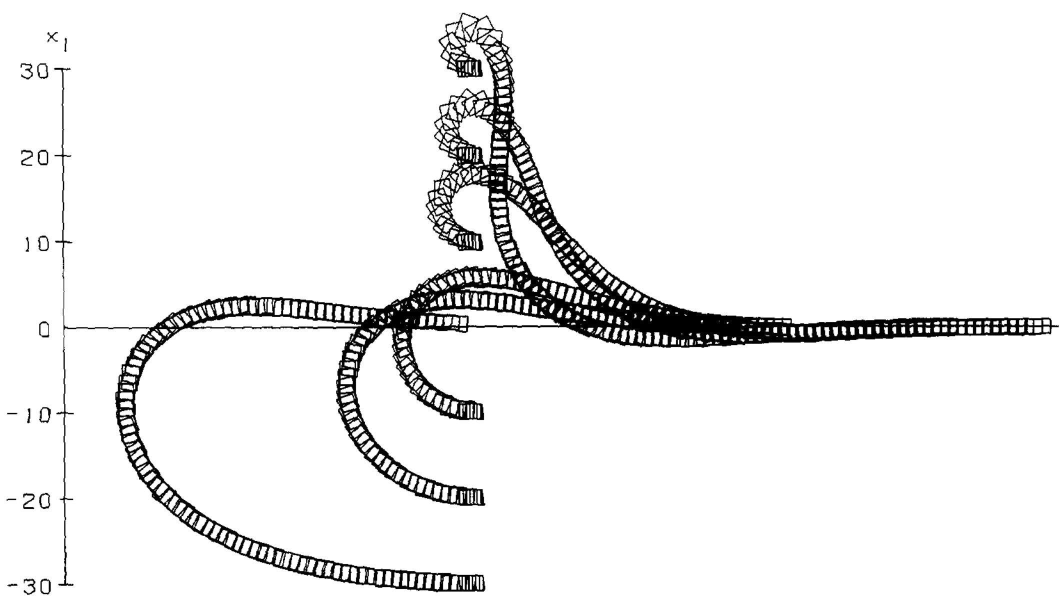

Fig. 15. Control result for cases 13–18 (original model).

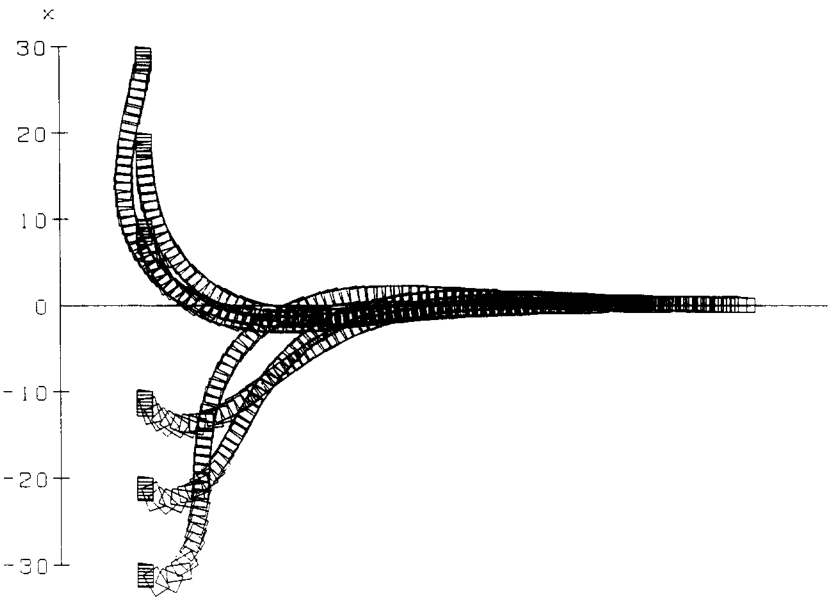

Fig. 16. Control result for cases 19–24 (original model).

5. Conclusion

This paper presented trajectory stabilization of a computer simulated model car via fuzzy control. Stability conditions of fuzzy systems have been given in accordance with the definition of stability in the sense of Lyapunov. First, we approximated a computer simulated model car, whose dynamics is nonlinear, by T–S (Takagi and Sugeno) fuzzy model. Fuzzy control rules, which guarantee stability of the control system under a condition, have been derived from the approximated fuzzy model. The simulation results show that the fuzzy control rules effectively realize trajectory stabilization of the model car along a given reference trajectory from all initial positions under a condition and that the dynamics of the approximated fuzzy model agrees well with that of the model car.

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