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On the Reliability of Unsaturated Hydraulic Conductivity Calculated from the Moisture Retention Curve

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Abstract. In comparison with direct measurements of unsaturated hydraulic conductivity, the methods of calculations from the moisture retention curve are attractive for their fast and simple use and low cost. These are the main reasons for their increasing use, mainly in spatial variability studies. On the other hand, it is known that their applicability is limited. The possibility of the use of the retention curve to indirectly determine hydraulic conductivities is analyzed as follows. The theoretical derivation of the relation $K(h)-\theta(h)$ is briefly discussed with regards to potential sources of inaccuracy. The sensitivity of the algorithm for $K(h)$ calculation is studied as a response to possible inaccuracies in the retention curve determination. Conclusions about the usability of calculated hydraulic conductivities are drawn.

Key words. Hydraulic conductivity, retention curve, capillary model, inaccuracy, response, sensitivity.

1. Introduction

The precipitous spread of simulation models in soil physics has brought about the question of reliable input characteristics. Their reliability is of key importance when the adequacy of the model is being assessed.

The basic relation describing the movement of fluids in porous media (Richards' equation)

$$\frac{d\theta(h)}{dh}\frac{\partial h}{\partial t}=\frac{\partial}{\partial z}\left\{K(h)\left[\frac{\partial h}{\partial z}+1\right]\right\} \tag{1}$$

includes two hydraulic characteristics of the porous medium: the retention curve, namely the relation of moisture content θ $[L^3/L^3]$ to the suction head $h[L]$, and the hydraulic conductivity function $K(h)$ $[L/T]$ (t is time, z is the vertical coordinate positive upward).

In the derivation of models based on Richards' equation, it is assumed that both characteristics are obtainable from direct experiments. That is generally true, but in reality hydraulic conductivities are rather difficult to measure. In the respective literature, the increasing use of methods where the unsaturated hydraulic conductivity $K(h)$ is derived from the knowledge of $\theta(h)$ can be seen. The approach is based on the capillary model theory and supplies smooth mathematical functions convenient for mathematical modeling or large spatial variability studies. Nevertheless, the reliability of calculated hydraulic conductivities has not yet been fully proved.

The basic theoretical work was presented by Childs and Collis-George (1950) followed by Burdine (1953) and Mualem (1976). The problem has been analysed in detail in the works of Brooks and Corey (1964) and van Genuchten (1978, 1980). Some authors even evaluate the saturated hydraulic conductivity (McCuen *et al.*, 1981) using the theoretical conclusions of Brutsaert (1967) and Brakensiek (1977).

It has to be emphasized that the theoretical adequacy of assumptions on which the capillary model theory is built up (not very strong itself), does not automatically guarantee its practical usability. In this, the relation between the sensitivity of the algorithm of the $K(h)$ calculation from $\theta(h)$ towards the inaccuracy in $\theta(h)$ determination is very important. The problem is analyzed in this paper.

2. Accuracy of the Retention Curve Determination

The inaccuracies connected with the determination of the retention curve include:

(a) The inaccuracy caused by an insufficient representativeness of the sample for the porous medium in question.
(b) The inaccuracy caused by errors of measurement.
(c) The inaccuracy of curve-fitting through the experimental data.

All these inaccuracies combined appear in the fitted approximation function, which is usually used for the prediction of the $K(h)$ relation instead of original experimental data.

3. The Adequacy of the Capillary Model Theory

The indirect determination of $K(h)$ is based on the substitution of the real material structure by a suitable model. The hydraulic conductivity is analyzed at the level of a single structure element of this model. The transformation to the continuum-scale level of all quantities as required in Richards' equation is accomplished by an averaging of the model element quantities.

To form the basis for further conclusions, the basic points of the capillary model theory are mentioned. In the case of capillary models (CpM) the structure element is the capillary. The model structure geometry is characterized by the known pore-size distribution $F(r)$ of the capillary radii. To find a mathematical expression of the relation $K(h) - \theta(h)$, the relation $K(h) - F(r)$, which is, in fact, the result of the CpM, has to be completed with the relation $F(r) - \theta(h)$. It can be assumed according to the CpM

$$F(r) = \theta(h)/\theta_s \quad \text{for } h = \mathbf{X}/r, \tag{2}$$

where θ is the moisture content when pores up to radius 'r' are filled, θ_s is the maximum moisture content (when all pores are filled), h is the suction head which in the CpM is equal to the capillary head in the equivalent pore of radius 'r' (for simplicity notation 'h' is taken as positive for unsaturated material), and $\mathbf{X}$ is a phenomenological constant.

According to the principal idea of the capillary model theory, the real material and its CpM behave equally when both media have an equal pore-size distribution $F(r)$. In the case of real materials, the actual pore-size distribution is not usually available. It is assumed that the indirect determination of $F(r)$ using relation (2) is sufficiently accurate, not only for the CpM but also for the real material which the CpM represents. The need to introduce this assumption is one of the weakest points of the capillary model theory. It is obvious that the CpM model and the real soil are, in fact, two different materials. In some applications this can be the reason for considerable discrepancies in $F(r)$ determination. Another problem of equating $F(r)$ and $\theta(h)/\theta_s$ is the retention curve hysteresis. Pragmatically, the characteristic $F(r)$ is taken to be equal to the drainage branch of the retention curve, which is the easiest to obtain.

4. *K*(*h*) Relation Introduced by Capillary Models

The basic relation for unsaturated hydraulic conductivity derived from the CpM can be written in the form

$$K(\theta_e) = c(\theta_s - \theta_r) \int_0^{\theta_e} \frac{d\theta_e}{h^2}, \tag{3}$$

where

$$\theta_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{4}$$

is the effective moisture content, θ_r is residual moisture content, and 'c' is a phenomenological constant. It is possible to theoretically evaluate this constant. Such a value, however, can never be taken for granted as a true value for the real soil. For that reason the constant 'c' is regarded as 'unknown' or empirical.

Because of 'c', the hydraulic conductivities calculated from relation (3) usually differ significantly from the real values. Therefore, this relation is mostly used for the determination of relative conductivities

$$K_r(\theta_e) = K(\theta_e)/K(1) \tag{5}$$

only, and then combined with the measured saturated conductivity K_s. Here, the multiplication constant 'c' is eliminated.

With θ_e^2 being the empirical tortuosity factor, we obtain the method of Burdine (1953) from formula (3). The relative conductivity is

$$K_r(\theta_e) = \theta_e^2 \int_0^{\theta_e} \frac{\mathrm{d}\theta_e}{h^2(\theta_e)} \Big/ \int_0^1 \frac{\mathrm{d}\theta_s}{h^2(\theta_e)}. \tag{6}$$

In the theory of Mualem (1976), the relative conductivity is

$$K_r(\theta_e) = \theta_e^b \left[\int_0^{\theta_e} \frac{\mathrm{d}\theta_e}{h(\theta_e)} \Big/ \int_0^1 \frac{\mathrm{d}\theta_e}{h(\theta_e)} \right]^2, \tag{7}$$

where b is an empirical constant. The following analysis is generally valid also for other models derived from the same principle (Childs and Collis-George, 1950; Millington and Quirk, 1960; Jackson *et al.*, 1965).

5. Sensitivity of the Algorithm for $K(h)$ Determination Towards the Inaccuracy in the $\theta(h)$ Determination

Let us introduce the term $S(\theta_e)$ in such a way that relation (3) can be expressed by the equation

$$K(\theta_e) = c(\theta_s - \theta_r)S(\theta_e). \tag{8}$$

An important feature of the function $S(\theta_e)$ is that it already includes all the necessary information about the calculated absolute hydraulic conductivities and is not influenced by the size of the unknown constant 'c' and by the artificially introduced tortuosity. Therefore, the function $S(\theta_e)$ is suitable for any analysis of hydraulic conductivities in absolute and/or relative values. Analogically, for K_r it can be written (from relations (6) and (7))

$$K_r(\theta_e) = \theta_e^b \frac{S(\theta_e)}{S(1)}, \quad K(\theta_e) = K_r(\theta_e)K_s. \tag{9}$$

As the result of the $K(\theta_e)$ and $h(\theta_e)$ dependence, the relationship $K(\theta_e)$ is partly loaded by inaccuracies caused by the theoretical inadequacy of simplifying assumptions and model images used to mathematize the relation $K(\theta_e)-h(\theta_e)$ and partly by the inaccuracy of the $h(\theta_e)$ determination. The inaccuracy $\delta h(\theta_e)$ in the $h(\theta_e)$ determination is transformed by the algorithm which relates $K(\theta_e)$ and $h(\theta_e)$ to the inaccuracy $\delta K(\theta_e)$ in the calculated $K(\theta_e)$. It is impossible to mathematically express the exact shape of the inaccuracy $\delta h(\theta_e)$, but it is still possible to study the transform $\delta h(\theta_e) \rightarrow \delta K(\theta_e)$ itself and draw conclusions about the influence of the inaccuracies in $h(\theta_e)$ determination on the calculated $K(\theta_e)$. Two cases of $\delta K(\theta_e)$ will be considered separately with the denotation: $\eta K(\theta_e)$, when the absolute hydraulic conductivities are treated (algorithm (8)) and

$\epsilon K(\theta_e)$, when the relative hydraulic conductivities are treated (algorithm (9)) to get the K (θ_e) prediction.

Let us define the inaccuracy function as the response of the algorithm for the calculation of $K(\theta_e)$ from $h(\theta_e)$ towards the variation $\delta h(\theta_e)$ of the retention curve $h(\theta_e)$ (see (8))

$$\eta K(\theta_e) = c(\theta_s - \theta_r)[S(\theta_e) - S^*(\theta_e)], \tag{10}$$

where $S^*(\theta_e)$ has $h(\theta_e) + \delta h(\theta_e)$ in the integrand instead of $h(\theta_e)$, e.g. in the case of basic formula (3)

$$S^*(\theta_e) = \int_0^{\theta_e} \frac{\mathrm{d}\theta_e}{[h(\theta_e) + \delta h(\theta_e)]^2}. \tag{11}$$

Relation (10) is meaningful for any chosen variation δh. To get the first image of the character of the transformation $\delta h \rightarrow \eta K$ through algorithm (8), we can choose the constant variation $\delta h = \Delta h$ for $\theta_e \in \langle 0, 1 \rangle$. Due to the shape of the retention curve near the saturation, and because of δh in the denominator of the integrand $S(\theta_e)$, the constant variation $\delta h(\theta_e)$ is transformed into the monotonically increasing function ηK with the tendency to rise sharply in the vicinity of saturation (see Figure 1a).

A different situation occurs when the combination of the calculated hydraulic

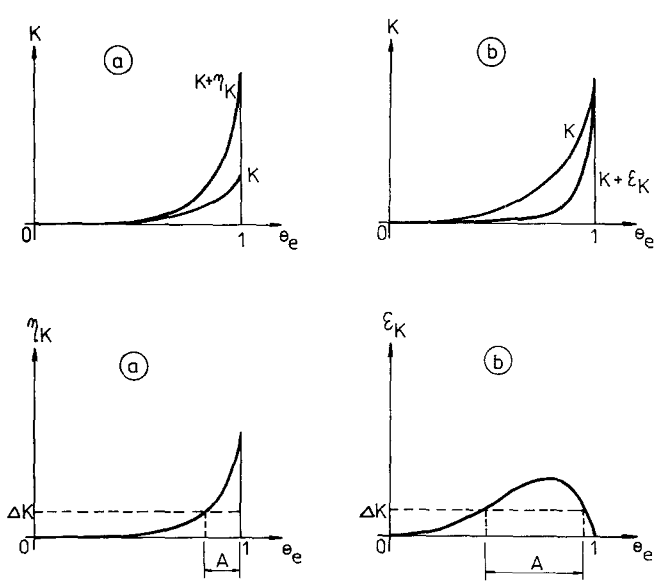


Fig. 1. The comparison of the response ηK and ϵK of the algorithm for unsaturated conductivity calculation, when applied to (a) the indirect determination of absolute hydraulic conductivity $K(\theta_e)$ in the whole-soil moisture interval, (b) the combination of the indirect determination of relative unsaturated conductivity $K_r(\theta_e)$ together with the experimental determination of saturated conductivity K_s.

conductivities $K_r(\theta_e)$ with the measured K_s is used for the $K(h)$ determination. In analogy with (10), it can be written (see (9)) as

$$\epsilon K(\theta_e) = K_s \theta_e^b \left[\frac{S(\theta_e)}{S(1)} - \frac{S^*(\theta_e)}{S^*(1)} \right]. \tag{12}$$

The comparison of responses (10) and (12) of the algorithms (8) and (9) on $\delta h = \text{const.} = \Delta h$ is shown schematically in Figure 1. It is evident that the introduction of the relative conductivities may have the desired effect of a decreased sensitivity in the vicinity of saturation but, on the other hand, at the same time it has the less favourable effect of a possible extention of the range of the critical sensitivity in which the maximum acceptable inaccuracy ΔK can take place. This fact may have serious consequences. The value of $K(\theta_e)$ in case (a) are usable at least in the interval 'A', meanwhile, the values of $K(\theta_e)$ in case (b) can be almost inacceptable, with the exception of measured value $K(1)$.

The decrease of the inaccuracy when comparing cases (a) and (b) is dependent on the fulfilment of relation (13)

$$\frac{S^*(\theta_e)}{S(\theta_e)} \rightarrow \text{const.} \tag{13}$$

Only if (13) were true for all $\theta_e \in \langle 0, 1 \rangle$, $\epsilon K(\theta_e)$ from (12) would be identically equal to zero.

To compare the responses ηK and ϵK of algorithms (8) and (9), it is better to express ηK and ϵK relatively, in relation to the saturated hydraulic conductivity. When integral $S(1)$ is used for the prediction of K_s, we get (see (8) and (10))

$$\frac{\eta K}{K_s} = \frac{S(\theta_e) - S^*(\theta_e)}{S(1)}. \tag{14}$$

When $S^*(1)$ is used for hydraulic conductivity prediction, then

$$\frac{\eta K}{K_s^*} = \frac{S(\theta_e) - S^*(\theta_e)}{S^*(1)}. \tag{15}$$

The response $\epsilon K / K_s$ can be written as (see (12))

$$\frac{\epsilon K}{K_s} = \theta_e^b \left[\frac{S(\theta_e)}{S(1)} - \frac{S^*(\theta_e)}{S^*(1)} \right]. \tag{16}$$

Using the relative expressions of functions ηK and ϵK, it is possible to compare responses of algorithms (8) and (9), with no need to estimate the constant 'c'.

The following examples with variable $\delta h(\theta_e)$ demonstrate the relevance of the sensitivity analysis on 'real' soils.

EXAMPLE 1. We have series of values $h_i(\theta_{ei})$ and K_s [Silt Loam G.E.3, Mualem, (1978)]; and we use the model of Mualem (with $b = 0.5$). Sensitivity effect is studied on the difference between the most commonly used ap-

Table I. Retention curve parameters for Silt Loam G.E.3 (BC means Brooks and Corey's method, VG means van Genuchten's method of approximation).

Method	θ_r	θ_s	α[1/m]	H[m]	n	λ
VG	0.1	0.396	0.414	—	1.861	—
BC	0.1	0.396	—	1.427	—	0.619

proximation of the retention curve, namely the relation of Brooks and Corey (1964)

$$\theta_e = \left[\frac{H}{h}\right]^{\lambda} \tag{17}$$

and van Genuchten's (1978) relation

$$\theta_e = \frac{1}{(1+(\alpha h)^n)^m}, \quad m = 1 - 1/n. \tag{18}$$

H, λ, α and n are fitting parameters (values given in Table I), the physical interpretation of some of them is discussed by the authors of Equations (17) and (18). The difference δh between both approximations for silt loam are shown in Figure 2a and the corresponding K_r predictions are shown in Figure 2b (to get

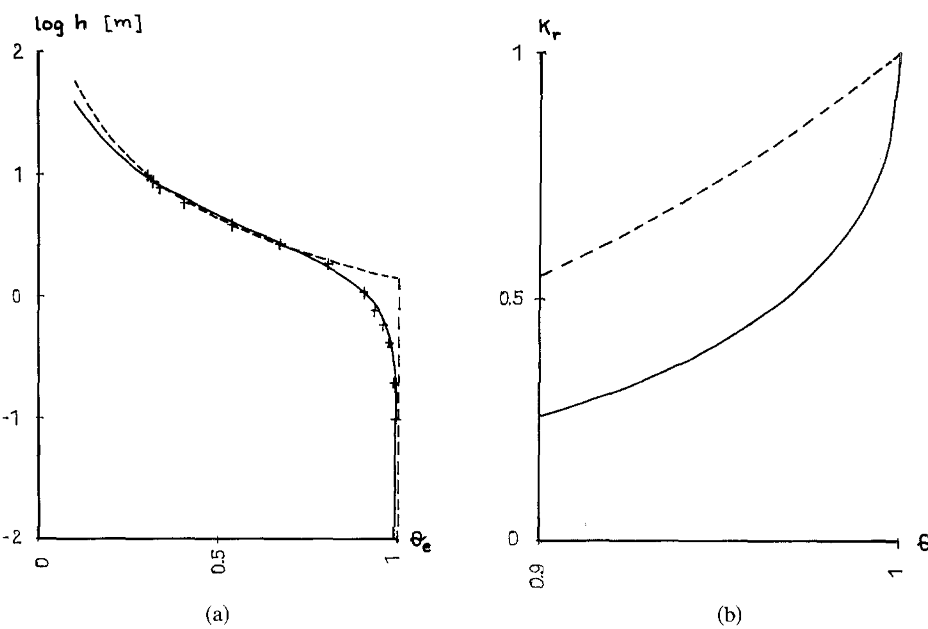


Fig. 2. (a) The typical difference between the Brooks–Corey approximation and the approximate retention curve of van Genuchten on example Silt loam G.E. 3. (b) Corresponding $K_r(\theta_e)$.

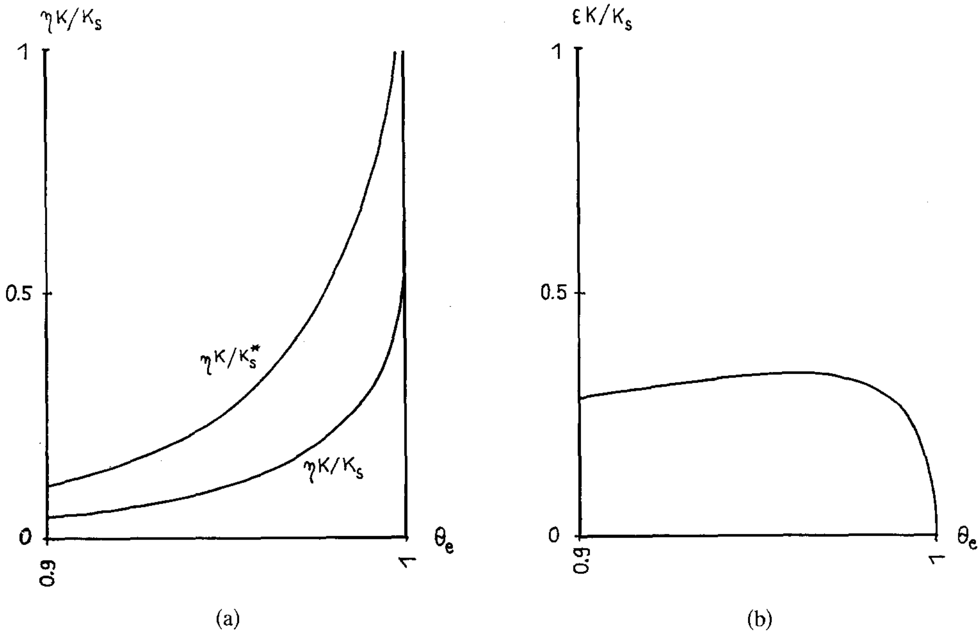


Fig. 3. Relative responses $\eta K/K_s$ and $\epsilon K/K_s$ calculated from various approximations of the same retention curve data. (a) Using expression (8). (b) Using the expression (9).

the comparable Brooks and Corey approximation, six experimental points near the saturated moisture content had to be ignored).

The responses of algorithms (8) and (9) to the variation δh calculated from relations (15) and (16) are shown in Figures 3a and 3b, respectively.

EXAMPLE 2. To adequately describe the actual role of possible uncertainties in the $h(\theta_e)$ determination near saturation, the retention curve is approximated by the modification of van Genuchten's expression, where the physical parameter θ_s is replaced by the parameter θ_m ($\theta_m \geqslant \theta_s$) optimized in the fitting process. The meaning of θ_s itself, as the maximum water content, remains unchanged. The retention curve thus consists of two parts: the linear part (from $h = 0$ to $h = h(\theta_s)$) and the nonlinear part (for $h > h(\theta_s)$) as shown in Figure 4. The retention curve is expressed by

$$\theta_e' = \frac{1}{(1 + (\alpha h)^n)^m}, \qquad \theta_e' = \frac{\theta_s - \theta_r}{\theta_m - \theta_r}\,\theta_e\,, \tag{19}$$
$$m = 1 - 1/n.$$

After substituting (19) into Mualem's relation (7) and integrating, we get the relative hydraulic conductivity

$$K_r(\theta_e) = \theta_e^{1/2}\,\frac{S(\theta_e)}{S(1)} \tag{20}$$

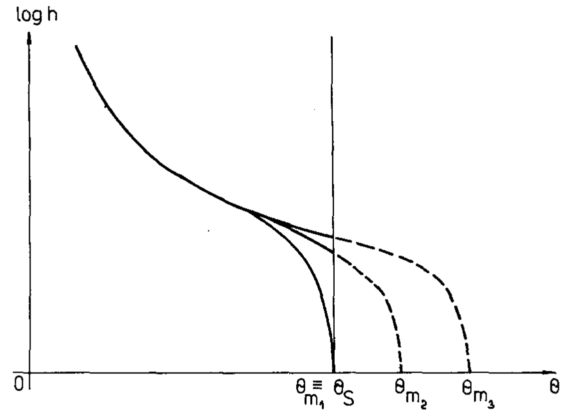


Fig. 4. Variations of the retention curve in the vicinity of saturation caused by the choice of different values of parameters θ_m.

and

$$S(\theta_e) = \alpha^2 \left[\frac{\theta_m - \theta_r}{\theta_s - \theta_r}\right]^2 [1 - (1 - \theta_e'^{1/m})^m]^2. \tag{21}$$

The following data were selected for demonstration purposes:

		K_s [m/sec]
Yolo light clay	(Moore, 1939)	1.230×10^{-7}
Silt loam	(Semotan, 1982)	3.889×10^{-8}
Fine sand G.E.#13	(Brooks and Corey 1964)	1.999×10^{-4}

The influence of the chosen variations of the retention curve (Tables II, III and IV) on the hydraulic conductivity prediction is shown in Figures 5, 6 and 7. From the relation $S(\theta_e)$, it is evident that even a small variation of the retention curve in the vicinity of saturation can be significantly magnified by the algorithm for the hydraulic conductivity determination. For some soils, this can produce errors in the orders of the determined $S(\theta_e)$ and, therefore, also in the values of $K(\theta_e)$ calculated from relation (8). When a combination of the indirect determination of relative hydraulic conductivity $K_r(\theta_e)$ from relation (9) and the experimental

Table II. Optimalized parameters α, n as the response to the variations of the parameter θ_m for Yolo Light Clay.

Number of curve	θ_r	θ_s	θ_m	α[1/m]	n	R_θ
1	0.19	0.499	0.499	2.651	1.651	0.9988
2	0.19	0.499	0.500	2.690	1.647	0.9989
3	0.19	0.499	0.510	3.110	1.614	0.9987
4	0.19	0.499	0.520	3.598	1.583	0.9977

Table III. Optimalized parameters α, n as the response to the variations of the parameter θ_m for Silt Loam Nova Ves.

Number of curve	θ_r	θ_s	θ_m	α[1/m]	n	R_θ
1	0.05	0.387	0.387	11.707	1.125	0.9938
2	0.05	0.387	0.388	12.130	1.124	0.9940
3	0.05	0.387	0.400	18.241	1.121	0.9957
4	0.05	0.387	0.420	33.547	1.117	0.9969
5	0.05	0.387	0.460	94.058	1.124	0.9976

Table IV. Optimalized parameters α, n as the response to the variations of the parameter θ_m for Fine Sand G.E. #13.

Number of curve	θ_r	θ_s	θ_m	α[1/m]	n	R_θ
1	0.20	0.377	0.377	2.061	6.359	0.9949
2	0.20	0.377	0.378	2.066	6.343	0.9945
3	0.20	0.377	0.400	2.162	6.009	0.9961
4	0.20	0.377	0.420	2.244	5.795	0.9969
5	0.20	0.377	0.460	2.395	5.504	0.9975

Rem: The correlation coefficient R_θ measures the agreement between the measured and calculated soil moisture contents.

value of K_s is used, the calculated $K_r(\theta_e)$ can be significantly incorrect in the whole soil moisture interval.

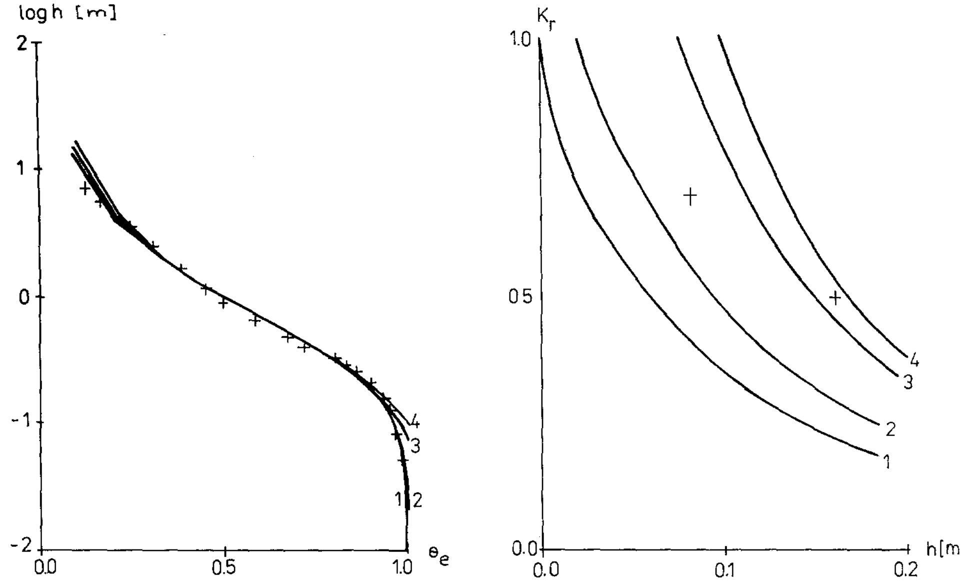
6. Summary and Conclusions

The algorithm for the indirect determination of hydraulic conductivity from the retention curve is used in two forms:

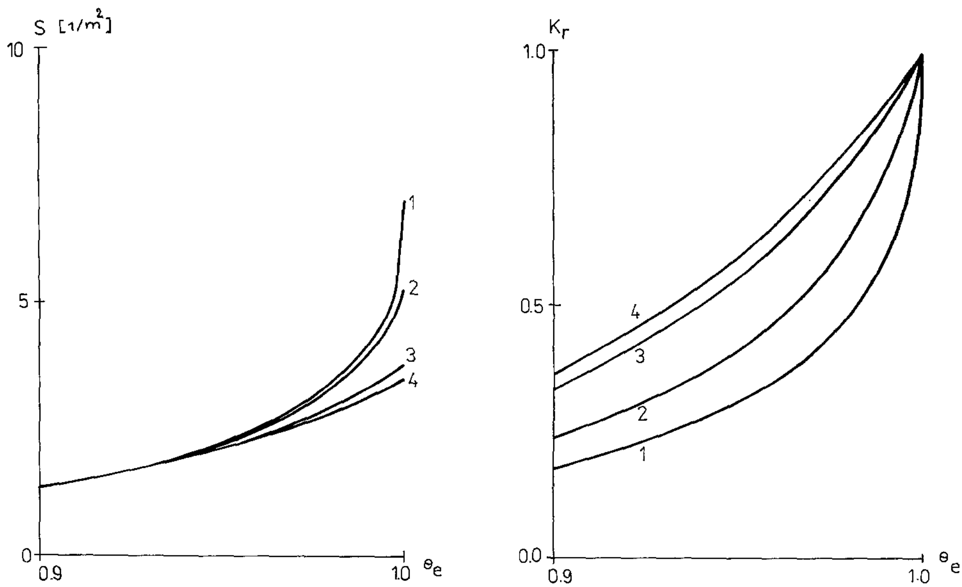
$$\text{(I)}\quad K^{\mathrm{I}}(\theta_e) = c(\theta_s - \theta_r)S(\theta_e) \tag{22}$$

$$\text{(II)}\quad K^{\mathrm{II}}(\theta_e) = K_r^{\mathrm{II}}(\theta_e)K_s, \quad K_r^{\mathrm{II}}(\theta_e) = \theta_e^b \frac{S(\theta_e)}{S(1)}. \tag{23}$$

The sensitivity of Algorithm I towards inaccuracies in the determination of the retention curve increases significantly in the vicinity of saturation. The increased sensitivity in that area, together with the fuzzy value of the constant 'c', make the usability of the Algorithm in this form almost impossible. In the use of Algorithm II, the need to determine the constant 'c' is eliminated. Here the sensitivity is different, the increased sensitivity being spread over the larger part of the interval $\theta_e \in \langle 0, 1\rangle$. The possible decrease of the sensitivity in comparison with the sensitivity of Algorithm I depends on the fulfilment of criterion (13). The



(a)



(b)

Fig. 5. The influence of chosen variations of the retention curve upon the calculated hydraulic conductivity $K_r(h)$ for Yolo light clay: (a) Variations of the retention curve $h(\theta_e)$ together with corresponding $K_r(h)$. (b) Changes of $S(\theta_e)$ and $K_r(\theta_e)$ produced by chosen variation of $h(\theta_e)$.

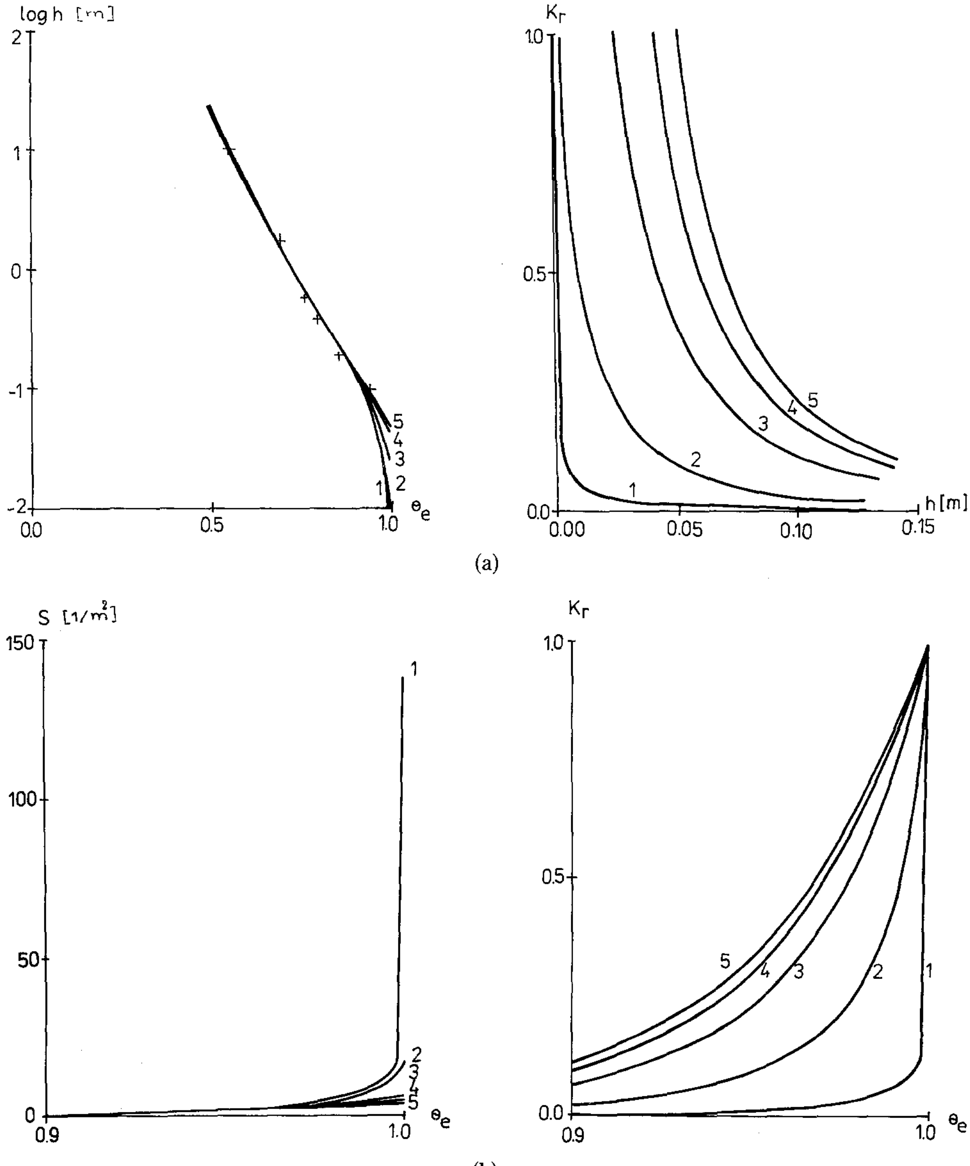


Fig. 6. The same as Figure 5 for Silt loam Nova Ves.

sensitivity of Algorithm II can influence the value of the calculated hydraulic conductivities over a substantial part of the interval $\theta_e \in \langle 0, 1 \rangle$. In both forms, the algorithm may supply poor predictions. A certain improvement can be given by introducing the following modification to Algorithm II.

If we assume that criterion (13) is valid with satisfactory precision, at least for certain sub-intervals of the whole effective moisture content interval

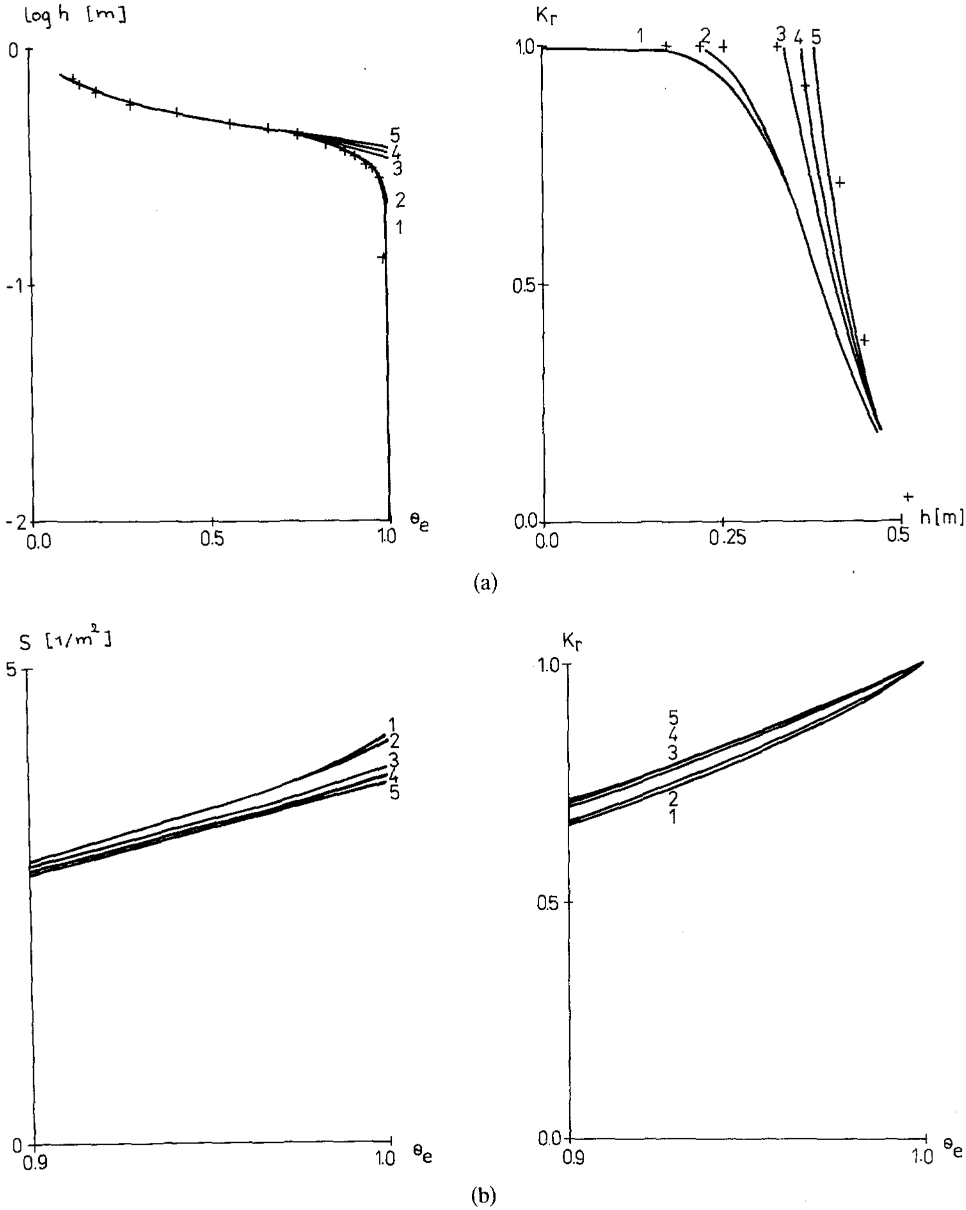


Fig. 7. The same as Figure 5 for Fine sand G.E.#13.

$$\frac{S^*(\theta_e)}{S(\theta_e)} \to \text{const.} \quad \text{for } \theta_e \in \langle 0, \theta_e^k \rangle, \quad \theta_e^k \leqslant 1, \tag{24}$$

then the values of $K(\theta_e)$ are not loaded in the interval $\langle 0, \theta_e^k \rangle$ by the error caused by the sensitivity of the algorithm to the inaccuracy in the retention curve determination. The extremely sensitive area is eliminated from the prediction.

Modified Algorithm II may then be written as

$$\text{(III)}\quad K^{\mathrm{III}}(\theta_e) = K_r^{\mathrm{III}}(\theta_e) K_s, \quad \theta_e \in \langle 0, \theta_e^k \rangle,$$
$$K_r^{\mathrm{III}}(\theta_e) = \frac{K^{\mathrm{II}}(\theta_e)}{K^{\mathrm{II}}(\theta_e^k)} \frac{K_k}{K_s} = \left[\frac{\theta_e}{\theta_e^k}\right]^b \frac{S(\theta_e)}{S(\theta_e^k)} \frac{K_k}{K_s}. \tag{25}$$

Algorithm III as well as Algorithm II are based on the original concept which assumes the multiplication of the calculated hydraulic conductivities by the constant matching factor, holding the condition $K(\theta_e^*) = K_*$. In Algorithm II, the values 1 and K_s correspond to the values of θ_e^* and K_*, while in Algorithm III, θ_e^* and K_* are equal to the values θ_e^k and K_k. For $\theta_e^k = 1$ Algorithm III becomes Algorithm II. The use of (25) requires another measured value of hydraulic conductivity $K_k = K(\theta_e^k)$ and a decision about whether the hydraulic conductivity in the interval $(\theta_e^k, 1\rangle$ is to be obtained by the interpolation between the measured values of K_k and K_s or whether it will also be measured. When Algorithm III is applied, it is necessary to determine the value of θ_e^k, i.e. the boundary between the calculated and measured hydraulic conductivities. There are three possibilities:

(1) To accept the arbitrary fixed convention for the θ_e^k (for example, $\theta_e^k = 0.9$ or $h(\theta_e^k) = 1\,m$) and to test its suitability on the extensive experimental material.
(2) To estimate θ_e^k from the analysis of sensitivity in relation to the validity of relation (24).
(3) For purposes of a large-scale application, to carry out the experimental calibration of θ_e^k (for example, in studies of spatial variability).

When $K(\theta)$, estimated by the indirect method, is to be used for the solution of a particular problem of soil hydrology (infiltration, redistribution, etc.), the sensitivity analysis as described in this paper should be used together with a similar analysis of Richards' equation algorithm for the given boundary and initial conditions.

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