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Capture and Rebound of Dust in Granular Bed Gas Filters

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SUMMARY

Experimental data for the efficiency of filtration of gases by fixed beds of granular solids are used to evaluate the reliability of the 'cell' and 'constricted tube' models for gas flow and aerosol transport. The dominant capture mechanisms are Brownian diffusion and inertial deposition. For Brownian diffusion, both models give sensible estimates for capture efficiency, but this process is shown to be insensitive to the model assumptions. Inertial deposition provides a much more sensitive test, and it is shown that neither model gives satisfactory predictions for the efficiency of inertial capture. Whether a dust particle adheres or rebounds on contacting a filter granule depends on the relative importance of kinetic and adhesion energies. An approach is proposed which enables the theoretical analyses to be applied to predict the limits of adhesion.

INTRODUCTION

The term 'granular bed filter' describes a device in which particles are removed from a fluid by passing it through a bed of an unbonded granular filter medium. Where granular bed filters are currently used for filtration of gases, they are applied to removing particles which are too small to be collected by conventional devices such as cyclones, *i.e.*, typically less than 5 to 10 μm in diameter. Rarely, as in some applications in the nuclear industry (e.g. [1]), they are used once to saturation. More often, granular bed filters are cleaned *in situ*, as in the 'Ducon' filter [2]), and the 'twist layer' filter [3]. Alternatively the medium may be displaced intermittently, as in the 'panel bed' configuration [4], or continuously, as in some more recent commercial devices (e.g., [5]). Interest in this general type of filter has revived in recent years, because of the need in advanced power generation schemes to filter gases derived from combustion or gasification of coal before they are admitted to a turbine [6 - 8]. Typical process conditions are 1100 to 1400 K at 10 to 20 bar, and a potential attraction of a granular bed filter lies in the possibility of using a cheap inert medium such as sand to withstand the aggressive environment.

This paper is concerned with two aspects of gas filtration in granular beds: *capture* or *collection*, *i.e.*, processes by which fine dust or aerosol particles carried by the gas are brought into contact with a granule in the filter, and *rebound* or *retention*, *i.e.*, whether a particle contacting a filter granule is retained and thereby removed from the gas. Only fixed-bed filters are considered, although fluidised beds might also be used [9]. Furthermore, only filtration in the depth of a 'clean' bed is discussed here; *i.e.*, the important questions of whether the dust forms a 'cake' on the upstream face of the filter and how dust deposition within the filter affects performance are not addressed, although some progress in understanding of these aspects has been made (see, e.g., [10 - 16]).

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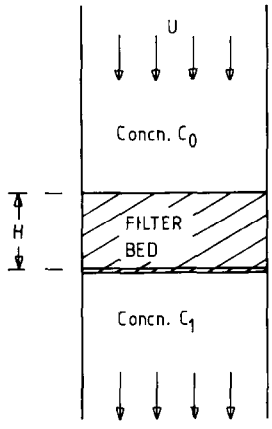

Fig. 1. Granular bed filter (schematic).

The filter is shown schematically in Fig. 1. The bed is of depth H, and comprises a granular collector of mean diameter D. The gas superficial velocity is U. The approaching gas carries aerosol particles of diameter d, which may be solid (dust) or liquid (mist). The overall penetration f through the filter bed is defined as

$$f = \frac{C_1}{C_0} \qquad (1)$$

where C_1 and C_0 are respectively the aerosol concentration in the exit and approach gas. For industrial purposes, concentration and penetration are usually expressed on a mass basis. However, for understanding of the processes occurring, it is necessary to consider either monodisperse aerosols or narrow size ranges within a heterodisperse aerosol.

CAPTURE

Consider a bed of clean filter granules. For an aerosol particle to be removed from a gas passing through the bed, it must first come into contact with a filter granule; this process is known as 'capture', 'collision' or 'collection'. For unambiguous experimental determination of capture rates, it is necessary to measure the penetration of an aerosol which is known to adhere to the filter element on contact; aerosols of liquids which 'wet' the filter medium have most commonly been used. Theoretical analyses of capture processes, discussed further below, have proceeded by calculating the gas flow field through a geometry representing an idealised

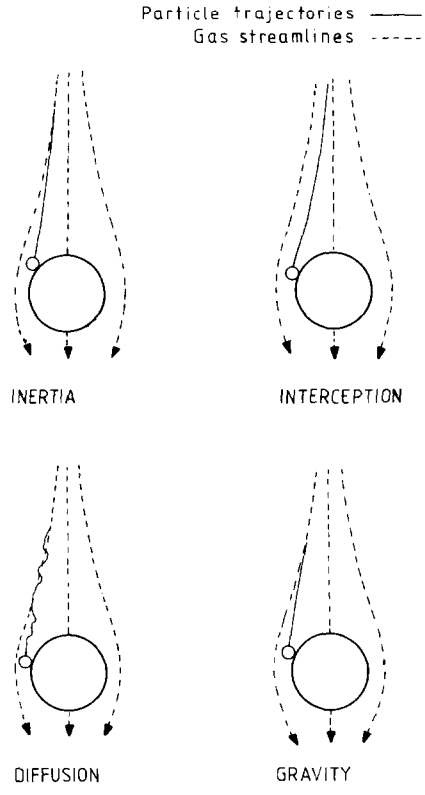

Fig. 2. Mechanical capture processes (schematic).

'unit cell' of the filter, and then analysing the motion of particles carried by the gas. Tien has argued [16, 20] that the inaccuracies in the experimental measurements are so great that the theoretical analyses should be regarded as at least equally reliable. However, the present author rejects this view for two reasons: Tien overstates the experimental difficulties [15, 21] and, while the experimental results obtained by different workers show considerable scatter, they show broad trends and, for some filtration regimes, agree very closely with the theoretical predictions. The approach taken here is therefore to use experimental results to identify any deficiencies in the analyses and to examine the possible causes for these deficiencies.

Capture processes

It is conventional [22] to distinguish between four mechanical capture processes, shown schematically in Fig. 2. Diffusional, gravitational and inertial capture all result from effects which cause the trajectory of an aerosol particle to deviate from the gas streamlines around one of the filter elements.

Diffusional capture results from transport of aerosol to the surface of the collector by Brownian migration. Gravitational capture arises from the settling of particles across the gas streamlines, and is most effective when the gas is in downflow. Inertial deposition arises from the inertia of the aerosol particle, which causes its trajectory to deviate from the gas streamlines. The fourth purely mechanical process is direct interception, which results from the finite size of the aerosol particle, so that a particle whose centre follows the gas streamlines may be captured if its centre passes within a distance $d/2$ from the surface of the collector. Other capture processes arise from electrophoretic, thermophoretic and diffusiophoretic effects [15] but are not discussed here.

Conventionally, capture rates are expressed by a 'single granule collector efficiency' E, defined as the ratio of the number of aerosol particles contacting the granule to the number of aerosol particles in the gas approaching the granule. In this discussion, approach gas volume is based on the superficial gas velocity through the filter U, because this leads to unambiguous definitions of the governing groups [17]. However, other workers have used different definitions, so that care is necessary in comparing published results. The rate of capture of aerosol particles by one filter granule r is given [18] by

$$r = \frac{EUC\pi D^2}{4} \tag{2}$$

where C is the local number concentration of aerosol particles in the gas. If the granules comprising the filter bed are arranged randomly with $H \gg D$, then the overall penetration follows [17, 18] as

$$f = f' \exp\left[\frac{-3E(1-\epsilon)H}{2D}\right] \tag{3}$$

where ϵ is the void fraction in the bed. Equation (3) refers to 'deep bed' filtration, *i.e.*, to processes occurring in the body of the filter. The factor f' is included to account for any anomalous processes occurring at entry to or exit from the bed. For example, if cake formation occurs, then f' will be less than unity and will decrease as the cake forms. Where more than one mechanism of capture operates, it is common to assume that the efficiencies of the component mechanisms are additive. This is clearly an oversimplification, but represents a reasonable approximation if the individual efficiencies are all small or if one mechanism is dominant [19].

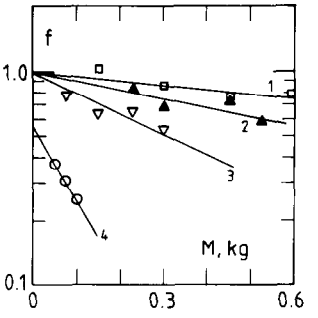

Fig. 3. Penetration f of di-ethyl hexyl sebacate aerosol through beds of closely sized glass beads as function of bed mass M in bed of diameter 7.62 cm. Data of Thambimuthu [23].

Data	U (cm/s)	D (μm)	d (μm)
1	1.5	1100	1.13
2	1.61	532	1.27
3	1.5	306	1.08
4	1.5	165	1.18

Determination of capture efficiency

Equation (3) provides the basis for experimental determination of E [17, 23]. In terms of the mass of collector granules in the filter M, eqn. (3) may be written

$$f = f' \exp\left[\frac{-3EM}{2D\rho_c A}\right] \tag{4}$$

where A is the face area of the filter and ρ_c the density of the collector granules. The value of E can then be determined by measuring penetration as a function of bed depth or bed mass, fitting the data by a linear regression of $\ln f$ as a function of H or M, and using a result which follows immediately from eqn. (3) or eqn. (4):

$$E = \frac{-2D}{3(1-\epsilon)} \frac{\mathrm{d}(\ln f)}{\mathrm{d}H} \tag{5}$$

$$= \frac{-2D\rho_c A}{3} \frac{\mathrm{d}(\ln f)}{\mathrm{d}M}$$

Figure 3 shows typical experimental results from the work of Thambimuthu [23] for

TABLE

Efficiency of mechanical capture processes

Mechanism	Collection efficiency E[a]	Source
Brownian diffusion	$\frac{4.36}{\epsilon}\left(\frac{\mathcal{D}}{UD}\right)^{2/3}$	Wilson and Geankoplis [25], Gebhart *et al.* [27], Balasubramanian and Meisen [26]
Inertial deposition	$\frac{(St')^{3.55}}{1.67 + (St')^{3.55}}$ where $St' = [B(\epsilon) + 1.14 Re_c^{0.5}\epsilon^{-1.5}]St$, $St = \frac{\rho_p d^2 UF}{9\mu D}$, $Re_c = \frac{U\rho D}{\mu}$, $B(\epsilon) = \frac{2(1-\alpha^{5/3})}{2 - 3\alpha^{1/3} + 3\alpha^{5/3} - 2\alpha^2}$, $\alpha = 1 - \epsilon$	D'Ottavio and Goren [28]
Direct interception	$6.3\epsilon^{-2.4}\left(\frac{d}{D}\right)^2$	Paretsky [12]
Gravitational settling:		Paretsky [12
upflow	$0.0375\left(\frac{u_t}{U}\right)^{0.5}$	
downflow	$0.0375\left(\frac{u_t}{U}\right)^{0.5} + 0.21\left(\frac{u_t}{U}\right)^{0.78}$	

[a]In the following expressions, u_t is the terminal velocity of an aerosol particle settling freely through the gas, and St' is a modified Stokes number. Other symbols are defined in the text. Note that the definition of Stokes number used here is twice the value used by D'Ottavio and Goren [28].

collection of di-ethyl hexyl sebacate aerosol by glass beads. It illustrates how the capture efficiency typically increases with decreasing filter granule size.

The Table summarises expressions for the efficiency of the four mechanical capture processes of Fig. 2. The dominant capture mechanisms are normally diffusion and inertial deposition [18]. The efficiencies of gravitational settling and direct interception are of less significance; the expressions for these mechanisms in the Table are purely theoretical, derived by Paretsky [12] from an analysis discussed below, but experimental results give no indication that these expressions are greatly in error [23]. The two dominant mechanisms are of more interest, and have therefore been more thoroughly investigated.

Diffusional capture dominates for small aerosol particles. The process is closely analogous to mass transfer in packed beds. From eqn. (2), the rate of collection per unit area of collector surface is $E_D UC/4$, where E_D is the efficiency of diffusional capture. Assuming complete retention of captured aerosol, the concentration is zero at the collector surface, so that C may be interpreted as the concentration difference between interstitial gas and collector surface, *i.e.*, as the mass transfer driving force. Thus, $E_D U/4$ is equivalent to the conventional

mass transfer coefficient k. Normally, granular bed filters operate at low values of the collector Reynolds number, $Re_c = U\rho D/\mu$ where ρ and μ are the density and viscosity of the gas respectively. Mass transfer coefficients for low Reynolds number takes the form

$$Sh = \text{fn}(Pe,\epsilon) \tag{6}$$

where Sh is the Sherwood number $dk/\mathcal{D}$, Pe is the Peclet number, $DU/\mathcal{D}$, and $\mathcal{D}$ is the Brownian diffusivity of the aerosol. For aerosol particles large compared with the mean path of gas molecules, $\mathcal{D}$ can be estimated from the Stokes–Einstein equation:

$$\mathcal{D} = \frac{Fk_BT}{3\pi\mu d} \tag{7}$$

where k_B is Boltzmann's constant, 1.380622×10^{-23} J K^{-1}, and F is the Stokes–Cunningham 'slip correction factor' which is typically close to unity except for sub-micron particles [18, 24]. Because

$$k = \frac{E_D U}{4} \tag{8}$$

it follows that

$$Sh = \frac{E_D UD}{4\mathcal{D}} \tag{9}$$

Because the values of $\mathcal{D}$ are much smaller than values for molecular diffusivity in gases, the Schmidt number, $Sc = \mu/\rho\mathcal{D}$, for capture by Brownian diffusion is very high, so that the process should be analogous to conventional mass transfer in liquids rather than in gases. A widely used correlation for mass transfer at high Sc and low Re_c is due to Wilson and Geankoplis [25]:

$$Sh = 1.09Pe^{1/3}\epsilon^{-1} \tag{10}$$

which rearranges to the expression for E_D given in the Table. Balasubramanian and Meisen [26] pointed out that the extensive data of Gebhart *et al.* [27] are consistent with this expression. Thambimuthu's results [23] confirm the applicability of the Wilson and Geankoplis correlation [25] to diffusional capture, as illustrated by the data in Fig. 4. These results show higher collection efficiency with downflowing gas compared with upflow, as predicted by Paretsky [12] (see the Table). They also show that, for high Pe, the collection efficiency rises sharply.

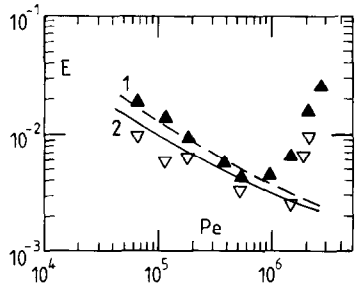

Fig. 4. Capture efficiency E for di-ethyl hexyl sebacate aerosol on glass beads as function of Peclet number Pe; data of Thambimuthu [23] with D = 306 μm. 1, Downflow; 2, upflow; curves show total capture efficiency by diffusion, gravitational settling and direct interception, evaluated using expressions in the Table.

This corresponds to capture by inertial deposition, so that Pe ceases to be an appropriate dimensionless group.

Inertial impaction becomes dominant for larger d and U, and is more problematic because there is no analogous molecular process. Correlations for the efficiency of inertial capture E_I must therefore be developed directly from measurements for aerosol particles which are retained on capture. Dimensional analysis suggests that such correlations should take the form

$$E_I = \text{fn}(St,Re_c,\epsilon) \tag{11}$$

where St is the collector Stokes number, $\rho_p d^2UF/9\mu D$ where ρ_p is the density of the aerosol particles. Tien [16] has summarised most of the correlations which have been proposed. That due to D'Ottavio and Goren [28], given in the Table, is one of the more recent, and is based on their own extensive experimental data. Considering first the limit of low collector Reynolds number, *i.e.*, $Re_c \to 0$, the effect of bed voidage is included in their correlation via a factor $B(\epsilon)$ defined in the Table. The form of this factor is suggested by the Happel cell model (see below) and over the range of interest for fixed beds, say $0.33 \leqslant \epsilon \leqslant 0.4$, it is approximated with 0.5% by the simpler form

$$B(\epsilon) = 4.19\epsilon^{-2.41} \tag{12}$$

The experimental results are then correlated in terms of the modified Stokes number

$$St^* = B(\epsilon)St \tag{13}$$

Equations (12) and (13) imply that the void fraction has a very strong effect on collection,

through its effect on the gas velocity approaching any individual collector granule. In a slightly different approach, Thambimuthu [23] argued that the impaction process should be controlled by the mean interstitial velocity U/ϵ, leading to the use of a different modified Stokes number:

$$St^+ = \frac{St}{\epsilon} \tag{14}$$

which implies a weaker effect of voidage. D'Ottavio and Goren's data also suggested a significant effect of Re_c, and the definition of the modified Stokes number

$$St' = [B(\epsilon) + 1.14Re_c^{0.5}a^{-1.5}]St \tag{15}$$
$$= St^*[1 + 1.14Re_c^{0.5}\epsilon^{-1.5}B^{-1}]$$

was suggested by boundary layer theory for flow around an isolated sphere at high Reynolds number. They then found that their own data were fitted well by the semi-empirical correlation

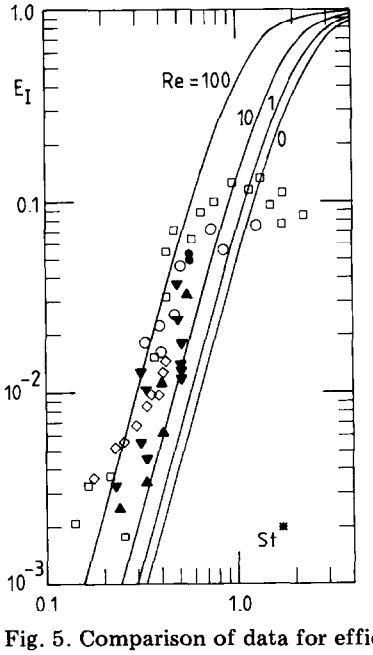

Fig. 5. Comparison of data for efficiency of inertial capture with correlation of D'Ottavio and Goren [28], evaluated for $\epsilon = 0.38$ when $Re_c = 0$.

$$E_I = \frac{(St')^{3.55}}{1.67 + (St')^{3.55}} \tag{16}$$

which has the form of eqn. (11) and shows, correctly, that $E_I \to 1$ as $St' \to 1$.

Because the form of eqn. (15) derives from purely theoretical arguments whose validity may be questioned, it is appropriate to test eqn. (16) against other data. Thambimuthu estimated the efficiency of inertial capture E_I from his own data by measuring the total efficiency E using eqn. (5), and then, for runs in which inertial deposition was a significant mechanism (cf. Fig. 4), subtracting out the efficiencies of the other mechanisms using the expressions in the Table. Using the same approach, he also obtained estimates for E_I from the data of Doganoglu [29], Phillips [29] and Yung *et al.* [31]. Figure 5 shows the data from these four sources plotted as functions of St^*, defined in eqn. (13). The curves in Fig. 5 represent D'Ottavio and Goren's correlation, eqn. (16). Data for St^* greater than about 1.0 show a decline in E_I which may be attributed to rebound (see below). Otherwise, E_I increases strongly with St^*, and the D'Ottavio and Goren correlation clearly describes this increase well. However, the strong dependence on bed voidage spreads out the data from beds with different ϵ. D'Ottavio and Goren took account of the resulting spread in their own data by the dependence on Re_c. However, the results in Fig. 5 do not show a systematic variation with Re_c and thus do not conform with eqn. (15). This conclusion is perhaps not surprising, given that the form of eqn. (15) is applicable to isolated spheres at high Reynolds numbers, while a fixed bed contains closely packed particles at low to intermediate Reynolds numbers.

In Fig. 6, the same data are plotted in terms of Thambimuthu's St^*, defined in eqn. (14). Again the effect of rebound is seen for St^* greater than about 0.05. Otherwise,

Author	Symbol	D (μm)	ϵ	Re_c	System
Thambimuthu [23]	●	165	0.33	1.26	Glass beads + liquid aerosol
	▲	306	0.34	2.3 - 4.2	
	▼	532	0.36	5.7 - 12.4	
Doganoglu [29]	○	110	0.40	0.35 - 1.4	Glass beads + liquid aerosol
Yung *et al.* [31]	◇	620	0.39	12 - 31	Iron shot + polystyrene latex
Phillips [30]	□	2000	0.38	14 - 130	Glass beads + potassium chloride

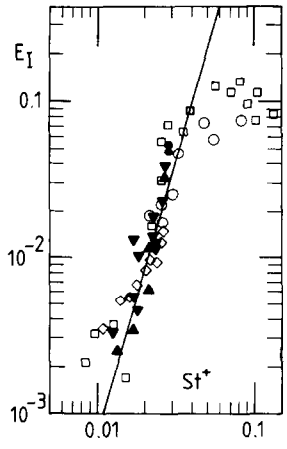

Fig. 6. Comparison of data for efficiency of inertial capture with eqn. (17). Symbols as for Fig. 5.

this form of correlation brings the data together well. Using the general form of eqn. (16), the data are fitted by

$$E_I = \frac{(St^+)^{3.55}}{1.1 \times 10^{-4} + (St^+)^{3.55}} \tag{17}$$

which is shown in Fig. 6 to represent a realistic correlation. Contrary to the findings of D'Ottavio and Goren, there is no evidence of an effect of Re_c.

For purposes of comparison, eqns. (16) and (17) can both be written in terms of St for a given value of ϵ. Taking the representative value $\epsilon = 0.38$ and, for D'Ottavio and Goren, $Re_c = 1.0$, they become

$$E_I = \frac{(St')^{3.55}}{1.72 \times 10^{-6} + (St')^{3.55}} \tag{16a}$$

$$E_I = \frac{St^{3.55}}{3.55 \times 10^{-6} + St^{3.55}} \tag{17a}$$

Comparison between eqns. (16a) and (17a) shows that the difference between these correlations is of detail only, *i.e.*, all the experimental data confirm the strong dependence of E_I on St. Unless further evidence of a significant dependence of E_I on Re_c or a stronger dependence on ϵ can be demonstrated, it is recommended that eqn. (17) be used for estimating E_I rather than the more complex form of eqn. (16).

Comparison with theoretical analyses

Two general approaches have been used to generate theoretical estimates for the efficiency of aerosol capture in granular bed filters, differing in the way the gas flow is handled. One approach uses the 'cell model' proposed by Happel [32] and Kuwabara [33], which takes the 'unit cell' as one collector granule surrounded by a concentric spherical shell of fluid corresponding to the associated interstitial volume. The Happel and Kuwabara models differ in detail in the condition imposed on the fluid on the boundary of the cell. The other approach uses the 'constricted tube model' of Payatakes *et al.* [34], in which flow passages through the bed are represented as a succession of pores with diameters at entrance and exit larger than at their centres. In the original model, the tube walls were assumed to have a parabolic profile. In each approach, the gas velocity field is obtained by solution of the Navier–Stokes equation, and the behaviour of aerosol carried by the gas is then calculated. For the cell model, this approach yields the capture efficiency directly. For the constricted tube model, the bed is divided into collection units of length

$$l = \left[\frac{\pi}{6(1-\epsilon)}\right]^{1/3} D \tag{18}$$

so that, if the efficiency of capture in an element of depth l is η, the overall penetration is given [34] by

$$f = f' \exp\left[\frac{H \ln(1-\eta)}{l}\right] \tag{19}$$

Comparison of eqn. (3) with the limiting form of eqn. (19) for small η shows that

$$\frac{\eta}{E} = \pi^{1/3}[0.75(1-\epsilon)]^{2/3} \tag{20}$$

$$\approx 0.86$$

In the comparisons discussed below, calculations from the constricted tube model have been expressed in terms of E, using eqn. (20).

For diffusional capture, analogous to mass transfer at high Sc as noted above, removal rates are calculated by solving the steady state diffusion equation, assuming constant concentration throughout the fluid entering the cell or tube [35]. Results from the cell model for the case of interest here, low Re_c and high Sc, have been reviewed by Tardos *et al.* [36] and Rajagopalan and Tien [37],

while mass transfer calculations from the constricted tube model have been given by Chiang and Tien [38]. All these theoretical calculations fall into the form

$$E_D = Pe^{2/3}\text{fn}(\epsilon) \quad (21)$$

consistent with the form derived from Wilson and Geankoplis (see the Table). Calculations based on the cell model give values for fn(ϵ) generally larger than $4.36/\epsilon$, while Chiang and Tien's constricted tube calculations are generally close to $4.36/\epsilon$. Thus, at least for diffusional capture, the constricted tube approach appears to be more realistic. Qualitative considerations suggest that this is not surprising. While the cell model may be a reasonable representation of average conditions within a structure of high voidage, such as a fibrous filter, it is not obviously a good representation of local conditions in a granular bed where particles are so loosely packed as to touch their neighbours. The fluid then flows through interconnecting passages of variable area, much closer to the constricted tube model. The fact that both models give predictions of the correct form, represented by eqn. (21), is therefore not surprising and is not evidence of particular success for either model. The condition of high *Pe* implies that the concentration boundary layer is thin. The average rate of mass transfer then depends on the average vorticity at the collector surface [39]. This in turn defines the form of dependence of *Sh* or E_D on *Pe* [40], so that any physically realistic model will lead to results of the form of eqn. (21). Only the form of fn(ϵ) then depends on the assumptions of the model, so that diffusional capture is not a sensitive test of the validity of a theoretical treatment.

Inertial deposition is more sensitive. For this mechanism, theoretical calculations are based on trajectory calculations obtained by integrating the equation of motion for an aerosol particle in the gas. The 'grazing trajectory', *i.e.*, the particle trajectory which just touches the surface of the collector, is calculated. All particles between the grazing trajectory and the collector are assumed to be collected, giving the impaction rate and hence the collection efficiency. Although conceptually simple, this approach has certain obvious difficulties:

(i) Particle trajectories are sensitive to the whole gas flow field, by contrast with Brownian diffusion, which is sensitive only to conditions at the collector surface. It follows that E_I will be much more sensitive than E_D to calculation of gas flow.

(ii) The equation of motion for an aerosol particle is usually based on the simple assumption that the instantaneous drag can be estimated from Stokes' law using the instantaneous relative velocity between gas and particle. Rajagopalan and Tien [37] summarise expressions for the effect on drag of the proximity of the collector surface. However, their estimates are still based on the assumption that the instantaneous flow is fully developed, ignoring the genuinely unsteady effects which can be significant in regions of high acceleration [40].

(iii) Even if the grazing trajectory can be located accurately, it is still necessary to calculate the rate of transport of aerosol by gas in the region bounded by this trajectory. The commonest assumption is that aerosol is uniformly distributed throughout the gas entering the cell or channel. While this assumption may be reasonable for particles small enough to be dominated by Brownian diffusion, it is more questionable for particles with significant inertia.

Tardos *et al.* [36] have reviewed analyses of inertial deposition derived from the cell model. Typical results are those of Paretsky *et al.* [41], who used the Happel model with creeping flow ($Re_c \to 0$). Their results for $\epsilon = 0.4$ are shown as curve 2 in Fig. 7, for comparison with experimental results as summarised by the empirical correlations of eqn. (17) (curve 1), Goren [42] (curve 4) and D'Ottavio and Goren [28] (curves 5). Equation (17) and eqn. (16) with $Re = 0$ are indistinguishable. Clearly the empirical results are all in broad agreement, but show no agreement with the predictions from cell model calculations. No doubt all the difficulties identified above contribute to the discrepancy but there are particular difficulties over (iii), the distribution of particles in the gas approaching the collector. When inertial capture is effective, particles escaping capture will be concentrated towards the centre of a gas passage. In a relatively dense structure like a packed bed, a passage between collector elements will, on average, be

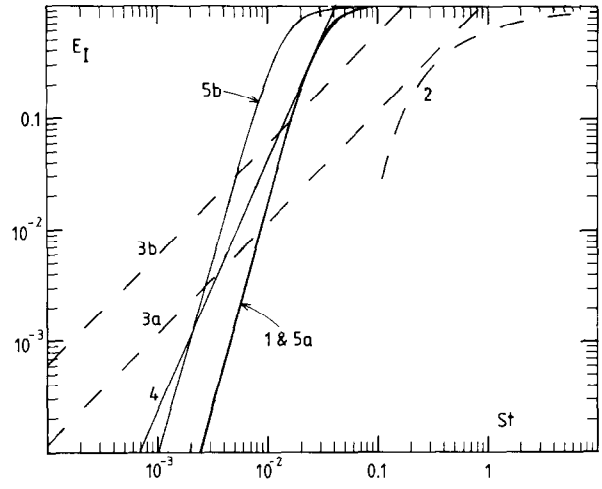

Fig. 7. Comparison of empirical and theoretical results for efficiency of inertial capture with $\epsilon = 0.4$. 1, Equation (17) (empirical); 2, Paretsky *et al.* [41] (cell model); 3, Pendse and Tien [20] (constricted tube model) (a) $Re_c = 0$; (b) $Re_c = 100$; 4, Goren [42] (empirical); 5, D'Ottavio and Goren [28] (empirical) (a) $Re_c = 0$; (b) $Re_c = 100$.

directed at the upstream surface of the next collector element. Thus, aerosol particles will be concentrated around the front stagnation point, rather than entering the cell distributed uniformly through the gas. Snaddon and Dietz [43] have attempted to account for this type of effect by including a 'flow intensification factor' to admit non-uniform aerosol distribution in the cell calculations. However, since this factor must be correlated empirically, it represents no obvious advantage over direct correlation of E_I.

Predictions from the constricted tube model are sensitive both to the assumed tube shape and to the method used to calculate the flow field [44], illustrating difficulty (i). Predictions for parabolic tube walls are poor [45]. Pendse and Tien [20] subsequently concluded that it is necessary to predict E_I as the geometric mean of values from two solutions, one obtained by a perturbation solution for flow through a tube whose walls form a half sine wave, the other by a collocation solution for parabolic walls. Notwithstanding Tien's rejection of experimental results as unreliable [16, 20], this *ad hoc* averaging procedure was justified by comparison with Doganoglu's data [29]. Converting to the form used here via eqn. (20), Pendse and Tien's result is

$$E_I = (1 + 0.04Re_c)St\pi^{-1/3}[0.75(1-\epsilon)]^{-2/3} \quad (22)$$

which is compared with the empirical correlations in Fig. 7. Equation (22) clearly shows too weak a dependence on St. It may be that suitable combinations of geometries and flow fields might improve the fit of predictions obtained from the constricted tube model. However, even to obtain the predictions in Fig. 7 has required sufficient *ad hoc* assumption that the model can no longer be regarded as fundamental. It is therefore difficult to see how this type of analysis can supplant empirical measurement for inertial capture.

It was noted above that Paretsky's cell model calculations for capture by gravitational settling and direct interception appear to be more reliable than for inertial deposition. Presumably this is because, as for diffusional capture, they depend mainly on conditions close to the collector surface so that they are less sensitive than inertial deposition to the simplifying assumptions made in the analysis.

REBOUND

In order for a solid dust particle to be removed completely from a gas in a granular bed filter, it must not only contact a filter granule but also be retained rather than rebounding or being re-entrained subsequently. For a particle which adheres on impact, the adhesion force is normally strong enough for fluid drag alone not to cause re-entrainment [46]. Dislodging by particles impacting subsequently may be cause of re-entrainment but, in general, initial adhesion is most critical. The forces which cause adhesion are all essentially short-range [47], and therefore do not generally contribute to capture [15]. If capture occurs by processes other than inertial deposition, captured particles are generally retained. However, once inertia becomes appreciable, some rebound is also likely to occur. As a rough initial guide [16], an impacting dust particle may rebound if St is sufficiently large (see Figs. 5 and 6). For dry particles with $St > 10^{-2}$, Tien [16] has given an empirical correlation for the efficiency of adhesion:

$$\gamma = 0.0318 St^{-1.248} \quad (23)$$

However, adhesion depends on the kinetic energy of the impacting particle [27] and is affected strongly by the composition and surface condition of both particle and collector; a mechanistic treatment is therefore necessary. Such an approach was initiated by Dahneke [48] and developed further by Stenhouse and Freshwater [46], Hiller and Löffler [49], Coury [14] and Clift [15].

Consider a 'dry' filter and dust, in which adhesion is dominated by van der Waals forces with no cohesion due to capillary effects. A particle approaching the collector with velocity v_i does so with kinetic energy

$$\sum_i = \frac{\pi \rho_p d^3 v_i^2}{12} \quad (24)$$

The impact velocity v_i should be evaluated [14, 15] by allowing for any relatively long-range attractive forces such as those which arise from electrostatic effects (see below) and also allowing for the additional drag of fluid on the particle due to proximity of the collector surface. In addition to Σ_i, the very short-range van der Waals forces give the particle a further kinetic energy Σ_v; in Dahneke's terms, Σ_v is the "depth of the potential well" into which the particle "falls" as it approaches the collector surface. In total, the kinetic energy of the particle on impact is therefore $(\Sigma_i + \Sigma_v)$. For a particle impacting normal to the surface, the energy at the instant of rebound is then $e^2(\Sigma_i + \Sigma_v)$, where e is the coefficient of restitution between dust and collector. In order to come away from the collector surface, the particle must now "climb out of the potential well", *i.e.*, its energy must exceed the energy for detachment, Σ_d. Thus the particle adheres if

$$d^3 v_i^2 < \frac{12}{\pi \rho_p} \left[\frac{\Sigma_e}{e^2} - \sum_v \right] \quad (25)$$

Inequality (25) suggests a critical impact velocity above which a particle will rebound from the collector, and is well supported by experimental evidence (e.g., 14, 45, 48, 49).

To apply these ideas to gas filtration, Coury [14] has proposed an approach which is a modification of one proposed by Stenhouse and Freshwater [46]. Estimates for v_i and hence Σ_i were obtained from calculations using the 'cell model' approach allowing for electrostatic attraction as in the analysis of Kallio and Dietz [51]. The cell calculations were found to give realistic predictions of the effect of electrostatic attraction on capture efficiency, because these effects are relatively short range (like Brownian diffusion) and therefore not as sensitive as inertial deposition to shortcomings in the basic model [15]. Electrostatic attraction was shown to be effective over distances from the collector surface within which the drag on the incoming particle is increased significantly above its value in an unbounded fluid, whereas van der Waals attraction operates over much shorter ranges [47]. This conclusion lies behind Coury's suggestion, noted above, that electrostatic and drag effects be included in Σ_i whereas van der Waals effects are included in Σ_v. This division also represents the distinction between Coury's approach and that of Stenhouse and Freshwater [46], who effectively took Σ_v as zero on the argument that viscous dissipation during the final stages of approach to the surface counterbalances the adhesion energy. For brittle materials, for which impact causes no permanent surface deformation, Coury [14] further suggested that Σ_d is given by the sum of the energies due to van der Waals and electrostatic attractions:

$$\sum_d = \sum_v + \sum_e \quad (26)$$

Inequality (25) then becomes

$$d^3 v_i^2 < \frac{12}{\pi \rho_p e^2} \left[\sum_e + (1 - e^2) \sum_v \right] \quad (27)$$

as the condition for retention on impact. This analysis shows how the potential energy associated with electrostatic attraction Σ_e can be partially dissipated before impact and yet contribute to Σ_d; thus, it explains why electrostatic attraction favours adhesion. Furthermore, the cell-model trajectory calculations predict that v_i is a minimum for impaction at the front stagnation point of the collector, and maximum close to the equator. Thus, it suggests that particles larger than the minimum size for complete

retention will rebound if they impact close to the equator but may adhere if they impact close to the front stagnation point; Taub [10] has reported essentially this effect in granular bed filters operated at relatively high velocities. Using Dahneke's estimate [48] for the coefficient of restitution of brittle materials like quartz, $e = 0.99$, Coury [14] obtained quantitative estimates for the limiting particle size below which all impacting particles should adhere. These estimates were in broad agreement with the experimental observations [15]. Thus, while this type of analysis clearly needs to be developed further, it serves to demonstrate how trajectory calculations may be used to predict conditions under which particles adhere or rebound.

The preceding results apply to dry brittle materials. When plastic deformation occurs, cohesion is enhanced because Σ_d is increased by a term which is not present in Σ_i or Σ_v. An even stronger effect will be observed if adsorbed or condensed liquid layers are present on dust or collector or both, because Σ_d will then be greatly increased by capillary effects. Therefore, dust particles impacting on a collector rendered 'sticky' by a non-volatile liquid should be much less liable to rebound. This effect has been observed qualitatively [14, 52], but it remains to be determined whether capillary-enhanced adhesion can be explained quantitatively by a result of the form of inequality (25).

CONCLUSIONS

Experimental results for gas filtration in granular bed filters can be used to test the validity of theoretical analyses for gas flow and aerosol transport through fixed beds. Aerosol capture by Brownian diffusion is controlled by processes occurring close to the collector surface, *i.e.*, the concentration boundary layer is thin. Rates of diffusional capture can therefore be estimated reliably by analogy with conventional mass transfer at high Schmidt number. A new correlation has been proposed, fitted to available data for inertial capture. Both the 'cell model' of Happel and Kuwabara and the 'constricted tube model' of Payatakes *et al.* lead to reasonable predictions of diffusional capture rates. However, this is not a good test of the models, because the thin concentration boundary layer renders their predictions insensitive to assumptions governing the gas flow field. To use the models to predict capture by inertial deposition requires particle trajectories to be calculated. These are sensitive to the whole gas flow field, and therefore represent a more critical test. In their current state of development, neither the cell model nor the constricted tube model gives a good prediction of the efficiency of inertial deposition.

Retention of a particle contacting a filter granule is governed by a balance between particle kinetic energy and adhesion energy. A simple mechanistic analysis shows how electrostatic effects aid adhesion, and how trajectory calculations can be used to estimate whether an impacting dust particle will adhere or rebound.

LIST OF SYMBOLS

A	cross-sectional area of filter
B	function of ϵ, defined in the Table
C	local number concentration of aerosol in gas
C_0, C_1	aerosol concentration in gas respectively entering and leaving filter
D	diameter of filter granule
$\mathcal{D}$	molecular or Brownian diffusivity
d	diameter of aerosol particle
E	capture efficiency of single filter granule
e	coefficient of restitution of particle from collector surface
F	slip correction factor
f	overall penetration of aerosol through filter, C_1/C_0
f'	penetration through entry and exit zones of filter
H	depth of filter bed
k	mass transfer coefficient
k_b	Boltzmann's constant
l	depth of collection unit in bed
M	mass of collector granules in filter bed
Pe	Peclet number, $DU/\mathcal{D}$
Re_c	collector Reynolds number, $UD\rho/\mu$
r	number of aerosol particles contacting one collector granule per unit time
Sc	Schmidt number, $\mu/\rho\mathcal{D}$
Sh	Sherwood number, $dk/\mathcal{D}$

St Stokes number, $\rho_p d^2 UF/9\mu D$
St' modified Stokes number defined in the Table and eqn. (15)
St^* $B(\epsilon)St$
St^+ St/ϵ
U superficial gas velocity
u_t terminal velocity of aerosol particle in gas
v_i velocity of impacting particle

Greek symbols

γ efficiency of adhesion on impact
ϵ void fraction in filter bed
η efficiency of capture in a single filter unit of depth
μ gas viscosity
ρ gas density
ρ_c density of collector granule
ρ_p density of aerosol particle
Σ_d energy barrier to detachment of particle from collector surface
Σ_i kinetic energy of particle approaching collector surface
Σ_v energy of adhesion of particle to collector surface

Subscripts

D diffusional capture
I inertial capture

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