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The Flow of Granular Materials Round Obstacles

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SUMMARY

Observations on the flow of mustard seed round obstacles inserted into a two-dimensional bed show flow patterns which are more readily explicable by the kinematic and stochastic models of particle flow than by plasticity theory.

INTRODUCTION

Stochastic methods for predicting the velocity distributions in the gravity flow of granular material have been proposed by Mullins [1] and Litwinitzyn [2]. In these methods the flow of a granular material is modelled as the counterflow of voids which diffuse upwards in 'biased random flight' [1]. The conclusions of these models are purely kinematic in the sense that the predicted velocity distribution is independent of the stress distribution within the material.

An alternative way of looking at Mullins' model has been presented in a later paper by Nedderman and Tüzün [3]. Their version is based on the idea that as the particles in one layer move, those in the layer above simply fall into the vacated space jostling one another for position. By considering three particles as shown in Fig. 1, they concluded that if one of the particles in the lower layer was moving faster there would be a tendency for the particle in the upper layer to move sideways as shown. It was therefore proposed that the horizontal velocity, u, was a function of the gradient of vertical velocity, $\partial v/\partial x$:

$$u = f\left(\frac{\partial v}{\partial x}\right) \tag{1}$$

This simplest non-trivial form of this equation,

$$u = -B\frac{\partial v}{\partial x} \tag{2}$$

was tested experimentally with some success.

The latter equation was also derived by Mullins from the original form of the model, and it is hardly surprising that the two forms give rise to identical conclusions since they do not differ in any significant way. Thus it may never be possible to decide which of the two starting hypotheses is the more realistic.

An important qualitative deduction can be made from the model. It is postulated that the cause of the motion of the particles in one layer is simply the departure of those in the layer beneath (Nedderman and Tüzün version) or the arrival of a void from below (Mullins version). In either case the velocity distribution at any level will depend solely on

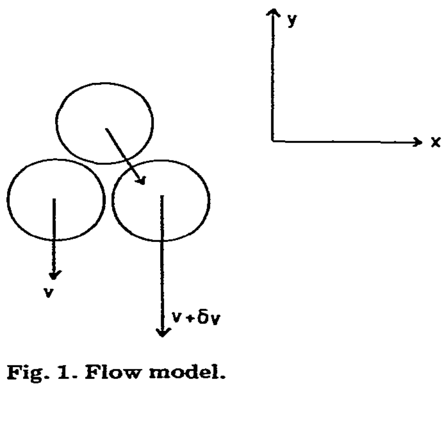

Fig. 1. Flow model.

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conditions below that level and will be totally independent of anything that may be happening above.

The major alternative method of predicting velocity distributions is based on plasticity theory, see for example Spencer [4], de Jong [5]. Here it is necessary first to calculate the stress distribution and then link this with strain increments by either an associated or a non-associated flow rule. There seems to be much dispute as to which flow rule is required, but whilst the choice affects the numerical values, qualitatively similar results must occur.

When gravitational effects dominate, the stress distribution calculations must be started at the top surface and work downwards. This can be done approximately by methods derived from Janssen's [6] method or more accurately by the method of characteristics as outlined by Sokolovskii [7]. In the latter case discontinuous stress fields are often (perhaps usually) predicted [8]. In a stress discontinuity the direction of the major principal stress changes discontinuously and whatever version of plasticity theory is invoked, the direction of the major strain increment must also be discontinuous. Thus from plasticity theory one must expect to find velocity fields that are discontinuous at least in their first derivatives. Furthermore, since the stress state depends only on conditions above the point of interest, plasticity theory will predict velocity profiles that only depend on upstream conditions.

A particular example of these deductions can be seen in the model proposed by Drescher *et al.* [9] for the flow in a wedge-shaped hopper. Here three zones were postulated as shown in Fig. 2. The material at the top of the hopper (zones A) was assumed to be in the form of two rigid colliding blocks

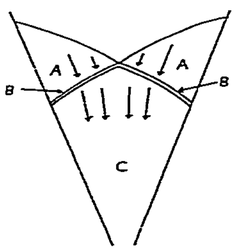

Fig. 2. Model of Drescher *et al.*

sliding along the walls. At the base of these blocks was a narrow rupture zone (B) in which plastic deformation gave a discontinuous change in velocity and voidage and from which the material emerged with a velocity profile that was maintained without change of shape throughout the radial flow zone (C).

Thus, without resorting to any calculations, we can deduce that the kinematic and stochastic models on one hand and plasticity theory on the other must predict radically different velocity distributions. The former models will predict velocities that depend primarily on conditions below the point of interest and, since velocities are assumed to be independent of stress, will be continuous even in the presence of discontinuous stress fields. The latter theory predicts velocities that are determined by conditions above the point of interest and which are commonly discontinuous.

With these ideas in mind, qualitative observations were made of flow patterns to see whether changes in geometry affected the velocities upstream or downstream. This work was carried out by two of us (S.T.D. and D.J.H.) as part of the requirements of the Chemical Engineering Tripos and further details can be found in our individual reports [10, 11]. Because of the limited time available, the scope of the work was somewhat restricted and work is continuing in the same field. Nonetheless we believe that these preliminary results are of interest.

APPARATUS

The main objective of the work was the observation of the flow patterns round an object placed in what would otherwise have been a plug flow region, particular attention being directed into seeing whether the disturbance to the flow field is primarily above or below the insert. The observations were made in a 'two-dimensional' bunker consisting of two vertical glass plates held 2.3 cm apart by wooden spacers. All the observations were made using mustard seed of mean diameter 2.28 mm to which a few black kale seeds had been added to provide visible marker particles. Previous work on similar bunkers had shown that the velocities of particles

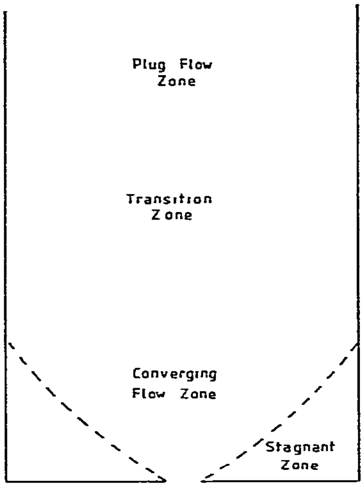

Fig. 3. Diagram of flow zones.

visible against the glass plates were similar to those in the bulk. This can easily be checked by integrating the measured velocity profile and comparing it with the measured volumetric flow rate. For high quality float glass and rigid spherical particles a discrepancy of less than 5% is normally found [12]. The same is not true if perspex walls are used, presumably due to retardation of the visible particles because of energy losses as they plough furrows through the relatively soft elastic surface, or possibly due to the effects of static electricity.

The bunker was constructed with two vertical rough wooden walls a distance 20 cm apart and a flat base with interchangeable orifices. With no inserts the flow pattern shown schematically in Fig. 3 was observed. Stagnant zones were found in the bottom corners with a converging flow zone between. Above the top of the stagnant zones the velocity profile died away gradually to give a plug flow region in the upper part of the bunker. Within this zone the velocity was found to be uniform except for a narrow shear region of thickness equal to a few particle diameters adjacent to the wall.

The flow round three inserts was investigated. These were made of perspex and consisted of a circular disc of diameter 10 cm, a square of side 10 cm and an equilateral triangle of side 10 cm. This latter object was inserted into the flow both with an apex pointing upwards and with an apex pointing downwards. In all cases the maximum dimension of the insert was half the width of the bin and preliminary experiments confirmed that the presence of the inserts did not affect the flow rate from the bin.

The principal aim of the work was the determination of streamline patterns for steady flow round the inserts and to this end a continuous flow was maintained by supplying the mustard seed to the top of the bunker from a storage hopper which was continually topped up with material. As the seed entered the bunker it passed through a 1 cm slot orifice extending the full width of the bunker, thereby ensuring that the material entered in a dilated state.

The streamline patterns were obtained by the simple expedient of following the dark kale seeds by eye and marking their positions on the glass sheets with a felt pen. This could easily be done at the low velocities (of order 2 cm/s or less) encountered in this work. These streamlines were subsequently traced onto paper to provide a permanent record. Later on, a more detailed study of one set of conditions was made by ciné photography. From the films, measurements of velocity as well as streamline patterns could be obtained.

RESULTS AND DISCUSSION

Figure 4 shows the streamline pattern obtained for flow round the square insert. Various features are worthy of comment.

A void space of roughly triangular cross-section was found beneath the insert. The void was bounded on its lower surface by narrow zones in which the material cascaded rapidly downwards in an irregular manner. The location of the lower boundary of the void space could not therefore be determined with any precision but was found to fluctuate by only a few degrees about the static angle of repose which had previously been measured as 32°.

The cascading zone was narrow, being of the order of only a few particle diameters in width, and the flow within it appeared turbulent. The horizontal position at which a given marker particle left this zone seemed to be independent of the original horizontal position of the particle. Thus it was not possible to trace streamlines through the cascading zone.

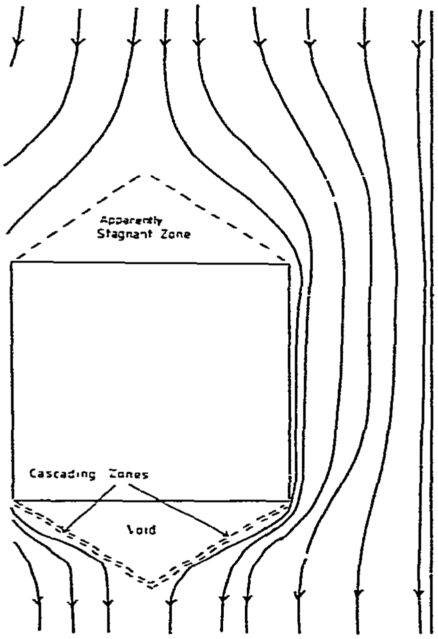

Fig. 4. Streamline pattern round square insert.

Plug flow seemed to be re-established almost immediately below the cascading zone. Thus the disturbance to the flow caused by the insert extended only a distance comparable with the void space depth, say a third of the insert width, below the insert.

In the lower part of the parallel sided channels between the insert and the wall, the streamlines were found to curve sharply towards the insert, feeding material into the cascading zone. Plug flow was however rapidly reestablished further up the channel.

On top of the insert there was a region in which the velocities were too small to be detected and the particles may even have been genuinely stationary. The apparent boundary of this stagnant zone was inclined to the horizontal at an angle considerably greater than the static angle of repose. Adjacent to the stagnant zone was a region of slow moving particles, and the streamlines in the bulk of the flow deviated from their plug flow arrangement at distances above the insert of the order of twice the insert width.

These observations are more simply explained by the kinematic and stochastic models than by plasticity theory. The former predict that the flow pattern depends primarily on what is happening below, whereas the latter predicts dependence on conditions above the plane of interest. If one acknowledges that particles cannot flow at an angle less than the angle of repose to the horizontal, one obtains a void space below an obstacle which may be considered to be an extension of that obstacle. The plug flow in the lower part of the bin is maintained up to the base of the void, and a non-plug flow region occurs in the passage of changing width between the void and the side wall. This flow pattern decays to plug flow again in the parallel sided channel and it is not until the top of the insert that departure from plug flow occurs again. Similarly, above the obstacle a velocity profile is observed which decays slowly to plug flow further up. Thus the departures from plug flow seem always to lie above the geometrical features causing them.

The streamlines shown in Fig. 4 were obtained at a flow rate that gave a plug flow velocity of 0.81 cm/s in the undisturbed region. Experiments at roughly twice that velocity showed only minor changes in the streamline pattern and consequently did not affect the conclusion that a disturbance is primarily above the cause.

Figures 5 and 6 show the streamline patterns round a cylinder and an equilateral triangle with its apex pointing upwards. The results are qualitatively very similar to those for the square insert. The slope of the bottom surface of the void space is as before, but the actual extent of the void beneath the cylinder is much smaller for geometrical reasons. Figure 5 shows clearly that the major disturbance to the plug flow caused by a cylindrical insert is above the obstacle.

The clearest demonstration of this effect was, however, obtained with the equilateral triangle with its apex pointing downwards. The top face of this insert was of the same size as that of the square insert, and to aid comparison the two sets of streamlines are superimposed in Fig. 7. It can be seen that above the triangular insert there was a much larger stagnant zone and that the streamlines deviated at greater heights above the insert. Indeed, in this case the lateral displacement of the streamlines is greatest some distance above the insert, so that by the time the particles have reached the level of the top of the insert they are already moving more or

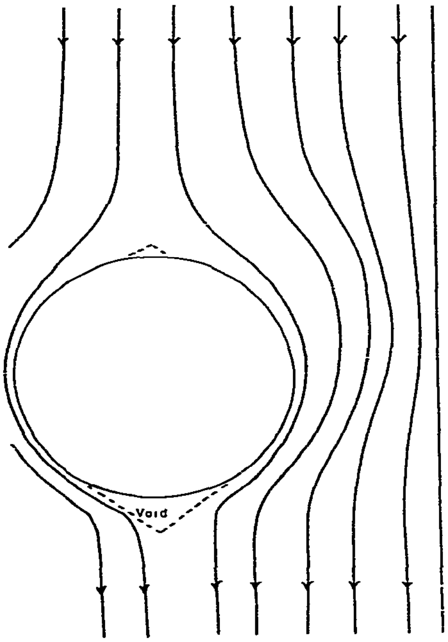

Fig. 5. Streamline pattern round circular insert.

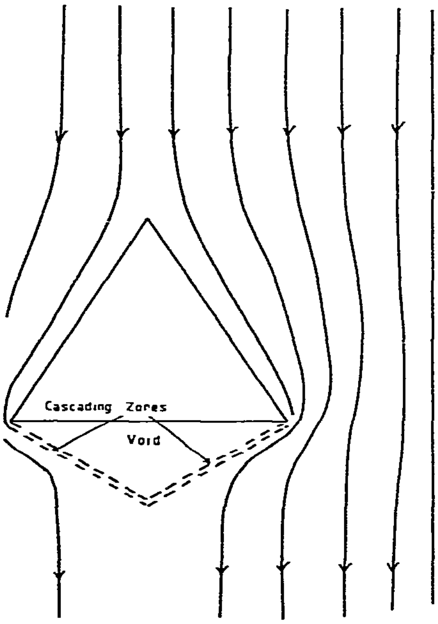

Fig. 6. Streamline pattern round triangular insert.

less parallel to the inclined wall. The observation that changes to the shape of the underside of the insert can affect the flow above the insert seems to be a convincing demon-

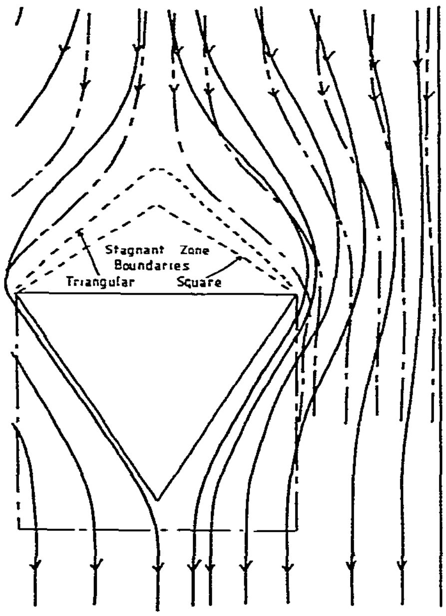

Fig. 7. Streamline patterns round square and triangular inserts; —·— square insert, —— triangular insert.

stration that the flow is controlled by factors below the plane of interest. Indeed, very approximately, the observed streamline patterns are much as one would expect for a fluid flowing in the opposite direction. Whereas for a fluid the wake lies behind the object, for a granular material the disturbance to the flow is ahead of the object.

VELOCITY PROFILES

In order to put the observations reported above on a firmer numerical footing, the velocity distribution between the square insert and the side wall was measured. Marker particles were photographed by a ciné camera and, by taking measurements from the film, the successive positions of particles could be found as a function of time. The velocities were obtained from the best-fit slopes to these curves. Velocity profiles were obtained at the levels shown in Fig. 8, which also defines the coordinate system. These velocity profiles are shown in Figs. 9 and 10. All the experiments were conducted at a flow rate that gave a velocity of 0.81 cm/s in the undisturbed plug flow region and hence an

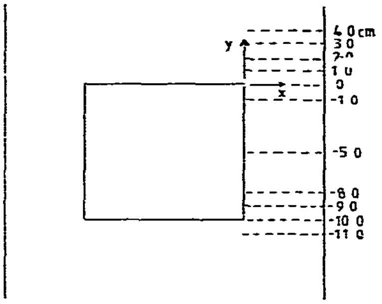

Fig. 8. Levels at which velocities were determined.

average velocity of 1.62 cm/s in the side channels.

It can be seen from Figs. 9 and 10 that there is a fair amount of scatter in the results, and it is not clear how much of this is due to genuine irregularities or imperfections in the experimental technique. It is however clear from Figs. 9 and 10 that the velocity is effectively constant across the width for the upper half of the channel $0 > y > -5$ cm. In the lower half of the channel the velocity near the insert rises rapidly and by the bottom is of the order of three times that near the bunker wall. This is entirely consistent with the curvature of the streamline, noted above, as the material moves sideways to feed the cascading zone.

The material at the top of the channel is effectively in plug flow, but above this level the velocity is progressively reduced from the side adjacent to the stagnant zone. Though no measurements were made at heights greater than 4 cm above the stagnant zone, it is clear from the streamline pattern that the reduction in velocity continues until plug flow is re-established at a height of about 10 cm.

Nedderman and Tüzün [4] obtained a value of 0.45 cm for the kinematic constant B of eqn. (2) in experiments on 1.5 - 2.0 mm glass ballotini. There is some indication from later experiments [13] that this constant is proportional to particle diameter, so a value of B some 10 - 20% greater is perhaps appropriate for mustard seed. Nonetheless the velocity distribution above the insert was predicted theoretically (as outlined in the Appendix) using a value of 0.45 cm and the results are shown where appropriate by the solid lines in Fig. 9. Calculations were also made with somewhat larger values of B but the results are barely distinguishable on the scale of Fig. 9.

It can be seen that the theoretical lines pass within the experimental scatter. Though this is not a sensitive test of the model nor of the precise value of B, the agreement is encouraging and lends weight to our conclusion that the kinematic and stochastic models predict the observed flow patterns far better than plasticity theory could.

CONCLUSIONS

The observations reported above on the flow of granular materials round inserts are qualitatively in agreement with those predicted by the kinematic and stochastic models. In the one case considered quantitatively, substantial agreement with theory was also found.

The alternative method of predicting velocity distributions presents formidable calculational problems and we are unaware of any predictions for geometrical situations as complicated as those investigated. The predictions for simpler situations, however, show features which were not observed in these experiments.

We therefore conclude that the kinematic and stochastic models are much more promising as a method of predicting the flow of granular materials than is plasticity theory.

LIST OF SYMBOLS

a	width of channel between obstacle and side wall
B	kinematic constant
n	integer
u, v	velocity components
v_p	plug flow velocity
x, y	Cartesian coordinates
α_n	eigen-value

APPENDIX

Theoretical prediction of velocity profiles

The basic conclusion of the kinematic theory,

$$u = -B \frac{\partial v}{\partial x} \quad (2)$$

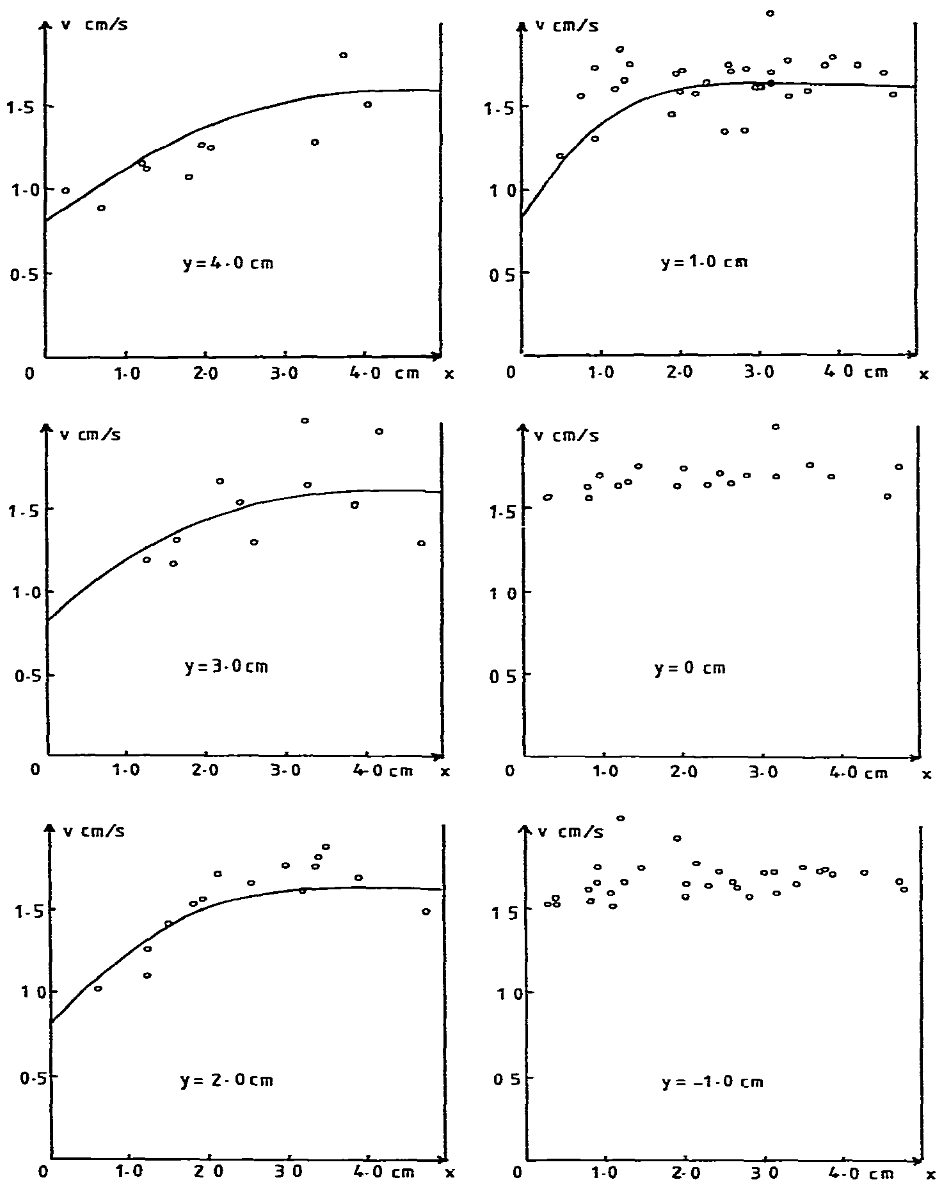

Fig. 9. Velocity profiles; ○ experimental values, —— theoretical predictions.

can be combined with the continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

to give

$$\frac{\partial v}{\partial y} = B\,\frac{\partial^2 v}{\partial x^2} \qquad (3)$$

For flow above the square insert with the coordinate system shown in Fig. 8, we have the following boundary conditions.

On $y = 0$ (the level of the top of the insert), $v = 0$ for $x < 0$; $v = 2v_p = -1.62$ cm/s for $x > 0$.

On the wall, $x = a = 5$ cm, $u = 0$ and hence $\partial v/\partial x = 0$,

and on the centre line, $x = -a$, $\partial v/\partial x$ is also zero by symmetry.

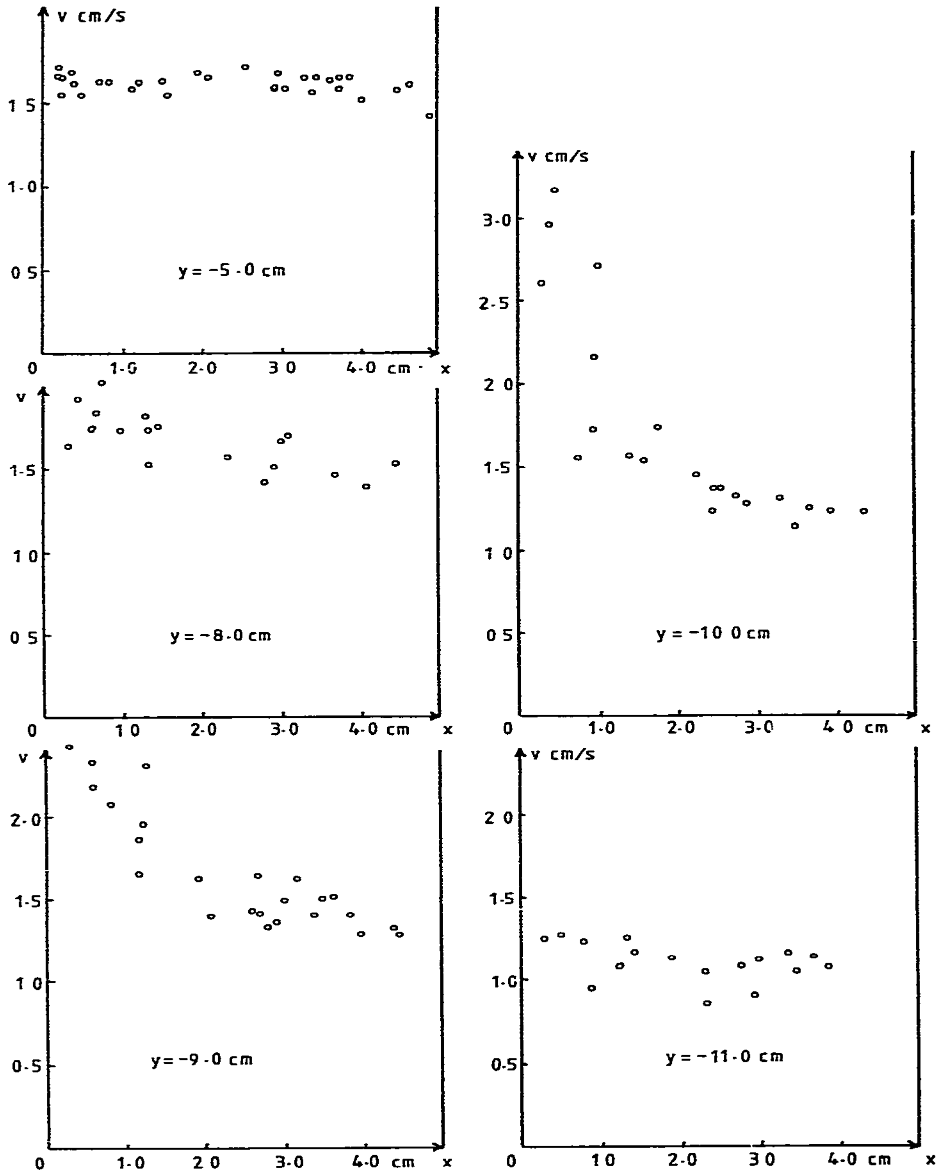

Fig. 10. Velocity profiles.

These boundary conditions make a separable solution possible and, using the techniques for Fourier analysis, the solution is found to be

$$v = v_p \left\{ 1 + \sum_{n \text{ odd}} \frac{2B}{\alpha_n a} \exp(-\alpha_n^2 y/B) \sin(\alpha_n x/B) \right\}$$

where $\alpha_n = n\pi B/2a$.

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