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HEAT TRANSFER EFFECT ON THE SPECIFIC COOLING LOAD OF REFRIGERATORS

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Abstract—The maximum possible specific cooling load that can be obtained from two-heat-reservoir refrigerators with a set of high-temperature heat sinks and low-temperature heat sources is analyzed. The refrigerators considered in this paper include (1) externally and internally reversible, (2) externally irreversible and internally reversible, (3) externally reversible and internally irreversible and (4) externally and internally irreversible refrigerators. The irreversibilities are assumed to be caused by heat transfer only. The specific cooling load, defined as the cooling load per unit total heat-exchanger surface area, is adopted as the objective function for the refrigerator performance analysis in this paper. Published by Elsevier Science Ltd.

Keywords—Specific cooling load, refrigerator, heat transfer effect.

INTRODUCTION

Among the important topics in thermodynamics has been the formulation of criteria for comparing the performance of real and ideal processes. For example, the Carnot cycle provides an upper bound on the COP (coefficient of performance) of all cyclic refrigerators operating between two fixed-temperature heat reservoirs. The work by Clausius, Kelvin and others carried out in this tradition identified the limits on work, heat transfer, thermodynamic efficiency, COP, energy effectiveness and energy figure of merit of various energy conversion devices. Since Gibbs, however, the focus has been directed toward state variables rather than the process variables of heat and work. An unavoidable consequence of this shift is the emphasis on equilibrium states and reversible processes. The use of reversible processes as standards of performance is not desirable because a reversible process must be carried out at an infinitesimally slow pace. Since power produced by a heat engine is work divided by time, a finite amount of work produced by the engine over an infinite time delivers no power. The need to develop a nontrivial amount of power in real energy conversion devices is one reason why the high-performance criteria of an ideal, reversible heat engine are seldom approached. Similarly, the cooling load provided by a refrigerator is heat divided by time. A finite amount of heat transferred in a reversible process requires an infinitesimally small temperature difference. Therefore, a finite amount of cooling load provided by the reversible refrigerator needs an infinitely large heat exchanger. In this instance, the specific cooling load (cooling load per unit total heat exchanger surface area) of a Carnot refrigerator is zero. Hence, there is a need to find a new bound in specific cooling load for comparing the performance of real refrigerators.

The consequence of incorporating finite-time processes into an otherwise ideal thermodynamic cycle was elegantly demonstrated by Curzon and Ahlborn [1]. They considered the case of finite rates of heat transfer to and from a Carnot heat engine. After maximizing the power output, they derived a simple expression for the efficiency that was different from the well known Carnot efficiency. Since finite-time thermodynamics was advanced in 1975, many authors have studied the effect of irreversibilities on the performance of heat engines. Some detailed literature surveys of the endoreversible heat engine were given by Sieniutycz and Salmon [2], Chen [3] and Sun and

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Chen [4]. Yet, only a few authors, including Leff [5], Sun [6, 7], Chen [8–10], Agrawa [11], Yan [12], Klein [13], Grazzini [14], Bejan [15, 16], Gordon [17] and Wu [18], have assessed the effect of finite rates of heat transfer on the performance of irreversible refrigerators.

In this paper, we shall examine the upper limit on the specific cooling load that can be delivered by a two-heat-reservoir refrigerator. We chose the specific cooling load (cooling load per unit total heat exchanger surface area) as the objective function for four types of refrigerators. The four cases are (1) externally and internally reversible (Carnot), (2) externally irreversible and internally reversible (endoreversible), (3) externally reversible and internally irreversible (exoreversible), and (4) externally and internally irreversible (real) refrigerators. The internal and external irreversibilities of the refrigerators are assumed to be caused by heat transfer only.

SPECIFIC COOLING LOAD ANALYSIS

The determination of the specific cooling load of a refrigerator is of considerable practical interest to practising engineers. Our investigation was undertaken to evaluate this quantity with the following assumptions:

1. The heat capacities of the heat source and heat sink are infinite, so that heat source and heat sink temperatures remain constant in the heat transfer processes.
2. The overall heat transfer coefficients (U_H and U_L) in the heat exchangers between the refrigerator and its surrounding heat reservoirs are constant.
3. The heat transfers are continuous and steady.
4. The irreversibilities are associated with the transfer of heat only.

Case 1: *externally and internally reversible* (*Carnot*) *refrigerator*

The Carnot refrigerator is a totally reversible cycle which is composed of four reversible processes–two isothermal and two adiabatic ones, as shown in Figs 1 and 2. In the heat addition process, heat is added to the refrigerator from the low-temperature heat source at T_L. If the temperature of the cold refrigerant (T_c) of the refrigerator is the same as the source, i.e. the temperature difference between the refrigerant and the heat source never exceeds a differential amount, an infinitely large heat exchanger surface area ($A_L = \infty$) is required in order to transfer a finite amount rate of heat flow (Q_L). Similarly, in the heat rejection process, heat is rejected from the warm refrigerant at T_w to the high-temperature sink at T_H. If the warm refrigerant is at the same temperature as that of the heat sink ($T_w = T_H$), the heat transfer surface area between the refrigerator and the heat sink must again be infinitely large ($A_H = \infty$) in order to reject a finite amount rate of heat flow (Q_L) from the warm refrigerant to the heat sink. Thus, the specific cooling

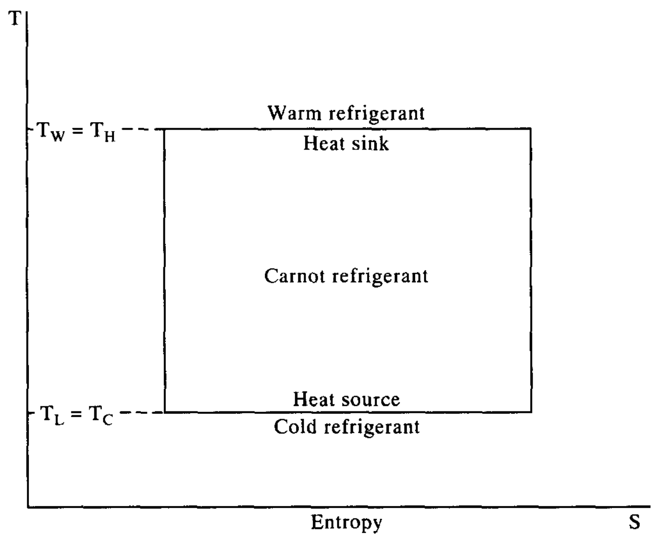

Fig. 1. Externally and internally reversible (Carnot) refrigerator T–s diagram.

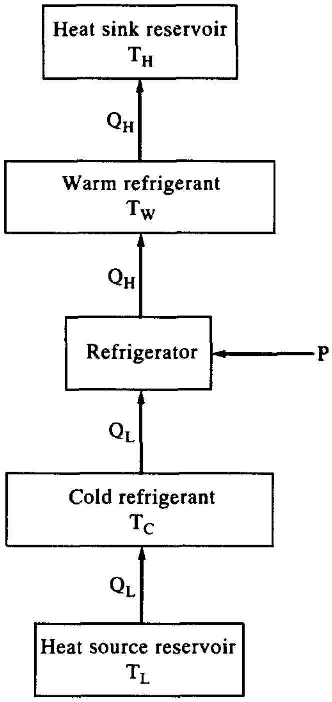

Fig. 2. Externally and internally reversible (Carnot) refrigerator.

load of the Carnot refrigerator $[Q_L/(A_H + A_L)]$ is equal to zero for the cycle producing any finite amount of cooling load. The COP of the Carnot refrigerator is the complete reversible Carnot COP:

$$\beta_{Carnot} = T_L/(T_H - T_L)\,. \quad (1)$$

Case 2. Externally irreversible and internally reversible (endoreversible) refrigerator

An endoreversible Carnot refrigerator is a modified Carnot cycle, as shown in Figs 3 and 4. The only irreversible processes in the cycle are the two heat transfer processes from the refrigerator to the heat sink and from the heat source to the refrigerator. To analyze this cycle, we assume that the temperatures of the heat sink, heat source, warm refrigerant in the heat rejection process, and cold refrigerant in the heat addition process are T_H, T_L, T_w and T_c, respectively. Thus heat flows

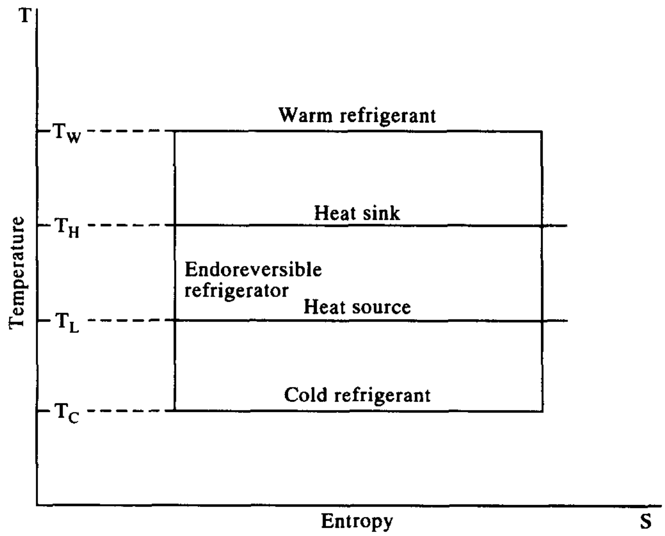

Fig. 3. Externally irreversible and internally reversible (endoreversible) refrigerator T–s diagram.

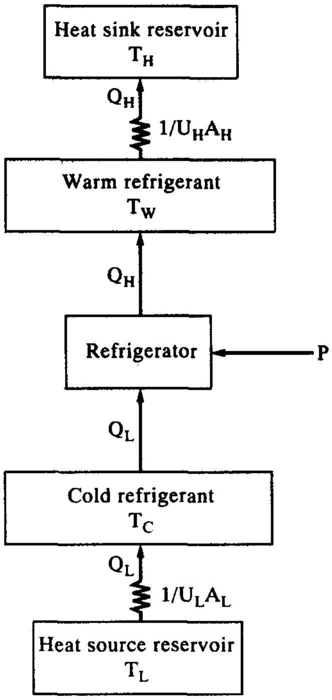

Fig. 4. Externally irreversible and internally reversible (endoreversible) refrigerator.

from the heat source to the cold refrigerant across a temperature difference of $(T_L - T_c)$ and heat flows from the warm refrigerant to the heat sink across a temperature difference of $(T_w - T_H)$.

The rate of heat flow (Q_L) from the heat source to the cold refrigerant in the low-temperature side heat exchanger of the refrigerator is proportional to the temperature difference $(T_L - T_c)$ and is given by

$$Q_L = U_L A_L (T_L - T_c) \,, \tag{2}$$

where Q_L is the cooling load of the refrigerator, U_L is the overall heat transfer coefficient and A_L is the surface area of the low-temperature side heat exchanger between the heat source and the refrigerator.

Similarly, the rate of heat flow (Q_H) from the refrigerator to the heat sink in the high-temperature side heat exchanger of the refrigerator is

$$Q_H = U_H A_H (T_w - T_H) \,, \tag{3}$$

where U_H is the overall heat transfer coefficient and A_H is the surface area of the heat exchanger.

The First Law of Thermodynamics requires that the power input (P) to the refrigerator be

$$P = Q_H - Q_L \,. \tag{4}$$

From the Second Law of Thermodynamics, the rate change of entropy of the refrigerant of the cycle requires

$$Q_H/T_w - Q_L/T_c = 0 \,. \tag{5}$$

The COP (β) and the specific cooling load (q_L) of the refrigerator are

$$\beta = Q_L/P \tag{6}$$

$$q_L = Q_L/(A_H + A_L) = Q_L/A_T \,, \tag{7}$$

where A_T is the total heat transfer surface area of the two heat exchangers.

Combining equations (2)–(7) yields

$$\beta = \{T_H/\{T_L - q_L(A_H + A_L)[(U_H A_H)^{-1} + (U_L A_L)^{-1}]\} - 1\}^{-1} \tag{8}$$

or

$$q_L = [T_L - T_H/(1 + \beta^{-1})]/\{(A_H + A_L)[(U_H A_H)^{-1} + (U_L A_L)^{-1}]\}\,. \qquad (9)$$

By either maximizing β in equation (8) or maximizing q_L in equation (9), we obtain the following optimal parameter relationship between the two heat exchangers:

$$A_H/A_L = (U_L/U_H)^{1/2}\,. \qquad (10)$$

The specific cooling load has the optimal value for any given COP value from equation (9) in the following form:

$$q_L = [T_L - T_H/(1 + \beta^{-1})]/[(U_H)^{-1/2} + (U_L)^{-1/2}]^2\,. \qquad (11)$$

It can be shown that equation (11) also presents the optimal COP value for any given specific cooling load.

Equation (11) verifies that the specific cooling load (q_L) is equal to zero if $\beta = \beta_{Carnot}$. Since a practising refrigeration engineer does not want to design a real refrigerator that delivers a specific cooling load of zero, the Carnot refrigerator COP is not a realistic bound which a real refrigerator may approach.

Equation (11) also indicates that q_L is a monotonic decreasing function of β. The specific cooling load approaches its maximum value when the COP is zero, i.e.

$$(q_L)_{max} = T_L/[(U_H)^{-1/2} + (U_L)^{-1/2}]\,. \qquad (12)$$

Combining equations (6), (7), (10) and (11), we obtain the optimal specific cooling load or the optimal COP for any given power input (P) and total heat transfer surface area (A_T) in the following forms:

$$q_L = \{[(T_H - T_L + DP/A_T)^2 + 4DPT_L/A_T]^{1/2} - [(T_H - T_L) + DP/A_T]\}A_T/(2DP) \qquad (13)$$

or

$$\beta = \{[(T_H - T_L + DP/A_T)^2 + 4T_L DP/A_T]^{1/2} - [(T_H - T_L) + DP/A_T]\}A_T/(2DP)\,, \qquad (14)$$

where $D = [(U_H)^{-1/2} + (U_L)^{-1/2}]$.

Equations (13) and (14) provide more general performance characteristic relationships for an endoreversible Carnot refrigerator for practising engineers in designing their real refrigerators. Notice that these optimal COP and specific cooling load are real bounds which are reachable if internal irreversiblities of the real refrigerators are minimized to zero.

Case 3. Externally reversible and internally irreversible (exoreversible) refrigerators

An exoreversible refrigerator is an externally reversible and internally irreversible system, as shown in Fig. 5. An ideal thermoelectric refrigerator is a typical exoreversible refrigerator, as shown in Fig. 5. The thermoelectric refrigerator is composed of two dissimilar semiconductors, p and n. The refrigerator is assumed to be insulated, both electrically and thermally, from its surroundings, except at the junction–reservoir contacts. The internal irreversibilities are caused by the Joulean electrical resistive loss and heat conduction loss through the semiconductors between the warm and cold junctions. The Joulean loss generates an internal heat I^2R, where R is the total internal electrical resistance of the semiconductor couple and I is the electrical current flowing through the couple. The conduction heat loss is $K(T_w - T_c)$, where K is the thermal conductance of the semiconductor couple.

Assuming that the material properties of the thermoelectric refrigerator are fixed, the traditional thermoelectric refrigerator analysis has the following expressions for the rates of heat transfer from the heat source to the refrigerator (Q_L) and from the thermoelectric refrigerator to the heat sink (Q_H), electrical power input (P), COP (β), Seebeck coefficient (α), internal electrical resistance (R) and heat conductance (K) as

$$Q_L = \alpha T_c I - 0.5I^2R - K(T_w - T_c) \qquad (15)$$

$$Q_H = \alpha T_w I + 0.5I^2R - K(T_w - T_c) \qquad (16)$$

$$P = I^2R + \alpha I(T_w - T_c) \tag{17}$$

$$\beta = Q_L/P \tag{18}$$

$$\alpha = \alpha_p - \alpha_n \tag{19}$$

$$R = \delta_n L_n/A_n + \delta_p L_p/A_p \tag{20}$$

and

$$K = k_n A_n/L_n + k_p A_p/L_p\ , \tag{21}$$

where α_n and α_p are the Seebeck coefficients of the n- and p-semiconductor legs, L_n and L_p are the lengths of the n- and p-semiconductor legs, A_n and A_p are the cross-sectional areas of the n- and p-semiconductor legs, δ_n and δ_p are the electrical resistivities of the n- and p-semiconductor legs and k_n and k_p are the thermal conductivities of the n- and p-semiconductor legs, respectively.

Equations (15) and (18) indicate that $Q_L = 0$ and $\beta = 0$ when $I = \{B_1 - [(B_1)^2 - 2B_1/(ZT_c)]^{1/2}\}/R$ and $I = \{B_1 + [(B_1)^2 - 2B_1/(ZT_c)]^{1/2}\}/R$, where $B_1 = T_c/(T_w - T_c)$ and $Z = \alpha^2/(KR)$.

To find the maximum cooling load from equation (15), taking the derivative of Q_L with respect to I and setting it equal to zero ($\partial Q_L/\partial I = 0$) gives

$$I_{m1} = \alpha T_c/R \tag{22}$$

$$(Q_L)_{m1} = [(T_c)^2/2 - (T_w - T_c)/Z]\alpha^2/R \tag{23}$$

and

$$\beta_{m1} = [T_c - 1/(ZB_1)]/(2T_w)\ , \tag{24}$$

where I_{m1}, $(Q_L)_{m1}$, and β_{m1} are the optimal current, cooling load and COP of the refrigerator at its maximum cooling load condition.

To find the maximum COP from equation (18), taking the derivative of β with respect to I and setting it equal to zero ($\partial\beta/\partial I = 0$) gives

$$I_{m2} = \alpha(T_w - T_c)/[(M - 1)R] \tag{25}$$

$$(Q_L)_{m2} = (T_w - T_c)M(MT_c - T_w)\alpha^2/[R(M + 1)(M - 1)^2] \tag{26}$$

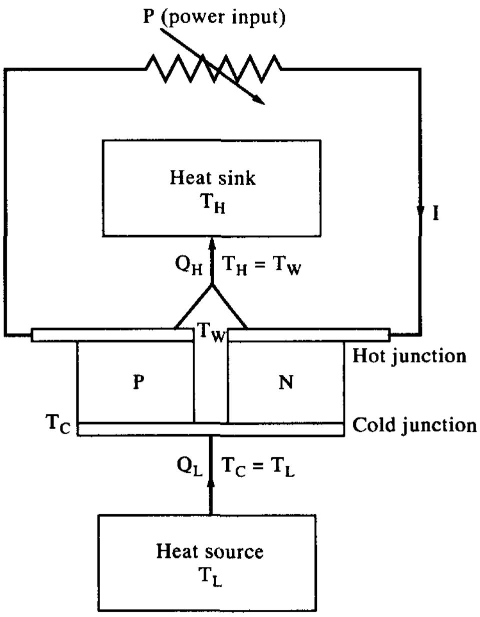

Fig. 5. Externally reversible and internally irreversible (exoreversible thermoelectric) refrigerator.

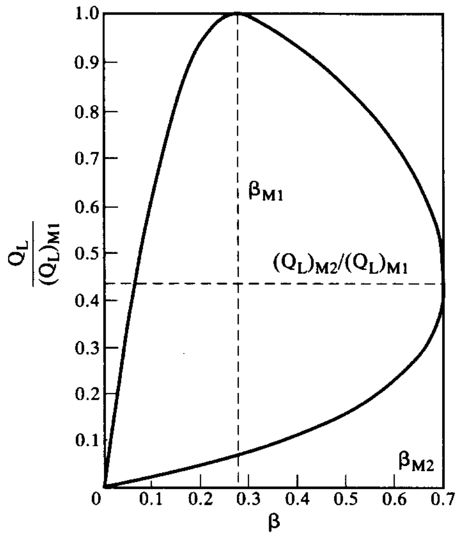

Fig. 6. Cooling load versus COP characteristics of the exoreversible thermoelectric refrigerator.

and

$$\beta_{m2} = B_1(M - T_w/T_c)/(M + 1)\,, \tag{27}$$

where $M = [1 + Z(T_w + T_c)/2]^{1/2}$; I_{m2} and $(Q_L)_{m2}$ are the optimal current, cooling load and COP of the refrigerator at its maximum COP condition.

The cooling load versus COP characteristic curve of the exoreversible thermoelectric refrigerator with $T_H = T_w = 300$ K, $T_L = T_c = 273$ K, $\alpha = 4 \times 10^{-4}$ V/K, $Z = 0.002$ and $\beta_{Carnot} = B_1$ is shown in Fig. 6.

To achieve external reversibility, the warm and cold junctions of the thermoelectric refrigerator must have the same temperatures as the heat sink and the heat source reservoirs, i.e. $T_H = T_w$ and $T_c = T_L$. The specific cooling load of the exoreversible refrigerator is zero, since it requires two infintely large surface area heat exchangers to transfer a finite amount of heat input and to produce a finite amount of cooling load.

Case 4. Externally and internally irreversible (real) refrigerator

Irreversibilities, such as friction, finite-rate heat transfer, heat leaks, free expansion, mixing, pressure drop, etc., do occur in real processes. In case 4, we consider only the finite-rate heat transfer, heat leaks and Joulean losses are accounted for in both internal and external irreversibilities in predicting the performance of the real refrigerator, as shown in Fig. 7.

The rate of heat transfer from the heat source at temperature T_L to the refrigerator cold junction at temperature T_c is

$$Q_L = U_L A_L (T_L - T_c)\,. \tag{28}$$

The rate of heat transfer from the refrigerator hot junction at temperature T_w to the heat sink at temperature T_H is

$$Q_H = U_H A_H (T_w - T_H)\,. \tag{29}$$

Combining equations (28), (15), (29) and (16) gives

$$T_w = [(U_H A_H T_H + I^2 R/2)(U_L A_L + I\alpha) + K(U_H A_H T_H + U_L A_L T_L + I^2 R)]/N \tag{30}$$

$$T_c = [(U_L A_L T_L + I^2 R/2)(U_H A_H - I\alpha) + K(U_H A_H T_H + U_L A_L T_L + I^2 R)]/N \tag{31}$$

$$q_L = U_L A_L (q_1 + q_2)/[N(A_L + A_H)] \tag{32}$$

$$q_1 = I^3 R\alpha/2 - I^2(T_L\alpha^2 + U_H A_H R/2 + KR) \tag{33}$$

$$q_2 = I U_H A_H T_L \alpha - K U_H A_H (T_H - T_L) \tag{34}$$

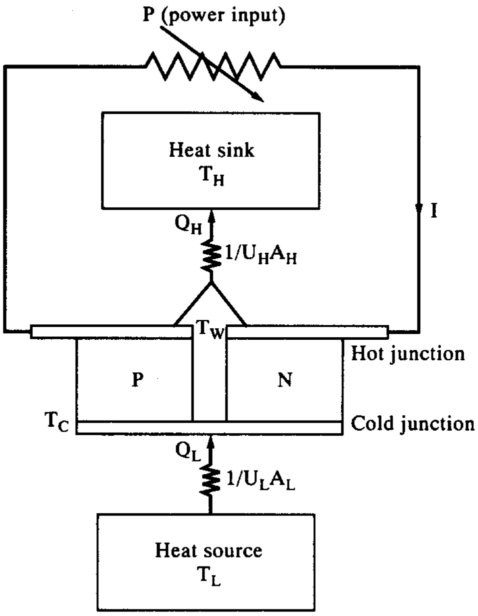

Fig. 7. Externally and internally irreversible (real thermoelectric) refrigerator temperature sink at T_H.

$$P = (P_1 + P_2 + P_3)/N \tag{35}$$

$$P_1 = I^3R(U_HA_H - U_LA_L)\alpha/2 \tag{36}$$

$$P_2 = I^2[(U_HA_HT_H + U_LA_LT_L)\alpha^2 + U_HA_HU_LA_LR + KR(U_HA_H + U_LA_L)] \tag{37}$$

$$P_3 = I\alpha U_HA_HU_LA_L(T_H - T_L) \tag{38}$$

and

$$\beta = q_L(A_H + A_L)/P\,, \tag{39}$$

where $N = (U_HA_H - I\alpha)(U_LA_L + I\alpha) + K(U_HA_H + U_LA_L)$.

The specific cooling load and COP of the refrigerator can be obtained by finding the optimum current and (A_H/A_L) first under the condition $(A_H + A_L = A_T)$ with a numerical calculation method. The specific cooling load versus COP characteristic of the real refrigerator is similar to that of the exoreversible refrigerator shown in Fig. 6.

CONCLUSION

For practical, space, weight and economic reasons, real refrigerators are designed to operate at their maximum specific cooling load. This paper utilizes a specific cooling load as an objective function for practising engineers to design a new refrigerator or to evaluate an existing one. The result gives a much more realistic approach to the cooling load and COP than those of the Carnot ideal reversible refrigerator. It also allows one to model these sources of irreversibilities as parameters to refrigerators so one can see qualitatively their impact on the performance of refrigerators.

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