Aerosol Science and Technology
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/uast20

Use of a Generalized Stokes Number to Determine the Aerodynamic Capture Efficiency of Non-Stokesian Particles from a Compressible Gas Flow

R. Israel [a] & D. E. Rosner [a]

[a] High Temperature Chemical Reaction Engineering Laboratory, Yale University, Chemical Engineering Department , New Haven, CT, 06520

Published online: 05 Jun 2007.

Use of a Generalized Stokes Number to Determine the Aerodynamic Capture Efficiency of Non-Stokesian Particles from a Compressible Gas Flow

R. Israel and D. E. Rosner
High Temperature Chemical Reaction Engineering Laboratory, Chemical Engineering Department, Yale University, New Haven, CT 06520

The aerodynamic capture efficiency of small but non-diffusing particles suspended in a high-speed stream flowing past a target is known to be influenced by parameters governing (a) small particle *inertia*, (b) departures from the Stokes drag law (associated with local particle Reynolds numbers greater than unity), and (c) carrier fluid compressibility (at nonnegligible free-stream Mach numbers). By defining an *effective Stokes number* in terms of the actual (prevailing) particle stopping distance, local fluid viscosity, and inviscid fluid velocity gradient at the target nose, we show that these effects are well correlated in terms of a "standard" (cylindrical collector, Stokes drag, incompressible flow, $Re^{1/2} \gg 1$) capture efficiency curve. We are thus led to a correlation that (a) simplifies aerosol capture calculations in the parameter range already included in previous numerical solutions, (b) allows rational engineering predictions of deposition in situations not previously specifically calculated, (c) should facilitate the presentation of performance data for gas cleaning equipment and aerosol instruments.

NOMENCLATURE

C_D	drag coefficient
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
d	diameter
l	stopping distance
M	Mach number
R	radius of curvature of the target
Re	Reynolds number
Stk	Stokes number
t_{flow}	flow time
t_p	particle stopping time
U	free stream velocity
u	fluid velocity along x coordinate
v	fluid velocity along y coordinate
$\mathbf{v}$	fluid velocity vector
v_s	settling velocity
x	coordinate along stagnation line
y	coordinate normal to stagnation line
γ	ratio of specific heats
η	capture efficiency of the target
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density of the fluid
ϕ	Re_p^2/Stk
ψ	non-Stokesian correction factor

Subscripts and Superscripts

c	compressible flow
crit	critical value
i	$M << 1$
eff	effective value
$M=0$	$M << 1$
p	pertaining to particle
stag	at the stagnation zone
∞	at free stream conditions
$'$	dummy parameter used for integration

0278-6826/83/01045–07$3.00

INTRODUCTION

The capture efficiency of a collector in a high-velocity flow field is mainly determined by an inertia parameter characterizing the particle motion, termed the Stokes number. However, for non-Stokesian particles the collection efficiency also depends on a Reynolds number based on particle diameter and the free-stream velocity. For a potential flow field approximation to the fluid motion about the collector, the collection efficiency of cylindrical and spherical targets have been calculated numerically as a function of both the Stokes and *particle* Reynolds numbers (Brun et al., 1955; Dorsch et al., 1955). The results obtained in the limit of negligible Mach number are reproduced here in Figures 1 and 2. As can be seen, for both Stokesian and non-Stokesian particles, the capture efficiency curves are qualitatively alike, providing the motivation to seek a similarity parameter that would reduce the dependence of the capture efficiency of both collectors to a single combination of the above variables. Likewise, it would be useful if the effects of nonzero Mach number on capture efficiency could also be correlated in terms of a Mach-number-dependent generalized Stokes number. Clearly, to predict the deposition rate of particles (Rosner et al., 1981) over a wide size spectrum (as would occur, e.g., in gas turbine and gas cleaning applications), a representation of the capture efficiency that dramatically reduces its dependence on geometric and fluid-dynamic parameters would greatly facilitate computations.

GENERALIZED STOKES NUMBER

Definition

The Stokes number can be viewed as the ratio of the characteristic stopping *time* t_p of the particle to the characteristic flow time around the collector. The stopping time (also known as the relaxation time) of the particle conventionally is computed applying the Stokes (linear) drag law to a particle initially moving with the free stream velocity. In generalizing the Stokes number, account must be taken of the fact that the drag on the particle may be non-Stokesian. In that case, the stopping *distance* is as follows:

$$l_p = \tfrac{4}{3}(\rho_p/\rho)d_p \int_0^{\mathrm{Re}_p} \frac{d\mathrm{Re}'}{C_D(\mathrm{Re}')\mathrm{Re}'}. \qquad (1)$$

A generalized Stokes number or the inertia parameter for particle capture by a cylinder or a sphere of radius R in an incompressible flow can then be written[1]

$$\mathrm{Stk}_{\mathrm{eff}} = \frac{l_p}{R} = \frac{8}{3}\left(\frac{\rho_p}{\rho}\right)\left(\frac{d_p}{d}\right) \int_0^{\mathrm{Re}_p} \frac{d\mathrm{Re}'}{C_D\mathrm{Re}'}. \qquad (2)$$

[1] The conjecture that non-Stokes particle capture can be correlated in these terms is mentioned in the important paper of May and Clifford (1967). Here we explicitly demonstrate its success for two collector geometries (cylindrical and spherical) and then extend the notion of $\mathrm{Stk}_{\mathrm{eff}}$ to include other important effects (mainstream Mach number, target geometry, etc.).

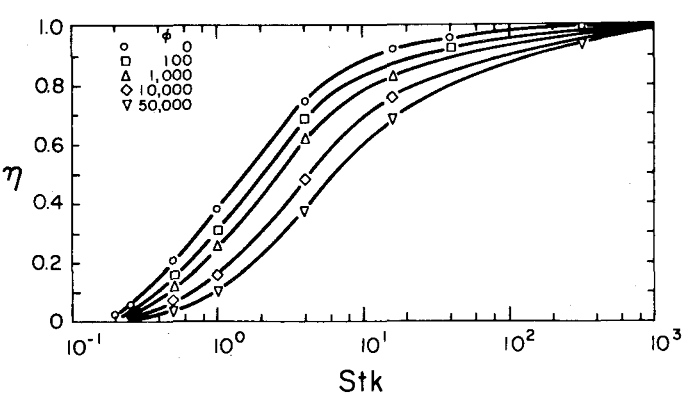

FIGURE 1. Capture efficiency for a cylinder as a function of particle inertia parameter Stk at several values of the non-Stokesian drag parameter ϕ; $\mathrm{Re}^{1/2} \gg 1$, $M^2 \ll 1$ (Brun et al., 1955).

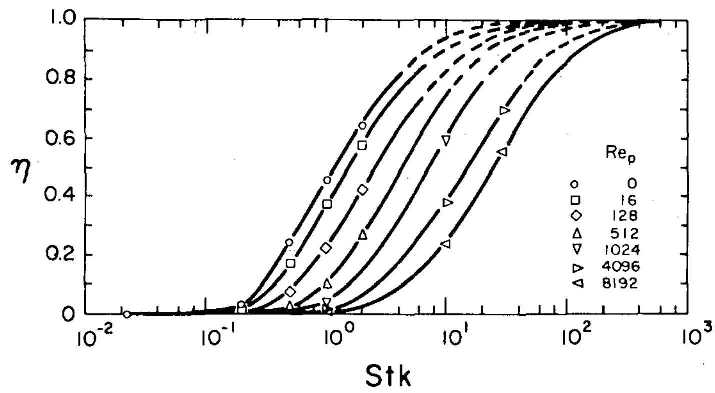

FIGURE 2. Capture efficiency for a sphere as a function of particle inertia parameter Stk, at several values of the particle Reynolds number; $Re^{1/2} \gg 1$, $M^2 \ll 1$ (Dorsch et al., 1955).

Equation (2) can be written in terms of the conventionally defined Stokes number:

$$\mathrm{Stk}_{\mathrm{eff}} = \psi(\mathrm{Re_p})\mathrm{Stk}, \quad (3)$$

where the non-Stokes drag correction factor ψ is given by

$$\psi(\mathrm{Re_p}) = \frac{24}{\mathrm{Re_p}} \int_0^{\mathrm{Re_p}} \frac{d\mathrm{Re}'}{C_D(\mathrm{Re}')\mathrm{Re}'} . \quad (4)$$

Note that in the Stokes limit $C_D \to 24/\mathrm{Re}$ and $\psi \to 1$, as required. For a given Reynolds number based on particle diameter and free-stream velocity, ψ can be computed by evaluating the integral on the right-hand side. The functional dependence of ψ on $\mathrm{Re_p}$ is presented in Figure 3. The generalized Stokes number can then be found by correcting the conventional Stokes number by $\psi(\mathrm{Re_p})$.[2] From Figure 3 it is obvious that the actual stopping distance, and hence the stopping time for a non-Stokesian particle, is far *less* than what Stokes drag would indicate. Physically, of course, this is due to the fact that the Stokes law *underestimates* the true particle drag force.

[2] This is equivalent to the practice, in micrometeorology applications (Aylor, 1978), of writing the Stokes number in terms of the particle settling velocity v_s; i.e., $\mathrm{Stk_{eff}} = v_s U/gR$ (in the absence of buoyancy corrections).

Application to Capture by Isolated Cylinders and Spheres at $Re^{1/2} >> 1$ and $M^2 << 1$

The numerical solutions for the collection efficiency of a cylinder and sphere, as given in Figures 1 and 2 for incompressible, inviscid flow,

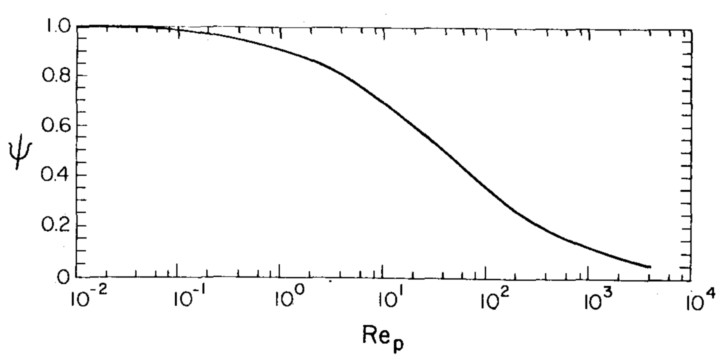

FIGURE 3. Non-Stokes particle drag correction factor relating $\mathrm{Stk_{eff}}$ to Stk [Eqs. (3) and (4)].

can now be transformed so as to show the variation of collection efficiency as a function of this generalized Stokes number. The transformation of the coordinates in Figure 2 for the sphere is straightforward, as both Stokes number and particle Reynolds number for a specified capture efficiency are given explicitly. In the case of Brun's plot for a cylinder, the capture efficiency is plotted against Stokes number for various values of a newly defined parameter ϕ, dependent both on the Stokes and Reynolds numbers. However, even in this case the particle Reynolds number can be derived for specified ϕ and Stokes number from the relation

$$\mathrm{Re}_p = (\phi \mathrm{Stk})^{1/2}. \tag{5}$$

Thus, in all cases, it is relatively simple to correlate the efficiency of the collector to our generalized Stokes number. The resulting values are plotted (in the transformed coordinates) in Figures 4 and 5. For each geometry, these data are seen to be acceptably represented by a single function $\eta(\mathrm{Stk}_{\mathrm{eff}})$. We shall show that a further generalization of the definition of $\mathrm{Stk}_{\mathrm{eff}}$ allows all results for both collector geometries to be reasonably well approximated by a single function[3] of $\mathrm{Stk}_{\mathrm{eff}}$. Choosing capture by an *isolated cylinder* as the reference condition, for $\mathrm{Stk}_{\mathrm{eff}} > 0.14$ we recommend[4] the curve fit

$$\eta(\mathrm{Stk}_{\mathrm{eff}}) \simeq [1 + 1.25(\mathrm{Stk}_{\mathrm{eff}} - \tfrac{1}{8})^{-1} - 1.4 \times 10^{-2}(\mathrm{Stk}_{\mathrm{eff}} - \tfrac{1}{8})^{-2} + 0.508 \times 10^{-4}(\mathrm{Stk}_{\mathrm{eff}} - \tfrac{1}{8})^{-3}]^{-1}, \tag{6}$$

which is accurate to better than 10% (rms) error and is shown dashed in Figure 4. Equation (6) will be used here to anticipate the simultaneous effects of non-Stokes drag and gas compressibility on the aerodynamic capture behavior of an isolated cylinder at high Reynolds numbers.

Compressibility Effect on Aerodynamic Capture by a Cylinder[5]

The incompressible fluid inviscid results cited pertain to the asymptotic limit $M^2 << 1$, where M is the mainstream *Mach number*. However, as is well known, actual gas flows about targets will depart from this limit at nonnegligible Mach numbers, even within the inviscid ($\mathrm{Re}^{1/2} >> 1$) limit. The effect of gas compressibility on the capture efficiency of an isolated cylinder has been considered by Brun et al. (1953) using the so-called Janzen–Rayleigh perturbation method. This method converges reasonably well up to the critical Mach number (~ 0.4), at which sonic speed is attained some-

[3] We redefine $\mathrm{Stk}_{\mathrm{eff}}$ so as to map all values of $\mathrm{Stk}_{\mathrm{crit}}$ to the same number despite the target geometry. This provides a useful, if approximate, single correlation of capture efficiency for different conditions of flow and geometry (see Figure 7 and the next section).

[4] The immediate vicinity of $\mathrm{Stk}_{\mathrm{crit}}$, where none of the available numerical calculations of η are reliable anyway, requires special treatment and will be discussed elsewhere.

[5] See Brun et al. (1953).

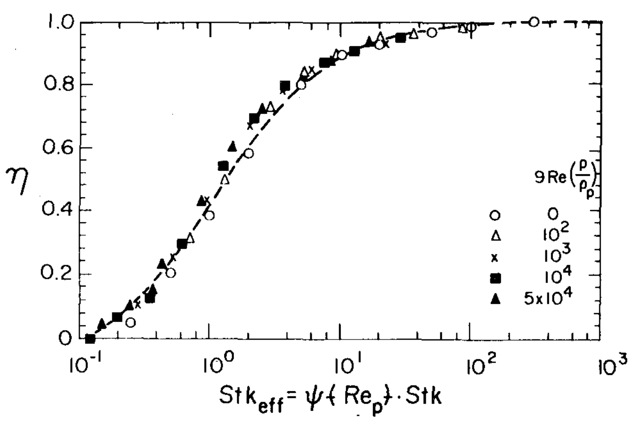

FIGURE 4. Correlation of capture efficiency behavior of a cylinder; $\mathrm{Re}^{1/2} \gg 1$, $M^2 \ll 1$.

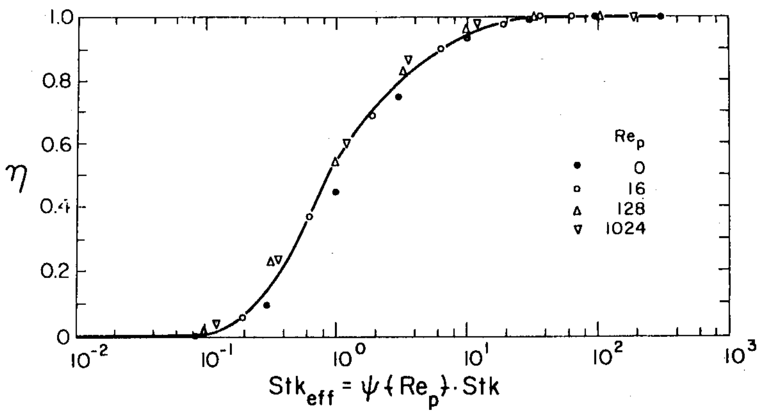

FIGURE 5. Correlation of capture efficiency behavior of a sphere; $Re^{1/2} \gg 1$, $M^2 \ll 1$.

where along the target. The actual particle trajectory numerical calculations of Brun et al. (1953) were carried out only at this particular Mach number (0.4), for a gas with $\gamma \equiv c_p/c_v = 1.4$.

In view of the ability of the "effective Stokes number" approach to deal with departures from Stokes drag, we consider here a further generalization of Stk_{eff} that should incorporate the dominant effect of gas compressibility on the aerodynamic capture of particles. For this purpose, consider first the Stokes drag limit and rewrite Eq. (2) in the instructive form

$$Stk = \frac{t_p}{t_{flow}} = \left(\frac{\tilde{\rho}_p d_p^2}{18\mu}\right) \Bigg/ \left[\frac{2}{(du_e/dx)_{x=0}}\right]. \qquad (7)$$

Here $(du_e/dx)_{x=0}$ is the inviscid forward stagnation point velocity gradient at the cylinder, known to be decisive in determining the critical Stokes number below which pure inertial impaction will not occur (Friedlander, 1977).[6] In defining an effective Stokes number to account for compressibility it is then clear from Eq. (7) that it is appropriate to base Stk_{eff} on the values of μ and du_e/dx actually prevailing in the crucial forward stagnation region. For isentropic flow of a gas with a viscosity dependent on a power ω of the absolute temperature, we can then write

$$\mu(\text{stag})/\mu_\infty = [1 + \tfrac{1}{2}(\gamma - 1)M^2]^\omega, \qquad (8)$$

where M is the free-stream Mach number. Moreover, according to Imai's calculations (Eser, 1943; Ehlers et al., 1948) using Janzen-Rayleigh perturbation theory:

$$\left(\frac{du_e}{dx}\right)_{x=0} = 2\frac{U}{R}\left[1 - \frac{5}{12}M^2 + \left(\frac{\gamma - 1}{240} + \frac{13}{80}\right)M^4 - \cdots\right], \qquad (9)$$

where the leading term, $O(M^2)$, has a coefficient that is independent of γ. We conclude that an appropriate effective Stokes number for capture by a cylindrical target in a compressible but subsonic gas flow is of the form[7]

$$Stk_{eff} = (Stk)_{M=0}\frac{(1 - \frac{5}{12}M^2 + \cdots)}{\{1 + [(\gamma - 1)/2]M^2\}^\omega}. \qquad (10)$$

[6] Note that the use of $2(du_e/dx)_{x=0}^{-1}$ for t_{flow} in the definition of Stk will have the property of mapping the *critical* Stokes numbers *for all target geometries* to the same value, $\frac{1}{8}$ when $Re^{1/2} \gg 1$. It should be remarked that even for compressible flows the tangential gradient $(\partial u/\partial x)_{x=0}$ can be shown to be equal to the local value of the spatial rate of fluid deceleration $(-\partial v/\partial y)_{x=0}$ (normal direction) in the immediate vicinity of a symmetrical forward stagnation point. On the basis of this check we have corrected a coefficient in the M^4 term reported by Brun et al., (1955).

[7] Evaluating the correction factor for $\gamma = 1.4, \omega = 0.67, M = 0.4$, we obtain 0.918, so that the critical value of the inertial parameter $Stk_{M=0}$ below which pure inertial deposition will not occur on a cylinder at $M = 0.4$ predicted to be

$[Stk_{M=0}]_{crit} = \frac{1}{8}/0.918 = 0.136,$

rather than the incompressible value 0.125. Convergence of the Janzen–Rayleigh series leading to Eq. (9) for a cylinder probably extends up to a free-stream Mach number of ~0.45. Interestingly enough, compressibility effects on a *sphere* are somewhat smaller—with the numerator of Eq. (10) containing the factor $1 - 0.25M^2 + \cdots$ (Lighthill, 1954).

Combining Eqs. (10) and (2) provides an approximate method for correlating the effects of both non-Stokes particle drag and gas compressibility on capture efficiency. Results of such a correlation are compared with the numerical calculations of Brun et al. (1953) in Figure 6. It is interesting to note that whereas Brun et al. (1953) concluded [from numerical calculations at or above $(\mathrm{Stk})_{M=0}$ of 0.5] the compressibility effect on η would become *small* near $\mathrm{Stk}_{\mathrm{crit}}$; in fact, the relative effect is very great there since the *compressible* theory predicts deposition only above $(\mathrm{Stk})_{M=0}=0.136$ (at $M=0.4$), whereas the *incompressible* theory predicts deposition above $(\mathrm{Stk})_{M=0}=0.125$. Thus, in contrast to the graph (Figure 6) of Brun et al., the present correlation approach leads us to conclude that $10^2(\eta_i-\eta_c)/\eta_i$ becomes 100% for all values of $(\mathrm{Stk})_{M=0}$ between 0.125 and 0.136.

DISCUSSION

While the range of aerosol *particle* Reynolds numbers considered above is quite large (up to nearly 10^4), it is apparent from Figures 4 and 5 that the aerosol collection efficiency for each target is well correlated by an effective Stokes number based on the actual (non-Stokes drag) stopping distance. This kind of reduction in the required number of "independent" parameters should also be valuable in dealing with particle Reynolds numbers not previously analyzed for either collector geometry. The success of this notion for these two particularly well-studied target geometries (isolated cylinder or sphere) supports the use of a generalized Stokes number to correlate the effects of non-Stokes particle drag in many other cases of practical interest. Moreover, by basing the effective Stokes number on the appropriate characteristic *flow* time [cf. Eq. (7)], the capture efficiency behavior of the sphere and cylinder (and other target geometries?) is adequately represented by a single composite graph [Figure 7 and curve fit to Eq. (6)] with a scatter not much different than the scatter in the individual non-Stokesian particle correlations for each of these two particular target geometries (Figures 4 and 5). Indeed, such a Stokes number [cf. Eq. (10)] includes the primary effects of fluid compressibility, thereby also allowing the correlation of Mach-number-dependent collection efficiencies.

CONCLUSIONS

Although not exact,[8] the use of a generalized Stokes number (based, in effect, on *actual*, non-Stokes, stopping time and appropriate flow

[8] In contrast, correcting Stokesian particle capture results for the effects of "slip" at appreciable particle Knudsen numbers (incorporating the Stokes–Cunningham factor SCF into the numerator of $\mathrm{Stk}_{\mathrm{eff}}$) *is exact*, since slip does not alter the underlying linearity of the relation between small particle drag and relative (particle-fluid) velocity. For non-Stokes *continuum* drag associated with $|\mathbf{v}_p-\mathbf{v}|d_p/\nu>1$, the particle momentum equation governing $\mathbf{v}_p$ is intrinsically nonlinear, and our use of the correction factor $\psi(\mathrm{Re}_p)$ can also be viewed as the result of performing a quasi-linearization of this vector equation.

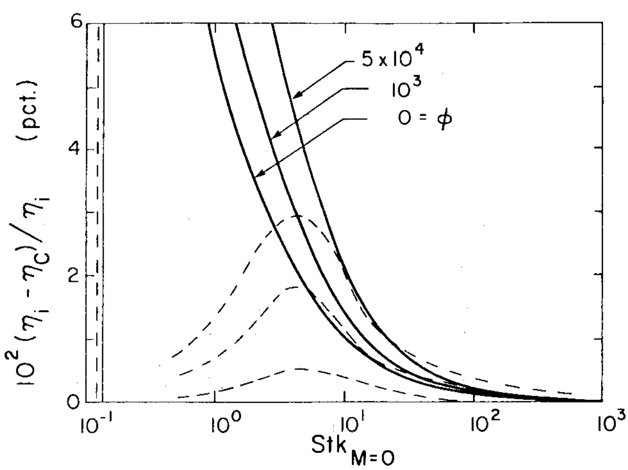

FIGURE 6. Compressibility and non-Stokes drag effects on the capture efficiency of an isolated cylinder; comparison of present correlation (shown solid) with the results of Brun et al. (1953 shown dashed).

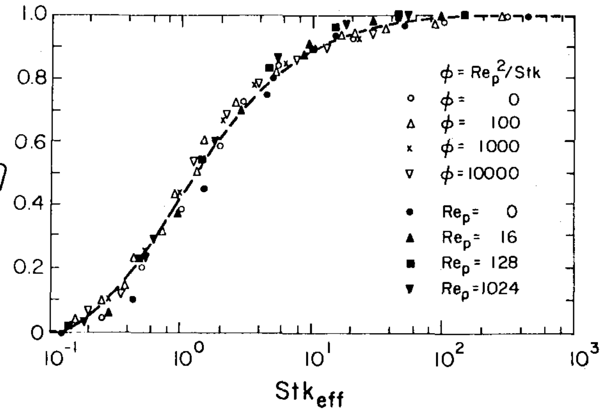

FIGURE 7. Composite correlation of capture efficiency of non-Stokesian particles by spheres (filled symbols) or cylinders (open symbols); $Re^{1/2} \gg 1$, $M^2 \ll 1$.

time) is seen to have a remarkable effect on suppressing the dependence of capture efficiency on many important geometric and fluid-dynamic parameters, including the Mach number.[9] This technique should prove very useful in making engineering predictions or correlating experimental data on both isolated and interfering collectors,[10] such as occur in gas-cleaning equipment and aerosol instruments. Moreover, even when the correlation is not "complete," it is still useful to employ a representation which minimizes the "apparent" differences between various geometric and flow situations. Finally, we anticipate that an appropriate Stk_{eff} will allow the correlation of other important features of inertial impaction, including particle kinetic energies upon impact, angular distributions of capture, etc. Further tests and extensions of these ideas are accordingly suggested and will, in part, be dealt with in separate reports.

This paper derives from a research program supported in part by AFOSR contract F49620-82K-0020 and NASA grant NAG 3-201 at Yale University–HTCRE Laboratory. It is a pleasure to acknowledge the computational assistance of A. Zydney, L. Martinez-Aparicio, and N. Anous, and helpful discussions with D. Aylor, J. Fernandez de la Mora, and D. Günes.

[9] Similar reasoning should correlate the cutoff behavior of cascade impactors run under compressible flow conditions (Flagan, 1982).

[10] This includes the limiting cases of staggered tube bundles (Whitaker, 1972; Rosner, 1982; Rosner and Atkins, 1982) and packed granular beds (Goren, 1982) at *target* Reynolds numbers, which need not be large.

REFERENCES

Aylor, D. (1978). In *Plant Disease—an Advanced Treatise* (J. Horsfall and E. B. Cowling, eds.), Academic, New York, Vol. 2, pp. 159–180.

Brun, R. J., Lewis, W., Perkins, P. J., and Serafini, J. S. (1955). NACA Rep. 1215.

Brun, R. J., Serafini, J. S., and Gallagher, H. M. (1953). NACA Tech. Note 2903.

Dorsch, R. G., Saper, P. G., and Kadlow, C. F. (1955). NACA Tech. Note 3587.

Ehlers, F. E., and Carrier, G. F. (1948). *Methods of Linearization in Compressible Flow*, Part I. Janzen–Rayleigh Method, Monogr. IV, Air Material Command Report No. F-TR-1180A-ND.

Eser, F. (1943). *Luftfahrtforschung* 20:220.

Flagan, R. D. (1982). Proc. ACS/IEC Winter Symp. on Aerosol Systems, *J. Colloid Interface Sci.* (in press).

Friedlander, S. K. (1977). *Smoke, Dust and Haze—Fundamentals of Aerosol Behavior*, Wiley, New York.

Goren, S. (1982). Paper 20f, presented at First Annual Meeting of the American Assoc. for Aerosol Research, Santa Monica, CA.

Lighthill, M. J. (1954). In *High Speed Aerodynamics and Jet Propulsion* (W. R. Sears, ed.), Princeton University Press, Princeton, NJ, Vol. 6, pp. 352–373.

May, K. R., and Clifford, R. (1967). *Ann. Occup. Hyg.* 10:83–95.

Rosner, D. E. (1982). *Energy, Mass and Momentum Transport in Chemically Reacting Fluids*, EXXON Research and Engineering Textbook Ser., Wiley, New York.

Rosner, D. E., and Atkins, R. M. (1982). Proc. Engineering Foundation Int. Conference on Experimental Research into Fouling and Slagging due to Impurities in Combustion Gases.

Rosner, D. E., Israel, R., and Zydney, A. (1981). ACS/IEC-81 Winter Symp. on Aerosol Systems.

Whitaker, S. (1972). *AIChE J.* 18:361–371.

Received 18 May 1982; accepted 20 September 1982