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Performances of Heat Exchangers with Tapered Fins

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ABSTRACT

The steady-state rates of deliberate heat dissipation from an array of vertical *duralumin fins, under natural convection conditions, have been measured when their base (which was either horizontal, with the fins protruding upwards, or vertical) was maintained at a uniform temperature of 40 K above that of the ambient environment. The 500-mm long tapered fins were each 1·6 mm at their tips and 4·4 mm thick at their base, and protruded 60 mm perpendicularly outwards from their 500 mm × 190 mm rectangular base.*

The optimal peak-to-peak pitch, S'_{opt} *of the fins, corresponding to the maximum steady-state rate of heat loss to the ambient air, was 14 mm ± 1 mm. This was smaller than that obtained under similar imposed conditions, with an array of rectangular fins, identical in dimensions except for their uniform fin thickness, and was almost invariant with respect to the orientation of the fin array for* $\theta = 40$ *K.*

The steady-state rates of heat dissipation from these two types of exchanger were compared. For the horizontally-based *finned systems, slight decreases in the rates of heat dissipation were observed for the tapered-fin arrays compared with those for the rectangular-fin arrays. However, for the orientation with vertical fins protruding outwards from the* vertical base, *increases in the rates of heat dissipation were observed with the tapered-fin arrays for* $8\ mm \leq S' \leq 15\ mm$*, but decreases occurred for larger inter-fin separations.*

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NOTATION

a_n	Constant value in eqns (2) and (3) depending on the heat-transfer coefficient as well as the emissivity of material of the finned surface and the temperature: $n = 1, 2, 3$ or 4
b	Fins' protrusion orthogonal to the heat-exchanger's rectangular base (see Figs 1 and 2), m
h	Average value of the coefficient for steady-state heat-transfers between the fins' surface and the environment, $W\,m^{-2}\,K^{-1}$
k	Thermal conductivity of the fin material, $W\,m^{-1}\,K^{-1}$
L	Common length of the vertical fins and the base, to which they are attached rigidly (see Figs 1 and 2), m
$\dot{Q}$	Steady-state rate of heat dissipation from the heat-exchanger, via the finned surface, to the surrounding air, W
S'	Uniform separation (i.e. the fin pitch) between the central planes of adjacent fins: this value applies for all the fins of the considered configuration (see Fig. 2), m
S'_{opt}	Optimal value of S' corresponding to the maximum value of $\dot{Q}$ for a given $(T_b - T_\infty)$, m
T_b	Steady-state temperature of the base's surface, which is exposed to the air (see Fig. 1), K
$T(x)$	Steady-state temperature at a distance x along the fin length (see Fig. 1), K
T_∞	Mean steady-state temperature of the surrounding ambient air, K
x	Distance along fin's centre-line, i.e. orthogonal to the base, measured in the direction from the tip to the root of the fin (see Fig. 1), m
y_b	Half fin thickness at the base of the fin (see Fig. 1), m
$y(x)$	Half fin thickness at a distance x along the fin from its tip (see Fig. 1), m
W	Total width of the fin array (see Fig. 2), m
α	Dimensionless parameter depending on the fins' profile
α_o	Value of α corresponding to the optimal fin profile
θ	Steady-state excess temperature of the exposed surface of the base of the fin array above that of the surrounding environment, K

OPTIMAL FIN SHAPE FOR LOSING HEAT RAPIDLY

Rectangular fins are used extensively to increase the rates of natural convection heat losses from systems, because such fins are simple, and cheap, to manufacture. However, it is well known[1–4] that it is possible to modify

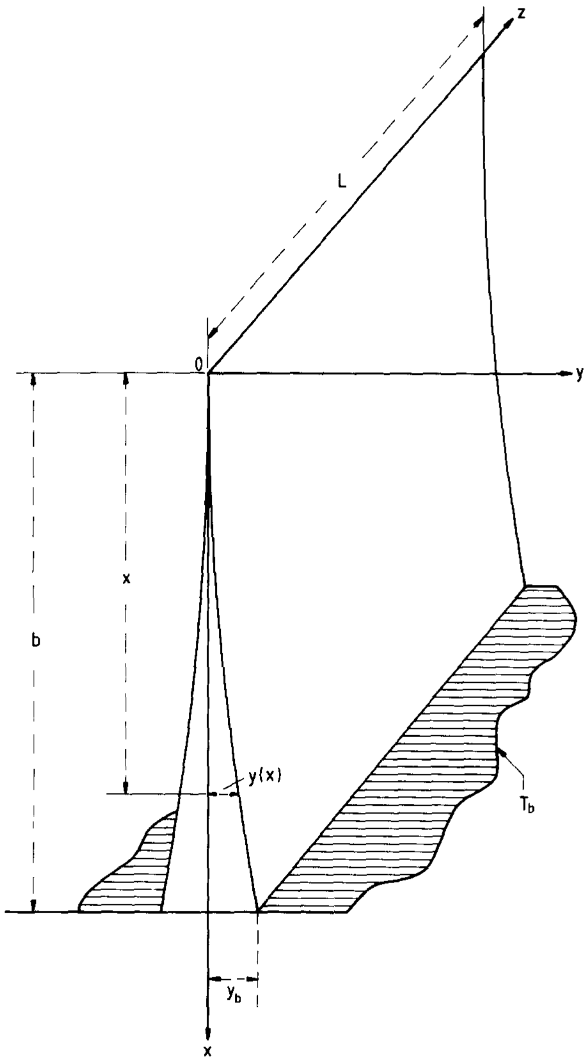

Fig. 1. The optimal fin.

the fins' profile in order to enhance further the rate of heat dissipation, yet simultaneously achieve an appreciable saving in the material required. The optimal fin (in the present context) is regarded as the one that needs the minimum amount of material to permit a given rate of heat dissipation to be obtained under specified circumstances, and so its use leads to lighter heat-exchangers. Nevertheless, the theoretical optimal fin shape (see Fig. 1) would be difficult and expensive to manufacture, and in any case would probably become deformed during service. Thus a truncated tapered fin, which closely approximates to the optimal shape near the fin's base, has been suggested for use in practice. However, few experimental studies have been undertaken to

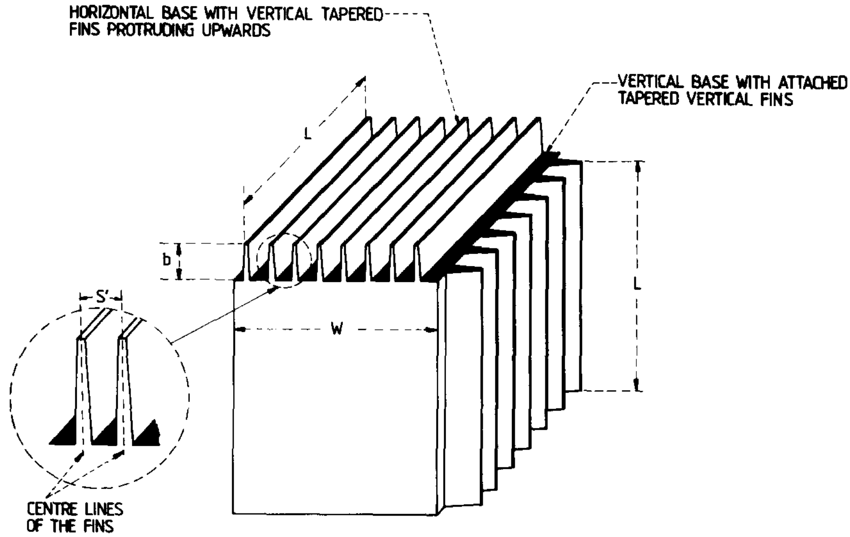

Fig. 2. The two most commonly employed orientations of open-ended, uniformly-spaced, vertical linearly-tapered fins: these are considered in this investigation. The geometric parameters influencing the heat-exchanger's performance are indicated.

investigate the heat-transfer performances of a parallel array of such tapered fins. So, in the present tests, experimental studies were undertaken for heat-exchangers with vertical tapered fins, protruding perpendicularly either outwards from a vertical rectangular base or upwards from a horizontal rectangular base (see Fig. 2).

PERTINENT PREVIOUS STUDIES

Steady-state heat-transfer performances of these two orientations of arrays of vertical duralumin fins (protruding from a duralumin base) have been determined previously.[5,6] However, only the *rectangular* fin shape was employed in these studies. Therefore the effect of using tapered fins, on the rate of heat dissipation, still needed to be ascertained.

Optimal-volume fin designs were deduced by Schmidt[1] for a predominantly 'convection' fin, and by Wilkens[7] for a predominantly 'radiation' fin. Cobble[8] proposed a method for determining the shape of an optimal-volume fin, so that the total rate of the simultaneous convection and radiation losses is maximised. It is assumed that the temperature profile along the central plane of each fin is given by

$$T(x) = T_\infty + (T_b - T_\infty)(x/b)^\alpha \tag{1}$$

where the index α (which is >0) is a dimensionless parameter dependent on the fin's profile. (For a rectangular fin, $\alpha \to 1$.) According to Cobble,[8] the optimal-volume fin has a half fin thickness, at a protrusion $(b-x)$ from the fin's base, of

$$y(x) = \frac{y_b \sum_{n=1}^{4} \frac{a_n}{(1+n\alpha_o)} \left(\frac{x}{b}\right)^{2+(n-1)\alpha_o}}{\sum_{n=1}^{4} \frac{a_n}{(1+n\alpha_o)}} \tag{2}$$

where

$$y_b = \left[\frac{b^2}{\alpha_o k}\right] \sum_{n=1}^{4} \frac{a_n}{(1+n\alpha_o)} \tag{3}$$

and α_o is the value of α for which the volume of the fin is the minimum corresponding to a certain preselected steady-state rate of heat dissipation.

Also, for the special case of a 'convection only' fin, $\alpha = \alpha_o = 1$, and

$$y(x) = y_b \left(\frac{x}{b}\right)^2 \tag{4}$$

where

$$y_b = \frac{hb^2}{2k} \tag{5}$$

These equations are applicable for the case of a single fin and may not be appropriate for a particular array of such fins.

EXPERIMENTAL INVESTIGATION

The apparatus used in this investigation was adapted from that described previously[5,6] (see Fig. 3). The tapered-fin array was manufactured from duralumin, which had a thermal conductivity of $160\ \mathrm{W\ m^{-1}\ K^{-1}}$ at 20°C. In order to reduce the radiation heat leak, the tapered fins and base surface components were highly polished to achieve a surface emissivity of $\leq 0{\cdot}1$, so that the heat loss from the tapered-fin assembly (at just above ambient temperatures) was predominantly via natural convection. Then the truncated optimal-fin profile (see Fig. 4) could be described approximately by eqns (4) and (5) with $h = 10\ \mathrm{W\ m^{-2}\ K^{-1}}$ and $b = 0{\cdot}24$ m. The assembly of tapered fins and spacer bars (as in Fig. 3) were pressed together by means of screwed nuts on threaded rods. The Dow Corning 340 heat-sink compound, between the pressed surfaces, ensured that only very low thermal contact resistances

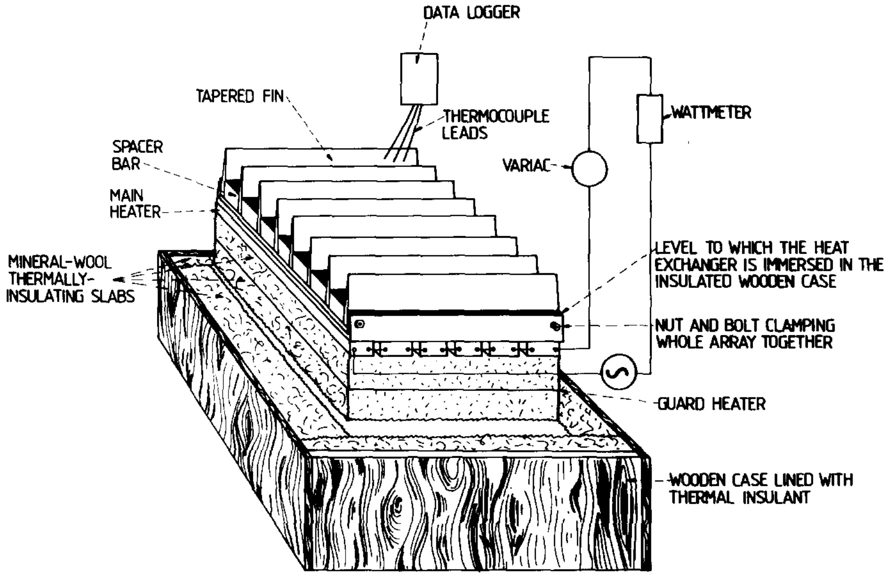

Fig. 3. Schematic representation of the main components of the test rig.

arose. The main heater was composed of six 100-W strip heating elements, which were attached to the back of the base of the fin assembly. A guard heater was installed behind, and parallel with, the main heater. Mineral-wool blankets were applied between the two heaters, as well as at the rear of the guard heater. The whole system was thermally well insulated with mineral-wool blankets, except for the finned surface, which was left bare and exposed to the ambient air. The power supplied to the heaters could be adjusted via a variac, and measured by an in-line wattmeter. The fin array's base temperatures were indicated by copper–constantan thermocouples and a data logger, and the base's surface in contact with the air was maintained at a steady state 40 ($\pm$0·4) K above the ambient temperature ($\sim$18°C). The test rig was supported by a metal framework in such a way that the heat-exchanger could be rotated through 90° in order to take up the two required commonly occurring orientations of the finned surface.

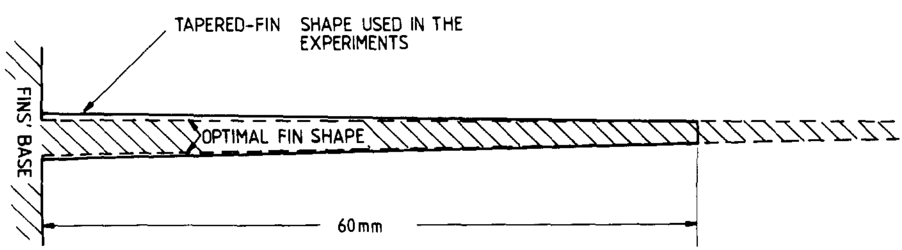

Fig. 4. Comparison of the truncated linearly-tapered fin employed and the optimal fin profile.

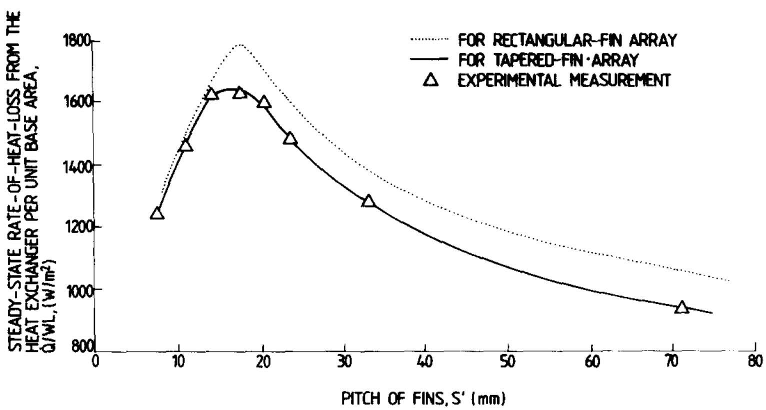

Fig. 5. Experimental results for the steady-state rate of heat loss through the air from an array of vertical fins protruding upwards from a *horizontal rectangular base*: $b = 60$ mm, $L = 500$ mm, $W = 190$ mm and $\theta = 40$ K.

OBSERVATIONS AND DEDUCTIONS

The total steady-state rates of heat dissipation, mainly due to natural convection from the finned surfaces of the vertically- or horizontally-based arrays of vertical linearly-tapered fins, with various fixed fin separations, have been measured, when their bases were maintained at 40 K above the ambient temperature. These rates of heat loss were accurate to $\pm 4\%$, due allowances having been made for the lateral (i.e. wild) losses from the base of the heat-exchanger. In Figs 5 and 6, the present experimental results are

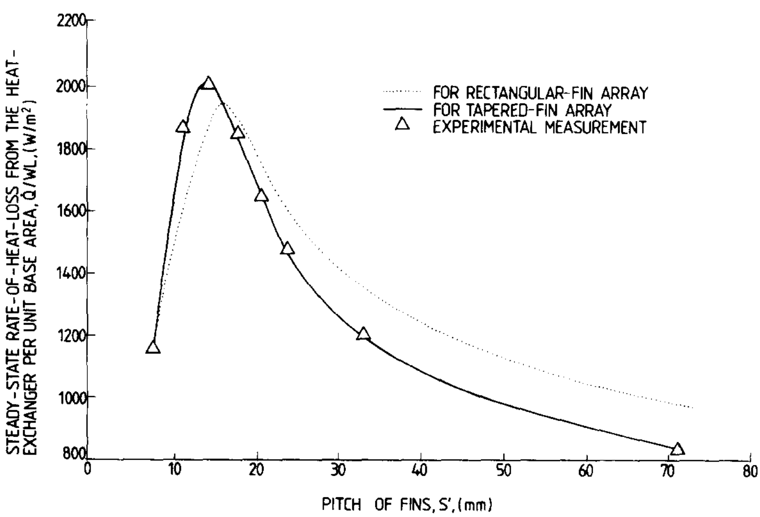

Fig. 6. As for Fig. 5 but with the fins protruding perpendicularly outwards from the now *vertical base*.

compared with those obtained with similar arrays, except for the use of highly-polished *rectangular* duralumin fins *each of the same volume* as each tapered fin. The relatively small heat-transfer contributions due to thermal radiation were almost identical with those for the corresponding rectangular fin array.[5,6]

For the array of tapered fins, the optimal fin pitch is 14 ± 1 mm and almost independent of the orientation of the fin array. For the *horizontally-based* fin array, the rate of heat dissipation from the tapered-fin array showed a slight *decrease* compared with that for the corresponding rectangular fin array of the same fin volume. However, the *vertically-based* tapered-fin array showed a slight *increase* in thermal performance for $8\,\text{mm} \leq S' \leq 15\,\text{mm}$, but still exhibited a drop in thermal performance for higher values of S'.

The tapered fin had approximately the optimal shape, as proposed in Ref. 8. Theoretically, the array of tapered fins should have exhibited a better performance than the assembly of rectangular fins. That this was not usually so was mainly due to the process of approximating to the optimal fin shape by the use of an easier to manufacture and more robust shape, namely that of the *truncated* linearly-tapering fin. However, the volume of the chopped-off optimal fin shape was small, but it corresponded to a large decrease of surface area. Thus the heat dissipation ability was diminished as a large portion of each fin surface's area had been sacrificed. Also, the proposed optimal profile was for a single fin, and this is probably not quite appropriate in the present investigation, which considered arrays of fins. The temperature drop from the base to the tip along a tapered fin in the present experiment was less than 3 K.

CONCLUSION

- For the linearly-tapered fin array, the optimal fin pitch, corresponding to the maximum steady-state rate of heat dissipation, was *reduced* compared with that for the rectangular-fin array, with fins of the same volume. The value of S'_{opt} was 14 ± 1 mm and almost invariant of the orientation of the array.
- For the horizontally-based heat-exchanger, a slight reduction of the steady-state rate of heat dissipation from the linearly-tapered fin array was observed compared with that from the rectangular-fin array (again of the same fin volume). However, a greater rate of heat dissipation was observed for the vertically-based tapered-fin array when $8\,\text{mm} \leq S' \leq 15\,\text{mm}$, but the rectangular-fin array still showed a better thermal performance for the larger inter-fin separations.
- The use of tapered (rather than rectangular) fins possesses no significant

advantage, and so it is not usually worth adopting complicated-shaped fins especially because of their higher manufacturing costs and greater vulnerabilities. The fin pitch (or spacing) is a much more influential parameter than the fin profile upon the rate of heat exchange that can be achieved.

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