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An Empirical Study on Intertemporal Decision Making Under Risk

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This study compares time preference in the cases of certainty and risk. We analyze both matching and choice behavior. We find that violations of the stationarity axiom are restricted to matching behavior, both for certainty and risk. We also compare the discounting of certain and risky outcomes as well as the discounting of gains and losses. In matching tasks, certain outcomes are discounted more than risky ones. We could not confirm these results in a choice task. Gains and losses are not found to be discounted at different rates.

(Experimental Study; Behavioral Anomalies; Time Preference)

1. Introduction

In many situations, a decision maker has to evaluate risky alternatives with future consequences. Investment decisions require estimating uncertain future profits and weighing them against present expenditures. The profitability of buying and selling stocks depends on future price movements whose probabilities need to be assessed and taken into account in present decisions. Insurance contracts protect against the financial consequences of possible future misfortunes at the cost of present insurance premiums. All these decisions have two aspects in common: the risk inherent in future outcomes and the time that will pass until these outcomes occur.

Now consider a lottery where, in period t, both the risk is resolved and a consequence occurs. Such a lottery will be denoted by L_t . There are two principal ways of eliciting the certainty equivalent of L_t in period 0, see Figure 1.

One possibility is to directly specify which certain amount of money makes a decision maker indifferent between receiving the lottery in period t, i.e., L_t , and receiving the certain amount now, denoted $CE_0(L_t)$. This elicitation procedure corresponds to arrow B in Figure 1. In order to assess this value, one has to simultaneously consider two things: the risk of the lottery and the time until the consequence occurs. Both your risk and your time preference will influence your present value. The amount will be lower the more risk averse you are and the more averse you are to waiting for the money.

Alternatively, risk and time preferences may also be elicited separately. In a first step, the certain amount $CE_t(L_t)$ is specified, i.e., in period t, the decision maker is indifferent between L_t and $CE_t(L_t)$ (see arrow A in Figure 1). This specification requires considering risk preference only, even if $CE_t(L_t)$ may depend on t. In a second step, the certain amount of money at t = 0, denoted $CE_0(CE_t)$, which makes the decision maker indifferent between $CE_0(CE_t)$ and $CE_t(L_t)$, is elicited. This specification (see arrow C in Figure 1) considers time preference only, that is, discounting $CE_t(L_t)$. Risk preference is irrelevant as both $CE_0(CE_t)$ and $CE_t(L_t)$ are certain.¹ From the economist's perspective, the same good cannot have two different prices. Thus evaluating L_t through *B* should be the same as evaluating L_t through A and C, i.e. $CE_0(L_t)$ should equal $CE_0(CE_t)$.

¹ Another possibility would be to first determine a lottery to be resolved and paid immediately that is equivalent to L_t and then determine its certainty equivalent. Then, however, the first step would still require that you take both your time and risk preferences into account.

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The goal of our paper is to study risk and time effects and to disentangle them if possible. Above we have used certainty equivalents to elicit the decision maker's preference. However, there are other possibilities which can be used in judging alternatives, like rating alternatives according to attractiveness, etc. In addition, it is well documented that judgment and choice can lead to differences, see Lichtenstein and Slovic (1971) and Tversky et al. (1988 and 1990). We therefore investigate how people *choose* between two risky future outcomes and, in addition, we analyse how people specify certainty equivalents in order to *match* risky future outcomes.

The paper will proceed as follows. Previous theoretical and empirical research on intertemporal choice under certainty and risk is reviewed in §2. Our research questions and hypotheses are specified in §3. The design and the results of the matching and the choice parts of our studies are presented in §§4 and 5. We conclude in §6.

2. Theory and Existing Experimental Evidence

One of the first experiments was conducted by Thaler (1981), who asked subjects to state how much they would require to make waiting for certain monetary receipt (payment) just as (un)attractive as receiving (paying) the money immediately. The exponential discounting model (Koopmans 1960) states that the present value $V_0(a_t)$ of a future receipt a_t in period t is

$$V_0(a_t) = \frac{v(a_t)}{(1+i)^t},$$
 (1)

where v is a period value function and i is a discount

rate. Note that neither v nor i depend on the period t in question. Using (1), Thaler assumed both the present and the future value function to be linear. He derived the discount rates to fit the observed responses. Based on different waiting periods (three months to 10 years) and different amounts (\$-250 to \$3,000), he found that discount rates decline as t increases ("short-long term asymmetry"), and they decline as the amounts increase ("magnitude effect"). He also found smaller rates for losses than for gains ("gain-loss asymmetry"). These results contradict (1).

The short-long term asymmetry is a direct violation of the axiom of stationarity. This axiom, introduced by Koopmans (1960), is central for deriving the exponential model (see, e.g., Meyer 1976). It states that when considering two outcomes a_s in period s and b_t in period t, time affects the decision maker's preference only through the difference s - t. The decision maker's preference will therefore remain unchanged if both outcomes are shifted into the future by some extra waiting time d:

$$a_s > b_t \Leftrightarrow a_{s+d} > b_{t+d}.$$
 (2)

If a decision maker prefers, for instance, one apple today to two apples tomorrow, he will also prefer one apple in ten days to two apples in eleven days since the time difference is one day in both cases. Stationarity implies that the same discount factor needs to be applied between periods s and t and periods s + d and t + d, thus, the discount factor is independent of time. Thus (1) cannot accommodate the short-long term asymmetry.

This asymmetry can be accommodated by the hyperbolic discounting model (Ainslie 1975):

$$V_0(a_t) = \frac{v(a_t)}{(1+t)^h},$$
(3)

where *h* is a parameter that determines the decision maker's time preference. Since the denominator of (3) is a power of *t*, the present value of a_t declines as a hyperbolic function of *t*. Hyperbolic discounting has been axiomatized by Harvey (1986). The axiom of stretching is central in the derivation of (3), just as stationarity is central in deriving (1). The axiom states that when considering two outcomes a_s in period *s* and b_t in period *t*, the decision maker's preference will remain unchanged if the same outcomes are presented in peri-

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ods d(s + 1) - 1 and d(t + 1) - 1, i.e., if both are stretched into the future by some factor $d \neq 0^2$:

$$a_s > b_t \Leftrightarrow a_{d(s+1)-1} > b_{d(t+1)-1}. \tag{4}$$

Note that axiom (4) is sensitive to changes of units in time. This is because the choice of time unit determines the meaning of adding up (or subtracting) 1. Imagine a decision maker who prefers, for instance, one apple today to two apples tomorrow. Under the stretching axiom (4), he should prefer one apple in 10 days to two apples in 21 days (stretch by d = 11). In contrast to stationarity, however, he may very well prefer two apples in 11 days to one apple in 10 days, which is the short-long term asymmetry.

Benzion et al. (1989) extended Thaler's study. In contrast to Thaler's study, Benzion et al. used a sample of 204 subjects that were knowledgeable in the theory of temporal discounting. The stimuli used included four different scenarios (postponing or expediting receipts or payments), four time delays (0.5, 1, 2, and 4 years), and four *certain* amounts (\$40, \$200, \$1,000, \$5,000). Benzion et al. confirmed all Thaler's findings concerning the short-long term asymmetry, the magnitude effect, and the gain-loss asymmetry. In addition, they found that implicit discount rates differed in the delay and speedup conditions ("delay-speed-up asymmetry"). For gains, their delay rates exceeded their speed-up rates, and the opposite was true for losses.

The short-long term asymmetry and the magnitude effect have also been verified by more recent studies. For gains, Kirby and Maraković (1995) have found strong evidence for hyperbolic discounting, i.e., for the short-long term asymmetry, in an experiment using real rewards. Chapman and Elstein (1995) have established the short-long term asymmetry and the magnitude effect not only for monetary but also for health consequences.

² Harvey called his axiom the property of "relative timing preferences," which he defined (Harvey 1986) to mean that the trade-off between two periods *t* and *s* depends only on their ratio t/s. This is equivalent to (4); see also Ahlbrecht and Weber (1995, Theorem 12). The +1 and -1 stem from the fact that Harvey denotes the first period he looks at (the present) t = 1, while we call it t = 0. Adding or subtracting 1 thus simply translates between notations. When interpreting Harvey's stretching axiom, we ignore the +1 and -1's.

Loewenstein (1988) confirmed the delay-speed-up asymmetry in two large studies and attributed it to reference point shifts.³ Shelley (1993) completed the picture by investigating delay, neutral, and speedup frames both for gains and for losses.⁴ She extended Loewenstein's reference point model to predict that gain rates should exceed loss rates in the delay frame, and loss rates should exceed gain rates in the speed-up frame. In the neutral frame, Shelley predicts that the gain-loss asymmetry will vanish. If Thaler (1981) and Benzion et al. (1989) had reported different discount rates for losses than for gains, this could be explained since all gain-loss asymmetries in their data were elicited in either delay or speedup frames. Shelley (1993) replicated the Benzion et al. (1989) study with 74 undergraduates. The stimuli used in her study included four delays (0.5, 1, 2, and 4 years), four outcomes (\$40, \$200, \$1,000, and \$5,000), and six scenarios (delay-, neutral-, and speed-up-frames for both gains and losses). She confirmed all Thaler's and Benzion et al.'s findings in the delay and speed-up frames. According to her predictions, she found no gain-loss asymmetry in the neutral frame, which is of relevance for our study.

We are aware of two studies that investigated the discounting of future *risky* outcomes, both of which considered neutral frames. Shelley (1994) investigated the gain-loss asymmetry for risky prospects using a subject pool of 30 MBA students. She framed her questions as managerial investment decisions whose risky return was modelled as a lottery with a p chance of a gain and a (1 - p)-chance of a loss.

³ Since the delay frame elicits an amount subjects are willing to accept while the speed-up frame elicits an amount subjects are willing to pay, the delay-speed-up asymmetry might also be traced to a willingness to pay/willingness to accept asymmetry (Knetsch and Sinden 1984; see also Kahneman et al. 1990, and Eisenberger and Weber 1995). In a third experiment, however, Loewenstein shows that the delay-speed-up asymmetry persists in choice tasks, which, he argues, cannot be influenced by a WTA/WTP asymmetry.

⁴ In a neutral frame, the decision maker is first asked to state how much he is willing to pay to receive an object immediately and then he is asked to state how much he is willing to pay to receive the object at a later point in time.

Subjects were asked to evaluate the attractiveness of the investments on a rating scale. The stimuli included gains ranging from \$60 to \$1000, losses ranging from \$-900 to \$-160, periods ranging from 0 to 2 years, and probabilities p = 0.6 and p = 0.4. Since managers have been found to think that they can influence magnitude and likelihood of future losses (March and Shapira 1987), this can lead to the hypothesis that they discount losses faster than gains. Shelley found evidence that discount rates for losses are higher than for gains but concluded cautiously as to the generality of her finding.⁵

Stevenson (1992) conducted a study that investigated time preference for certain and risky future gains using a subject pool of 107 undergraduates. In a first session, subjects were asked to evaluate risky future gains on a rating scale. In a second session, the same subjects gave their strength of preference between two riskless investments on a rating scale. Probabilities ranged from 0.1 to 1.0, amounts from from \$60 to \$610, and periods from two months to eight years. Stevenson confirmed the short-long term asymmetry for risky future gains and found individual discount rates to be higher for certain outcomes than for risky outcomes ("certainty-risk asymmetry"). The latter finding is counterintuitive for economists who are used to charging risk premia for risky future outcomes (i.e., to discount risky returns more heavily than certain ones). From the behavioral point of view, however, Stevenson claims that considering the risk of a future prospect distracts from the time issue and thus leads to lower discount rates than for certain outcomes. Figure 2 summarizes the results of the existing studies on how risky outcomes are discounted in a neutral⁶ frame.⁷

Figure 2 Existing Results on the Discounting of Risky Future Outcomes



3. Research Questions, Subjects, and Procedure

3.1. Research Questions and Hypotheses

We restrict our attention to neutral frames since Loewenstein (1988) and Shelley (1993) have explained the delay-speed-up asymmetry as a reference point effect. Also, we do not investigate a magnitude effect for risk since it is not evident what is to be understood by the "magnitude" of a lottery. Our aim is replicate and extend the findings presented in Figure 2.

We test the following.

(1) Short-term long-term asymmetry, for both certain and risky future outcomes and for both gains and losses. We expect to replicate Thaler's (1981), Benzion et al.'s (1989), and Stevenson's (1992) findings and establish a short-long term asymmetry for risky losses.

(2) *Certainty-risk asymmetry*, for both gains and losses and for both short- and long-term decisions. We expect to replicate Stevenson's (1992) finding on the certaintyrisk asymmetry for gains and extend it to losses.

(3) *Gain-loss asymmetry*, for both certain and risky future outcomes and for both short- and long-term decisions. We expect to replicate Shelley's (1993) findings for certain outcomes. Since Shelley (1994) hypothesized that the gain-loss asymmetry she had detected for risky outcomes may have been due to the management investment frame, we expect to find no asymmetry for risky outcomes.

We perform both *matching* and *choice*-based tests of the asymmetries. We expect to confirm our hypotheses through both matching and choice tasks.

⁵ "Generalizations of the tendency to discount loss faster than gain to particular decision makers or decision situations should be made with caution, however. Risk propensity is known to vary with context."—Shelley (1994, p. 150–151).

⁶ Since the results by Thaler (1981), Loewenstein (1988), and Benzion et al. (1989) refer to speed-up or to delay frames, they have been omitted from Table 2.

⁷ All the effects discussed so far refer to single future outcomes. For an overview on empirical work that considers whole sequences of outcomes, see Loewenstein and Prelec (1993).

3.2. Subjects and Procedure

Participants were 132 students from the University of Mannheim. Of these, 80 were third-year students specialising in banking and finance and 52 were second year students enrolled in finance classes.⁸ They were paid a flat DM 20 fee for participation.⁹

Each subject worked through an interactive computer program. The program started with a warm-up period of approximately 15 minutes that presented the format of the questions and allowed subjects to get accustomed to the response requirements. Subjects were assured that there were no right or wrong answers but that the purpose of the study was to elicit their personal preferences. The supervisor checked that comprehension was assured for each individual subject.

There were two parts: Part 1 consisted of 24 matching questions; Part 2 consisted of 42 choice questions that were derived from the individual responses in part one. There was a break of approximately 15 minutes between the two parts. The number of questions was similar to or less than the number of questions in the studies presented in the last section.¹⁰

Half the subjects were asked to state certain gains that match the attractiveness of a given gain lottery and to choose between gain lotteries to be played and paid at time *t*. These lotteries gave a *p*-chance of winning an amount *x*. The other half was given the same matching and choice tasks with the amounts to be lost, not won. In both parts, the order of the questions was determined by a random process.¹¹ In the following, we present the

⁸ We present the combined results of both groups, as we could not find any systematic difference between them.

⁹ DM = Deutsche Marks. At the time of the experiment the exchange rate was DM 1.50 for 1 US-\$. Following Bohm (1994) an incentive compatible scheme might have been preferable. He showed that incentive compatibility significantly reduces preference reversals. We were not able to pay subjects according to their performance, for there did not seem to be a suitable way to have people pay us money in one or two years from now.

¹⁰ Benzion et al.'s (1989) study consisted of 64 questions. Stevenson's (1992) study consisted of 96, respectively, 144 questions in two separate sessions. Shelley's studies consisted of 96 questions (1993), respectively, 128 questions (1994).

¹¹ Clearly, the elicitation of $CE_t(L_t)$ must precede the elicitation of $CE_0(CE_t)$. If the random process happened to pick a question that turned out to require elicitation of some (still) unknown variable first,

design and the results of each part of the study in consecutive order.

4. Part 1—Matching

4.1. Design

Subjects were presented with a lottery L_t over monetary consequences which was to be resolved and paid at time t. Subjects were asked for the certain amount $CE_0(L_t)$ now which they judged to be equivalent to L_t , and the certainty equivalent at time t ($CE_t(L_t)$). Then they were presented with the individual certain amount CE_t $= CE_t(L_t)$, and they were asked for the certain amount now ($CE_0(CE_t)$) which would make them indifferent. Thus subjects had to perform tasks B, A, and C as shown in Figure 1. Figure 3 displays a typical screen of the matching tasks in Part 1.

Part 1 was designed from a factorial design with two points of time (t = 6 months and t = 24 months) and with four lotteries L_t (x = DM 12, p = 0.99), (x = DM250, p = 0.5), (x = DM 25,000, p = 0.01), and (x = DM250, FM), where FM denotes the event that, at time t, the Temperature in Fairbanks, Alaska, exceeded the temperature in Murmansk, Russia. Our motivation for the fourth lottery was to also investigate time preference for an event lottery thus considering the case of ambiguity (see Camerer and Weber 1992 for an overview on decision making under ambiguity). We chose the temperature event since we felt it allowed students to form an idea about which probabilities might be involved (e.g., using their knowledge about geography), while on the other hand it did not allow exact assessment of these probabilities.

4.2. Results

We excluded 25 subjects (19%) from the analysis because they gave one or more nonsensical¹² responses. This seemingly high rate of unusable responses is not

this question was deferred, and the random process picked another question to be answered next.

¹² We considered certainty equivalents equal to or above the highest possible or equal to or below the lowest possible outcome of the lottery in question as nonsensical. Out of 25 people, 13 were excluded for they considered a 0.99-chance of receiving x DM equal to x DM. The remaining 12 showed stronger violations of dominance.



Figure 3 Computer Screen for Matching Tasks in Part 1

Please enter the amount and confirm with "return"

uncommon for experiments on intertemporal decision making.¹³ The following results for both Part 1 and Part 2 are therefore based on 55 subjects who handled gains and 52 who handled losses.

Table 1 displays means and medians of subjects' responses. For three lotteries the means and medians are close. For the DM 25,000 lottery they diverge, as there was a wide variation of subjects' responses, with $CE_0(L_t)$ ranging from DM 0.50 to DM 15,000. In addition, for this lottery two responses make the mean of $CE_{24}(L_{24})$ exceed the mean of $CE_6(L_6)$ by a margin of more than one hundred.¹⁴ Although this shows that outliers distort the means of the observed variables, we have not excluded any subject solely on the basis of extreme responses. Rather, we do not base our tests on the means of the implicit discount rates but conduct tests (such as sign and median tests) which are less sensitive to outliers. For each subject, the risk free rate *i* was calculated at which $CE_t(L_t)$ was discounted to $CE_0(CE_t)$:

$$CE_0(CE_t) = \frac{CE_t(L_t)}{(1+i)^t}.$$
 (5)

The discount rate *i* describes the discounting of a certain future outcome¹⁵ and corresponds to arrow C in Figure 1. We compare *i* to two different discount rates r_1 and r_2 , which both describe the discounting of the risky future outcome L_i .

The expected value $E(L_t)$ can be discounted to $CE_0(L_t)$ in one step (Figure 1, arrow B):

$$CE_0(L_t) = \frac{E(L_t)}{(1+r_1)^t}.$$
 (6)

Alternatively, L_t can first be transformed into a certain future outcome $CE_t(L_t)$ (arrow A in Figure 1) which is then discounted to $CE_0(CE_t)$ (arrow C in Figure 1):

$$CE_0(CE_t) = \frac{E(L_t)}{(1+r_2)^t}.$$
 (7)

The rates r_1 and r_2 both measure the decision maker's risk and time preference. Here, r_1 is the discount rate in case time and risk preferences are elicited simultaneously. The discount rate r_2 results when time and risk preferences are assessed separately. Note that r_2 contains *i* in the sense that *i* corresponds to arrow C while r_2 corresponds to A and C.

The Short-Long Term Asymmetry. Table 2 displays for how many subjects six-month discount rates calculated via (5), (6), and (7) are above, equal to, or below the corresponding 24-month rates. Z-values derived from a sign test are given for each entry, significant entries are shaded.¹⁶

The data strongly support the short-long term asymmetry, for both certainty and risk. The only entries in Table 2 which do not support the asymmetry are those

¹³ For instance, Benzion et al. (1989, p. 275) report 78 out of 282 (=28%) unusable responses, and Shelley (1993, p. 811) 14 out of 88 (=16%). Thaler (1981, p. 203) does not report how many subjects participated to begin with but only reports that he received "about twenty usable responses."

¹⁴ Two subjects stated $CE_6(L_6) = 2,000$, $CE_{24}(L_{24}) = 5,000$ and $CE_6(L_6) = 150$, $CE_{24}(L_{24}) = 3,000$. Excluding these responses reduces the mean of $CE_6(L_6)$ to 931.43 and the mean of $CE_{24}(L_{24})$ to 942.09.

¹⁵ This interpretation assumes that subjects do consider the future amount to be certain. We take this assumption as a working hypothesis, for we cannot know to what extent subjects' responses may have been influenced, contrary to the experimental design, by a feeling that all future is uncertain.

 $^{^{16}}$ In order to simplify the tables, we visualize entries that are significant at 5% (1%) level lightly (darkly).

		Gains (N = 55)			Losses $(N = 52)$				
Lottery	Certainty Equivalents	t = 6 n median	nonths mean	t = 24 median	months mean	t = 6 n median	nonths mean	t = 24 median	months mean
x = DM 120	$CE_{0}(L_{t})$	110.00	104.82	100.00	91.20	-110.00	-102.92	-100.00	-96.50
p = 0.99	$CE_{t}(L_{t})$	112.00	107.72	110.00	103.73	-115.00	-108.18	-115.00	-105.73
	CE_0 (CE_t)	105.00	97.42	95.00	80.35	-105.00	-95.40	-95.00	-84.89
x = DM 250	$CE_{0}(L_{t})$	110.00	107.50	100.00	95.85	-115.00	-111.54	-100.00	-103.85
p = 0.5	$CE_{t}(L_{t})$	120.00	120.44	120.00	112.75	-120.00	-116.67	-122.50	-115.63
	CE_{0} (CE _t)	114.00	107.40	90.00	83.42	-110.00	-102.35	-100.00	-92.16
x = DM 25000	$CE_{0}(L_{t})$	235.00	975.87	240.00	860.11	-248.50	-825.56	-250.00	-797.94
p = 0.01	$CE_{t}(L_{t})$	250.00	936.65	250.00	1053.29	-250.00	-931.37	-255.00	-975.87
	CE_{0} (CE _t)	228.00	863.78	220.00	861.48	-244.00	-686.46	-232.50	-631.78
x = DM 250	$CE_{o}(L_{t})$	110.00	107.30	100.00	97.04	-112.00	-111.92	-105.00	-108.73
Event	$CE_{t}(L_{t})$	120.00	116.82	120.00	116.30	-115.00	-113.02	-117.50	-110.43
	CE_{0} (CE _t)	105.00	99.18	95.00	116.30	-104.00	-99.23	-100.00	-89.11

 Table 1
 Matching Tasks—Means and Medians of Subjects' Responses

that refer to risk-adjusted rates for the lottery (x = DM 25,000, p = 0.01). Since risk-adjusted rates describe both risk and time preferences, we tested for possible differences in the assessments of $CE_6(L_6)$ as opposed to $CE_{24}(L_{24})$. We found none; in addition, within the three subgroups of subjects with $CE_6(L_6) < (=, >) CE_{24}(L_{24})$, we were able to confirm the short-long term asymmetry, as well.

The Certainty-Risk Asymmetry. Reconsider Figure 1. The lottery L_t and the certain amount $CE_t(L_t)$ are equally attractive. In order to examine whether certain and risky outcomes are discounted differently, we therefore need to compare the present value $CE_0(L_t)$ of L_t with the present value $CE_0(CE_t)$ of the equivalent $CE_t(L_t)$. According to Stevenson's hypothesis, $CE_0(L_t)$ will be higher than $CE_0(CE_t)$. If both risk and time are considered simultaneously (as in the elicitation of $CE_0(L_t)$), it is possible that the risk distracts from the necessity to discount. This is not possible if risk and time are separately taken into account (as in $CE_0(CE_t)$). Table 3 displays the number of subjects with $CE_0(L_t)$ above (equal to, below) $CE_0(CE_t)$. Z-values derived from a sign test are given, significant entries are shaded. The hypothesis that risky outcomes are discounted less than certain ones is supported for losses, for both short- and long-term rates. For gains, the picture is not as clear. In all entries, more subjects state preferences in favor of the hypothesis than against it. However, only four entries are significant, while four entries are not. In summary, our data do not reject Stevenson's finding of a certainty-risk asymmetry for gains. They clearly extend this asymmetry to losses.

The Gain-Loss Asymmetry. Since each subject dealt either only with gains or only with losses, we conduct this analysis through a between-subjects comparison. Suppose that, according to our hypothesis, gains and losses are discounted at equal rates. If we order individual implicit discount rates obtained both from the gain and the loss groups into a single ranking, among those rates that are below the median, equally many rates should stem from the gain and loss groups. This rank test is known as the median test by Westenberg and Mood (Gibbons 1971). Table 4 displays how many discount rates below the median of the common empirical distribution of gains and

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		Gains $(N = 5)$	5)	Losses (N = 52)		
Lottery	i	r ₁	r ₂	i	r ₁	r ₂
x = DM 120	36 : 2 : 17	41:0:14	46:0:9	41 : 1 : 10	43:0:9	44:1:7
p = 0.99	Z=2.56	Z=3.64	Z=4.99	Z = 4.30	Z=4.71	Z=5.13
x = DM 250	35 : 2 : 18	39:1:15	38:2:15	38:2:12	34 : 0 : 18	41:3:8
p = 0.5	Z = 2.29	Z=3.24	Z=3.10	Z=3.61	Z = 2.22	Z=4.58
x = DM 25000	35 : 3 : 17	32 : 1 : 22	28 : 2 : 25	42 : 1 : 9	25 : 0 : 27	23 : 0 : 29
p = 0.01	Z = 2.43	Z = 1.35	Z = 0.54	Z = 4.58	Z = -0.23	Z = -0.83
x = DM 250	38:2:15	37:0:18	39:1:15	40:2:10	35 : 2 : 15	41:1:10
Event	Z=3.10	Z=2.56	Z=3.24	Z=4.16	Z = 2.77	Z=4.30
Overall	144 : 9 : 67	149 : 2 : 69	151 : 5 : 64	161 : 6 : 41	137 : 2 : 69	109 : 5 : 54

Table 2 Results for Matching Tasks Show Short-Long Term Asymmetry

The upper left entry is to be read as follows:

for 36 subjects the 6 month rate was above the 24 months rate,

for 2 subjects they were equal,

for 17 subjects the 6 month rate was below the 24 months rate.

	Gains ((N = 55)	Losses ($N = 52$)			
Lottery	t = 6 months	t = 24 months	t = 6 months	t = 24 months		
x = DM 120	30:12:13	40:9:6	33:9:10	36:8:8		
p = 0.99	Z = 2.29	Z = 4.58	Z=3.19	Z = 3.88		
x = DM 250	27 : 11 : 17	32:10:13	26:12:14	35:7:10		
p = 0.5	Z = 1.35	Z = 2.56	Z = 1.66	Z = 3.47		
x = DM 25000	28:6:21	28:8:19	30:5:17	37:7:8		
p = 0.01	Z = 0.94	Z = 1.21	Z = 1.80	Z=4.02		
x = DM 250	30:9:16	29:4:22	35:6:11	38:8:6		
Event	Z = 1.89	Z = 0.94	Z = 3.33	Z=4.44		
Overall	115 : 38 : 67	129 : 31 : 60	124 : 32 : 52	146 : 30 : 32		

Table 3 Results for Matching Tasks Show Certainty-Risk Asymmetry

The upper left entry is to be read as follows:

for 30 subjects the present value of the lottery was above the present value of its certainty equivalent, for 12 subjects they were equal,

for 12 subjects they were equal,

for 13 subjects the present value of the lottery was below the present value of its certainty equivalent,

	t = 6 months			t = 24 months			
Lottery	i	r ₁	r ₂	i	r ₁	r ₂	
x = DM 120	24 : 27	19:16	25 : 24	29 : 24	22 : 20	24 : 23	
p = 0.99	Z = -0.42	Z = 0.17	Z = 0.14	Z = 0.69	Z = 0.31	Z = 0.15	
x = DM 250	24 : 21	25 : 23	24 : 29	29 : 24	25 : 23	28 : 24	
p = 0.5	Z = 0.45	Z = 0.29	Z = -0.69	Z = 0.69	Z = 0.29	Z = 0.55	
x = DM 25000	17 : 22	27 : 25	27 : 23	29 : 24	23:23	26 : 26	
p = 0.01	Z = -0.8	Z = 0.28	Z = 0.57	Z = 0.69	Z = 0.00	Z = 0.00	
x = DM 250	25 : 27	26 : 25	25 : 27	27 : 26	28 : 20	28 : 25	
Event	Z = -0.28	Z = 0.14	Z = 0.28	Z = 0.14	Z = 1.15	Z = 0.41	
Overall	90 : 97	97 : 89	101 : 103	114 : 98	98 : 86	106 : 98	

Table 4 Results for Matching Tasks Show No Gain-Loss Asymmetry

The upper left entry is to be read as follows:

among the individual rates below the median of the common empirical distribution of gain and loss rates

24 rates stem from the loss group and

27 stem from the gain group.

losses stem from the loss group.¹⁷ Z-values are given for each entry.

No entry supports a gain-loss asymmetry at the 5% level. For the certainty case, this finding replicates the results of Shelley (1993). For the case of risk, this finding needs to be compared to the results of Shelley's (1994) subsequent study. Shelley (1994) found some evidence for a gain-loss asymmetry for risk. Our finding suggests, in line with Shelley's own remarks (see Footnote 5), that the management investment frame used in her study might be something special.

5. Part 2–Choice

5.1. Design

In Part 2, subjects were presented with choices between a lottery at time t, denoted L_t , and a lottery L' at time t', denoted $L'_{t'}$. Subjects could state indifference or pref-

¹⁷ Discount rates that equal the median are disregarded by this median test.

erence for either of the two alternatives. Figure 4 shows a typical computer screen in Part 2.

To explain how the short-long term asymmetry was tested, recall that the exponential discounting model builds on the stationarity axiom (2), which cannot accommodate a short-long term asymmetry. In contrast, the hyperbolic discounting model builds on the stretching axiom (4), which implies a short-long term asymmetry. Short-long term asymmetry was tested by asking whether stationarity (2) or stretching (4) is a better description of subjects' actual choices. Let us, for example, assume that a subject prefers L_0 (DM 120 for sure) over L'_{6} (0.5 chance of DM 250, nothing otherwise). The preference obeys stretching if the decision maker will still prefer the sure amount over the lottery after we have stretched the pair t = 0, t' = 6 month by some arbitrary factor. Stretching by a factor 2 (factor 5) yields L_6 vs. L'_{18} (L_{24} vs. L'_{54}).¹⁸ To characterize a wider variety of

¹⁸ Taking six months as the length of one period, the time interval t = 0 months t = t' = 6 months is [0, 1]. A stratch by a factor of 2 (see

^{= 0} months -t' = 6 months is [0, 1]. A stretch by a factor of 2 (see





Please choose by moving the cursor to your preferred alternative and confirm with "return"

behavior, assume a decision maker prefers L_0 over L'_6 , is indifferent between L_6 and L'_{18} , and prefers L_{54} over L'_{24} . We would characterize this behavior by saying that under stretches, the decision maker's preference changes towards the later alternative.¹⁹ The stationarity axiom can be tested similarly.

In Part 2, three categories of questions tested the short-long term asymmetry. Category CR presented choices between a certain and a risky alternative, category CC presented choices between two certain alternatives, and category RR presented choices between two risky alternatives. For all three categories, five pairs were presented: a choice between L_0 and L'_{6} ; to test stationarity, choices between L_6 and L'_{12} , as well as between L_{24} and L'_{30} ; to test stretching, choices between L_6 and L'_{54} .

Category CR consisted of 20 choices constructed from four basic choice pairs: the four lotteries L' from Part 1 and L equal to the individual $CE_0(L'_6)$. In category CC, the one basic choice pair was $L = CE_6$ (x = 250, p = 0.5) and $L' = CE_0(CE_6$ (x = 250, p = 0.5)). Category RR was derived from the one choice between a 0.5 chance of obtaining CE_6 (x = 250, p = 0.5) and nothing otherwise and a 0.5 chance of obtaining $CE_0(CE_6$ (x = 250, p = 0.5)) and nothing otherwise.

A fourth category of questions was designed to test the certainty-risk asymmetry. Suppose that, according to Stevenson's hypothesis, risky outcomes are discounted less than certain ones. Shifted into the future, a risky gain will become relatively more attractive than a certain gain, thus more subjects should prefer the risky gain over the certain gain. The opposite should hold for losses. If certain and risky outcomes are discounted at equal rates, however, subjects' preferences should remain constant as waiting time increases. Gain-loss asymmetry can therefore be tested by having subjects choose between alternatives L_t and L'_t , then increasing t and having subjects choose again. We defined 12 choices: the four lotteries from Part 1 vs. $CE_6(L_6)$ were presented in t = 0, in t = 6 months, and in t = 24 months.

We did not test the gain-loss asymmetry for choice, because no subject handled both gains and losses. If the same choice were given to subjects twice, with outcomes once framed as a loss and once framed as a gain, discrepancies between both choices might shed light on the empirical status of the gain-loss asymmetry for choice. We suspected, however, that subjects would have realized that the same choice had already been given to them before, only with a minus sign prefixed to the amounts.

5.2. Results

The Short-Long Term Asymmetry. Table 5 displays how many subjects stated stable preferences or preferences changing toward the earlier or later alternatives in categories CR, CC, and RR.

Consider the first two columns, which refer to the test of stationarity. In all entries, there are more subjects with stable preferences under shifts, thus obeying the stationarity axiom, than there are subjects whose preferences change under shifts. For 8 of the 12 entries for shifts, this is significant at a level of at least 5%. These

Equation (4)) yields [1, 3] or t = 6 months, t' = 18 months and a stretch by a factor of 5 yields [4, 9] or t = 24 months, t' = 54 months. Note that this calculation is sensitive to the choice of time unit.

¹⁹ The example can be denoted as "EIL" (preference for Earlier-Indifferent-Later alternative). Response patterns EEL, EIL, EII, ELL, IIL, ILL are all interpreted as changing toward the later option. Response patterns LLE, LLI, LIE, LII, LEE, IIE, and IEE are interpreted as changing toward the earlier alternative. Note that a few response patterns (e.g., ELE) do not fit into this scheme.

			Shi	fts	Stretches	
Category	Alternatives		Gains	Losses	Gains	Losses
	certain	x = DM 120	7:29:9	2:26:10	26 : 21 : 1	0 : 19 : 27
		p = 0.99	Z = 1.94	Z = 2.27	Z = -0.14	Z = -1.18
CR	amount	x = DM 250	10:28:8	6:26:10	24:22:2	5:14:27
		p = 0.5	Z = 1.47	Z = 1.54	Z = -0.58	Z = -2.65
	vs.	x = DM 25000	6:33:9	6:32:10	10:35:4	4:32:9
		p = 0.01	Z = 2.60	Z = 2.31	Z = 3.00	Z = 2.83
		x = DM 250	6:27:11	7:29:10	20:29:4	6:18:20
		Event	Z = 1.51	Z = 1.77	Z = 0.69	Z = 1.09
CC	two certain amounts		8:32:8	3:33:5	29:20:0	4:12:30
			Z = 2.31	Z = 3.90	Z = -1.29	Z = -3.24
RR	two lotteries		8:29:10	5:29:8	23:27:2	3:18:23
			Z = 1.57	Z = 2.47	Z = 0.28	Z = 1.21
Overall			45 : 178 : 55	29:175:53	132 : 154 : 13	22 : 113 : 136

Table 5 Results for Choice Tasks Show No Short-Long Term Asymmetry

The upper left entry is to be read as follows:

7 subjects' preferences changed towards the earlier alternative,

29 subjects' preferences were stable,

9 subjects' preferences changed towards the later alternative

entries are shaded. Z-values are given. Thus, contrary to our hypothesis, the data support the stationarity axiom and reject a short-long term asymmetry.

Next, consider the last two columns, which refer to the test of the stretching axiom. Only for the DM 25000 entry in category CR, for both gains and losses, are there significantly more stable than changing preferences under stretches. In contrast to these two entries which support the stretching axiom, the DM 250 loss entry in category CR as well as the loss entry in category CC report significantly more changing than constant preferences. Z-values are given. All four significant entries—both those supporting and those contradicting the stretching axiom—are shaded. Thus, contrary to our hypothesis, the stretching axiom is not supported. The choice based results clearly imply that stationarity is the better description of choice behavior. Much to our own surprise, we did not detect a short-long term asymmetry for choice.

Further, the data for stretching (last two columns in Table 5) show that the nonstable preferences change in a systematic manner: for gains, more preferences change toward the earlier alternative; for losses, more preferences change toward the later alternative. This finding is significant at the 5% level in all entries except for the DM 25,000 lottery in category CR. In order not to overload Table 5, we have restrained from giving Z values for this test. These systematically changing preferences may be explained by impatience. Under stretches, the time interval between both alternatives increases. Then for gains, impatience implies that preferences change toward the earlier; for losses toward the

Table 6 Results for Choice Tasks Show No Certainty-Risk Asymmetry

Alte	ernatives	Gains	Losses	
	x = DM 120	5: 39 : 5	9:28:12	
	p = 0.99	Z = 2.71	Z = 1.00	
certain	x = DM 250	4:32:12	6:32:9	
amount	p = 0.5	Z = 2.31	Z=2.48	
vs.	x = DM 25000	5:37:8	8:34:4	
lottery	p = 0.01	Z = 3.39	Z = 2.48	
	x = DM 250	8:35:5	4:34:4	
	Event	Z = 3.18	Z = 3.24	
Overall		22:143:30	27 : 128 : 31	

The upper left entry is to be read as follows:

5 subjects' preferences changed toward the certain alternative, 39 subjects' preferences were stable,

5 subjects' preferences changed toward the risky alternative

later option. If the increase of the time interval causes this response pattern, this is evidence that subjects' preferences between two alternatives remain stable as long as the time interval between them remains constant. This is precisely the stationarity condition which corresponds to stable preferences under shifts.

The Certainty-Risk Asymmetry. Table 6 displays how many subjects had constant response patterns or patterns changing towards the risky or certain option.²⁰

For all entries, more subjects had constant preferences than changing preferences. Z-values are given; significant entries are shaded. Contrary to our hypothesis, we did not find a certainty-risk asymmetry for choice, for neither losses nor gains. Like for the short-long term asymmetry, the certainty-risk asymmetry appears to be a behavioral phenomenon which is restricted to matching tasks. The causes for this finding are yet to be explored.

6. Summary and Conclusion

This study has investigated intertemporal choice and compared the cases of certainty and risk, both via matching and choice tasks. Figure 5 summarizes the results for discounting certain or risky gains or losses. See Figure 2 for the results prior to our study.

We found that the results are sensitive to the elicitation procedure used. For matching tasks, the short-long term asymmetry—a larger discount rate for the first period than for subsequent ones-persists both for risk and certainty. This was not confirmed through choice tasks. At this point we can suggest that exponential discounting may be a valid description of pairwise choice, while hyperbolic discounting more accurately describes pricing behavior. Using matching, gains and losses were discounted at equal rates. This supports earlier findings by Shelley (1993) that gain-loss asymmetries are only due to reference point shifts. Finally, we have established that with matching, risky outcomes are discounted less than equally attractive certain ones. This supports earlier findings by Stevenson (1992), who suggested that the risk of a future lottery distracts from the necessity to discount. Again, this finding is not supported in the choice part of our study.

Being puzzled by the discrepancies between matching and choice data ourselves, we have only guesses to offer as to what might be the cause of our findings. Systematic differences between choice and matching have been reported before. Lichtenstein and Slovic (1971) established the famous preference reversal phenomenon, which states that people often attach a higher value to one of two options, but choose the other when being offered a choice between the two. Tversky et al. (1990) report preference reversals in an intertemporal context. In pairwise comparisons, they found that subjects tend to choose a short-term option over a long-term option but tend to price the long-term option higher. Note, however, that the discrepancy between choice and matching which we have detected is not that lower valued options are chosen over higher valued options.²¹

²⁰ Response patterns are classified as in Footnote 19.

²¹ Although we can detect evidence in our data for preference reversals. Note that subjects had to choose between $CE_0(L_6)$ vs. L_6 with the $CE_0(L_6)$ that had been elicited individually in the matching part. All strict preferences in this choice could be interpreted as preference re-





Rather, the key message of this paper is that general principles which have been found to govern the evaluation of alternatives by matching do not seem to produce valid predictions for choice.

Now, what could be a possible reason for the discrepancy between choice and matching? In decision making under risk, it is well known that subjects edit alternatives before choosing between them (Kahneman and Tversky 1979). Prospect theory suggests that in an editing process, subjects cancel common outcomes of both alternatives. A common consequence in an intertemporal setting might be a common delay in both alternatives to choose between. Following prospect theory, subjects would cancel such common delays. If they do, they exactly cancel out the effects of shifting alternatives. It is no surprise then, that choices remain stable under shifts. In contrast, there can be no cancellation in matching tasks, since in matching, each alternative is considered individually.

Of course, one must also look for alternative explanations. Tversky et al. (1988) report a prominence effect which states that in choice tasks, people concentrate on the most prominent dimension of the two alternatives. In matching tasks, however, this dimension is given less implicit weight. This may be due to a compatibility bias, which means that the weight of a stimulus feature is determined by its compatibility to the response task. Fischer and Hawkins (1993) have analyzed the prominence effect and different types of compatibility biases. Although at first glance it may seem worthwhile to elaborate on such behavioral phenomena in order to explain our results, we do not see how our findings could be traced either to the prominence effect or to compatibility biases. Future research will investigate the causes of the discrepancies between choice and matching. Also, other forms of judgments (e.g., attractiveness ratings) will be considered to check the stability of the results found here.

The empirical status of the certainty-risk asymmetry is not settled, either. It seems limited to matching, though an answer to the question why still needs to be found. If there is a certainty-risk asymmetry, this has important implications (e.g., for financial intermediaries). Many financial products involve risky future returns. Customers who plan to invest in these products are more likely to focus on the risk of the return than on the time horizon over which this (risky) return will be realized. They will therefore tend to choose investments with a (too) moderate risk and which yield their return only after (too) long time horizons. Professional advice to customers may take this into account. It may seem worthwhile for future research to explore the empirical status of a certainty-risk asymmetry for this special investment context.22

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versals, using a strict interpretation of this phenomenon. On the overall level, 74% of the responses revealed preference reversals for gains (67% for losses). In accordance with Tversky et al.'s (1990) finding, more preference reversals are due to strict preferences for the shortterm $CE_0(L_6)$ (gains: 48%; losses: 49%) than for the long-term L_6 (gains: 26%; losses: 18%).

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