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Problems in the application of dimensional analysis and scale-up of mixing operations

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Abstract—Dimensional analysis is a tool to quickly and reliably elaborate a dimensionless frame in which the experiments are favourably presented, because it is condensed as compared with the dimensional one. Further advantage lies in the 'scale-invariance' of the dimensionless frame, thus enabling the only reliable scale-up. There are only two real problems in dealing with dimensional analysis. The first is the construction of a complete list of relevant parameters which describe the process ('relevance list'). The second is the determination of the process characteristics and the establishment of the real operational number(s) particularly in case of large scale factors. This paper discusses possible pitfalls and their avoidance in the field of mixing. © 1998 Elsevier Science Ltd. All rights reserved.

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1. INTRODUCTION TO DIMENSIONAL ANALYSIS

Mixing technology is a branch of mechanical chemical engineering where the dimensional analysis and the scale-up based on it have been in use from its very beginning. Therefore, this field offers prime examples of how to proceed, but also where and how to omit mistakes in order to gain universal rules for scale-up by minimizing experimental work.

1.1. *The nature of dimensional analysis*

Dimensional analysis is based upon the recognition that a mathematical formulation of a physical technological problem can be of general validity only when the process equation is dimensionally homogenous.

The aim of the dimensional analysis is to check whether the physical content in the examination can be formulated in a dimensionally homogeneous manner or not. The procedure necessary to accomplish this consists of two parts:

(a) First, all physical parameters necessary to describe the problem are listed. This so-called 'relevance list' of the problem consists of the quantity in question and of all parameters which influence it. In each case only *one* target quantity must be considered; it is the only dependent variable. On the other hand, all the influencing parameters must be primarily independent of one another.

(b) In the second step the dimensional homogeneity of the physical content is checked by transforming it to a dimensionless form. (Each physical content which can be transformed into dimensionless expressions is obviously dimensionally homogeneous!)

By performing this operation one obtains two essential advantages inherently tied to the dimensional analysis:

(1) The amount of the dimensionless numbers is smaller than the amount of the quantities contained in them. Nevertheless, the problem gets described equally comprehensively. The *pi-theorem* states: Every physical relationship between n physical quantities can be reduced to a relationship between $m = n - r$ mutually independent dimensionless groups, whereby r stands for the rank of the dimensional matrix, made up of the physical quantities in question and generally equal to the number of the basic quantities contained in them.

(2) According to the *Theory of models* two processes may be considered completely similar if they take place in similar geometrical space and if all the dimensionless numbers necessary to describe them have the same numerical value. Clearly, the scale-up of a desired process condition from model to industrial scale can be accomplished reliably only if the problem was formulated and dealt with according to the dimensional analysis, because only its dimensionless representation is independent of scale ('scale-invariant') and thus presents the basis for a reliable scale-up.

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2. HOW TO OBTAIN A RELIABLE DIMENSIONLESS PRESENTATION (PI SPACE)

2.1. *Dimensional analysis*

The intuitive trials of Froude (1870) and Reynolds (1883) were followed by the dimensional analytical procedure of Lord Reyleigh (1915) which was later taken up and popularized by Buckingham (1914), Bridgman (1922) und Langhaar (1951). For problems whose physical laws are accessible only by experiments, the dimensional analysis represents the only possible aproach to a reliable pi space.

2.2. *The Navier–Stokes' differential equations*

These equations enabled many researchers (Nusselt, Prandtl, Schmidt, etc.) at the beginning of this century to obtain dimensionless numbers in order to describe processes in the field of momentum (hydrodynamics), heat and mass transfer. They were gained by setting up differential equations for chosen boundary conditions and converting them to a dimensionless formulation without solving them analytically. This approach also enabled Damköhler (1936) to obtain a set of dimensionless numbers for an adiabatic chemical reaction taking place in a catalytic furnace. They were able to describe the process far better than those obtained by the dimensional analysis (Zlokarnik, 1991).

2.3. *Making physical laws dimensionless*

This produces dimensionless numbers which are best fitted to describe them, for example, the Euler number $Eu \equiv \Delta p/(\rho v^2)$ and the Newton number $Ne \equiv F/(\rho v^2 l^2)$ result from corresponding physical laws in which they serve as coefficients. Where such a possibility exists it should of course be taken advantage of.

In this context, we mention the important relationship between the applicability of the dimensional analysis and the available knowledge of the problem. Pawlowski (in: Zlokarnik, 1991) outlined the following five steps:

1. The physics of the basic phenomenon is unknown.
   → Dimensional analysis cannot be applied.
2. Enough is known about the physics of the basic phenomenon to compile a first, tentative relevance list.
   → The resultant pi set is unreliable.
3. All the relevant physical variables describing the problem are known.
   → The application of dimensional analysis is unproblematic.
4. The problem can be expressed in terms of a mathematical equation.
   → A closer insight into the Π relationship is feasible and may facilitate a reduction of the set of dimensionless numbers.
5. A mathematical solution of the problem exists.
   → The application of dimensional analysis is superfluous.

3. SETTING UP A RELEVANCE LIST

An actual problem in applying dimensional analysis is setting up a relevance list. The process under consideration can be described by a complete set of dimensionless numbers only if the dimensional parameters influencing it are also completely listed. In doing so one has to be aware of the linear independency of these parameters. One quantity is linearly dependent on the others if it can be replaced by them, e.g. owing to the equation $\nu \equiv \eta/\rho$, only two of these three material properties may be listed; the third one is linearly dependent of the other two.

The relevance list may be formed only of relevant quantities. Consider the relevance list for the stirrer power in liquid aeration: The stirrer power P_0 in a non-aerated liquid has no business being there. The quotient $P/P_0 \equiv Ne/Ne_0$ is not helpful because Ne_0 is a constant. Similar is the case in the often encountered expression ν/ν_{water}, which contains the (constant) kinematic viscosity of water. Though a dimensionless expression, it has no significance whatsoever.

The relevance list must also include the universal physical constants (such as universal gas constant, R or the acceleration due to gravity, g) if these constants influence the process concerned. Some of the relevant parameters may be dimensionless by origin: angles α, volume fraction φ, etc. If relevant for the process, they have to be listed.

Each relevance list may contain only one target quantity. It is the only dependent variable. For the overall view the parameters influencing it will be subdivided into geometric, physical and operational ones.

In some operations the *target parameter* must be chosen with care. Let us recall the often made blunder of choosing $k_L a$ as the target parameter in surface aeration. Surely, the mass transfer through the liquid surface can be pursued solely by measuring the concentration of the dissolving gas in the liquid volume and by calculating $k_L a$ from the reading $c(t)$. Nevertheless, the target parameter in this operation is $k_L A$ and it can only be obtained by multiplying the measured $k_L a$ by the liquid volume $V\colon k_L A = k_L a\ V$.

In mass and heat transfer operations, the definition quantities $k_L a \equiv G/(V\,\Delta c)$ and $h \equiv Q/(A\,\Delta\vartheta)$ exclude the constituting parameters from being introduced into the relevance list. The only justification for the introduction of e.g. temperature difference $\Delta\vartheta$ into the relevance list for the heat transfer coefficient h is given by additional introduction of the temperature coefficients of density β and viscosity γ, resp. In these cases the temperature difference $\Delta\vartheta$ is an additional parameter from which the Grashof number Gr, etc. is formed.

Furthermore, it has to be considered whether the target quantity is an intensive or extensive one, because this also has an influence on the operational parameters. For example, $k_L a$ is volume-related and as such an intensive parameter. Thus, it affects the relevance list.

The application of an intensive quantity such as $k_L a$ is advantageous also if the material system cannot be regarded as completely homogeneous. Then a certain dependency on geometric parameters D, H, etc. must be expected.

There is no point in overloading the relevance list by listing all *geometric parameters*. Each relevance list is valid only for a specific geometrical configuration (type of stirrer, form of the vessel, installation conditions), therefore only the so-called characteristic geometric parameter is to be introduced, and with its help all the other geometric parameters are transformed to dimensionless numbers. In mixing technology, this parameter is obviously the diameter of the stirrer, as the diameter influences the stirrer power by d^5.

Those *physical properties* which are unknown and cannot be measured are problematic. One has to be reminded that liquid properties governing the coalescence behaviour and foaming in gas/liquid contacting are still not known. At present, the only satisfactory method is to specify the material system precisely (e.g., as a 5% NaCl solution) and to keep it unchanged when measuring differently scaled models.

It can be assumed that in most mixing operations the physical properties of the gas phase (ν_G and ρ_G) can be neglected as compared to the liquid ones. Nevertheless, if their influence is to be examined, these quantities will enter material numbers (ν_G/ν_L, ρ_G/ρ_L) and not process related (Re, Fr, etc.) ones: The latter have to be formed by the physical properties of the liquid.

In listing the *operational quantities* the acceleration due to gravity g *is often forgotten*. This is all the more surprising in view of the fact that the relevance of this quantity is easily enough recognized if one asks the following question:

Would the process function differently if the process took place on the moon instead of on Earth?

If the answer to this question is affirmative, g is a relevant variable.

The gravitational acceleration g can be effective solely in connection with the density, i.e., in the form of gravity $g\rho$. When inertial forces play a role, the density ρ has to be listed additionally. Thus it follows that:

(a) In cases involving the ballistic (Galilean) movement of bodies, the formation of vortices in stirring, the bow wave of a ship, the movement of a pendulum and other oscillation processes affected by the Earth's gravity, the relevance list comprises $g\rho$ *and* ρ. (In forming e.g. the Froude number, they reduce to g alone: $Fr \equiv n^2 d\ \rho/g\rho = n^2 d/g$)

(b) Creeping flow in a gravitational field is governed by the gravity $g\rho$ alone.

(c) In heterogeneous material systems with differences in density (sedimentation or buoyancy movements), both the difference in gravity $g\Delta\rho$ *and* ρ are crucially important.

It is often overseen that in mixing operations taking place in material systems with density differences the compound $g\Delta\rho$ is indispensable. Without $\Delta\rho$ the gravitation exerts no influence and in a gravitation-free space $\Delta\rho$ loses any significance. Therefore, both quantities belong inseparably together and form the same number Fr' or Ar.

An *intermediate quantity* is one whose introduction into the relevance list replaces two or more parameters thus simplifying it. One of the best known intermediate quantities is the superficial velocity used in bubble columns and fluidized beds, where it replaces the volumetric throughput q as well as the column diameter D, because of $v \propto q/D^2$.

In mixing operations, a prime example for intermediate quantities is given in the homogenization of liquid mixtures with different densities and viscosities; Zlokarnik (1970). A pursuit of the mixing process by Schlieren optics shows that the quick coarse equalization is succeeded by a very slow molecular homogenization, taking place in a material system already exerting the physical properties of the uniform mixture.

The original relevance list

$$\{\theta; d, \varphi; \rho_1, \rho_2, \nu_1, \nu_2; n, g\Delta\rho\}$$

contains nine parameters and results in a pi space of six numbers. Due to the above observation it can be restricted via introduction of two intermediate physical properties (density ρ' and viscosity ν' of a homogeneous mixture)

$$\nu' = f(\nu_1, \nu_2, \varphi) \text{ and } \rho' = f(\rho_1, \rho_2, \varphi)$$

to a six parametric relevance list

$$\{\theta; d; \rho', \nu', g\Delta\rho, n\}.$$

Now, the process can be described by a mere three parametric space

$$\{n\theta, Re, Ar\}$$

$$Re \equiv nd^2/\nu',\ Ar \equiv d^3\ g\Delta\rho/(\rho'\nu'^2).$$

4. GENERATION OF DIMENSIONLESS NUMBERS

Generating dimensionless numbers and possibly their transformation represents an extremely easy undertaking compared to the drawing up of a reliable and as accurate as possible relevance list. This has been made possible by matrix calculations proposed by Pawlowski (in: Zlokarnik, 1991), which replaced the ardous handling of linear equation systems. The cited literature offers detailed examples of how to handle this technique in order to quickly obtain the complete set of dimensionless numbers.

To put it precisely: There are no problems nor limits in the application of dimensional analysis and in scale-up techniques associated with it. However, there may be problems concerning the generation of a complete relevance list and limitations with regard to procurring the model material systems or with aquiring reliable measurement data. This will be discussed in the following.

5. MEASURING TECHNIQUES AND SOURCES OF ERROR

Some of us surely still recall the problems encountered in measuring the mixing power with laboratory appliances, whose friction losses were of the same magnitude as the power consumption of the stirrer.

At present, some target quantities of little sensitivity still exist. In the following two important ones will be discussed.

5.1. *Mixing time θ*

Measuring this target quantity on a laboratory scale and in the turbulent region ($Re > 10^4$) amounts to some seconds and the concurring result is $n\theta = \text{const}$. This is not in agreement with the physics of the process, because the equalization must finally be limited by the molecular diffusivity and then the consequence is $\theta = \text{const}$.

From this point of view, it would surely be more sensible to formulate the target number as a combination $n\theta/Re \equiv \theta\nu/d^2$ or $\theta\nu/D^2$ resp., because this number turns out to be a constant at $Re \to \infty$, and signals, that θ enhances with D^2, (see Fig. 1). Without doubt, macromixing is an extremely scale-dependent process, but this does not corresponds to $n\theta = \text{const}$. [It was Kipke (1982), who has first drawn the attention to this problem.] This interrelation is proven by measurements at different liquid height/vessel diameter ratios (H/D), Fig. 2.

From Fig. 2 it can be determined that for the cross-beam stirrer the correlation

$$n\theta \propto (H/D)^{2.6}, \quad Re = 10^3\text{–}10^5$$

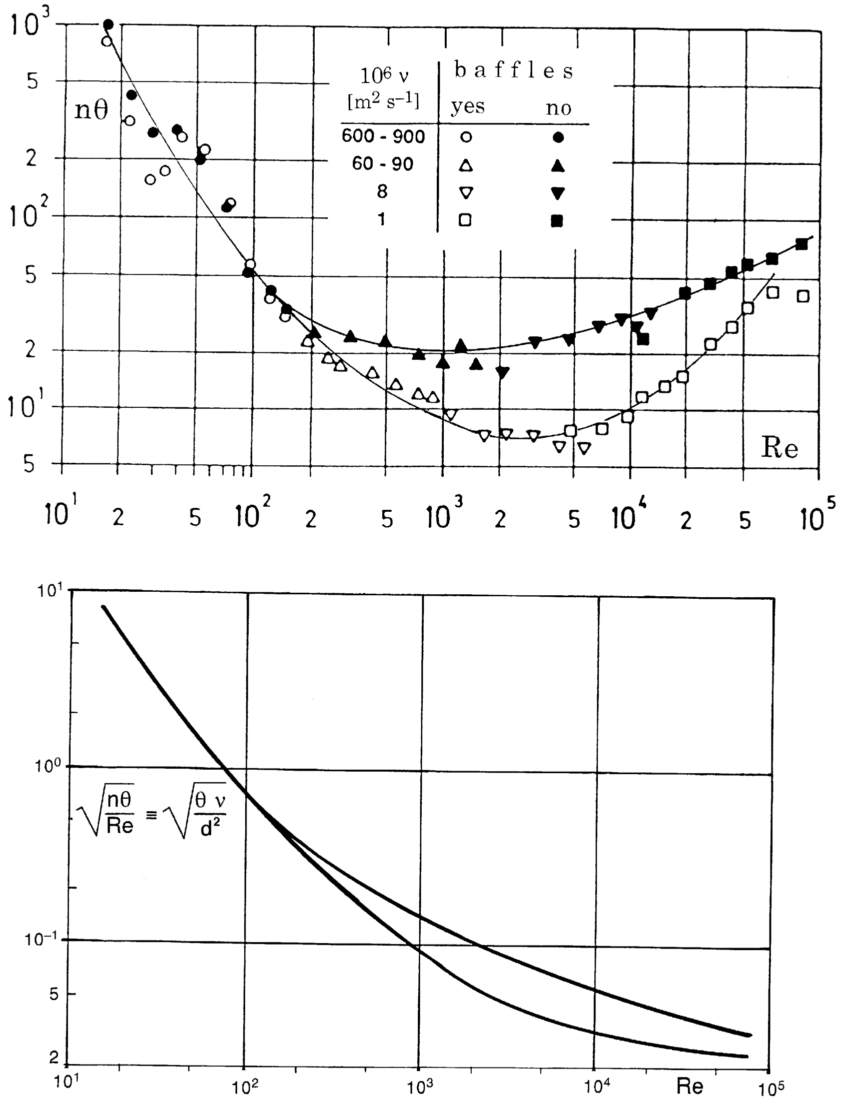

Fig. 1. Mixing time characteristics of a leaf stirrer in a vessel with and without baffles. Taken from Zlokarnik (1967). Upper plot: usual presentation $n\theta = f(Re)$, plot bellow: the proposed presentation $n\theta/Re = f(Re)$.

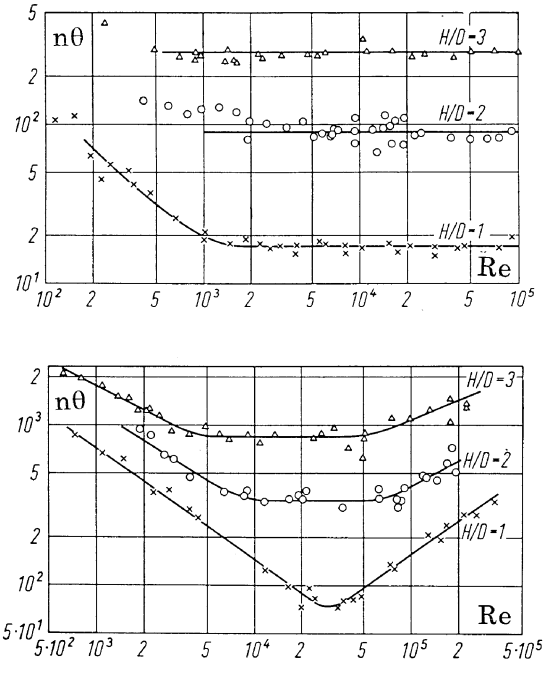

Fig. 2. Mixing time characteristics of a cross beam stirrer (upper plot) and a propeller stirrer (underneath) in a vessel with baffles and different H/D ratios. Taken from Zlokarnik (1967).

holds, whereas for the propeller stirrer the correlation $n\theta = f(H/D)$ depends strongly on the Re value:

$$n\theta \propto (H/\mathrm{D})^{0.85}, \quad Re = 10^3$$

$$n\theta \propto (H/D)^{1.5}, \quad Re = 10^4\text{–}10^5.$$

Certainly, these are aspects by which the scale-up of homogenization processes appears in a different light. These problems deserve more attention payed by the industrial R&D.

In this context, it has to be remembered that the otherwise excellent visual determination of the mixing time by a chemical decolorisation method cannot be precise in larger vessels. Thick water layers glow blue and white painted inner walls of large steel vessels distort the perception of the colour change. This behaviour has been found in the evaluation of the homogenisation characterisics obtained by mixing with rising up gas bubbles; Zlokarnik (1968). Whereas it was possible to obtain the same correlation in transparent vessels of laboratory size ($D = 0{,}44$–1.0 m), the industrial vessel of $D = 1.8$ m with white painted inner wall gave 1/3 lower values.

In this process the final correlation can be compressed to the expression

$$\theta/D = 4.75(H/q)^{1/2}(d/g)^{1/4},$$

which testifies that here the dependency $\theta \propto D$ exists.

Consequently, one solution to this problem certainly consists in conducting mixing time measurements in transparent vessels of constant diameter and to change the liquid height instead.

Table 1. Correlation between 1 s criterion and the corresponding time number $t^* \equiv t(g/D)^{1/2}$ when D is changed by the scale factor of $\mu = 1{:}10$

D (m)	t (s)	t^*	t^*	t (s)
0.2	1	7.07	7.07	1.00
0.5	1	4.47	7.07	1.58
1.0	1	3.16	7.07	2.23
2.0	1	2.23	7.07	3.17

5.2. *Complete suspension of solid particles according to the 1-s-criterion*

According to a proposition (Zwietering, 1958), a complete suspension of solid particles in a liquid is accomplished when none of the particles remain longer than 1 s on the vessel floor. Although easily measured, this is a very inaccurate criterion displaying a low sensitivity. Who is really able to determine visually whether it was 0.7 or 1.3 s? In addition, this quantity is a dimensional one and therefore not convenient for scale-up; Einenkel (1980) has drawn attention to this circumstance. In any case, this criterion is of hardly any relevance because of its low sensitivity.

Fom Table 1 it is obvious that replacing t by a dimensionless time $t^* \equiv t(g/D)^{1/2}$ would result in no advantage. The left side of this table indicates how t^* changes when $t = 1$ s is kept constant. By strongly changing the diameter from $D = 0.2. \rightarrow 2.0$ m the value of t^* alters by only a factor of $7.07/2.23 = 3.17$. The right-hand side of the table views this situation from an opposite point of view. It shows how the time t would have to change to satisfy a given $t^* =$ idem.

A sensible way out of this dilemma was found by replacing the 1 s criterion by the relative suspension layer criterion $h^* \equiv h_s/H = 0.9$ (the layer h_s of suspended particles reaches 90% of the liquid height H, Einenkel (1980)).

6. ACCURACY OF MEASUREMENT AND SCALE-UP FACTOR

A frequently posed question is how accurately the measurements have to be to safeguard the scale-up. This question is quickly answered; Zlokarnik (1991): The larger the scale-up factor $\mu \equiv l_T/l_M$, the more precise the measurements on the laboratory scale have to be. If this cannot be achieved, then the measurements have to be performed on a bigger pilot plant scale to diminish the scale-up factor.

In this context, too, the author has to refer to his own measurements of mass transfer by surface aerators (Zlokarnik, 1979). They were performed on three different scales (aerator diameter $d = 90$, 180, 270 mm) and resulted in the finding that sorption number Y depends solely on the Froude number Fr:

$$Y \equiv f(Fr) \rightarrow \frac{k_L A}{d^3}\left(\frac{v}{g}\right)^{1/3} = f\left(\frac{n^2 d}{g}\right).$$

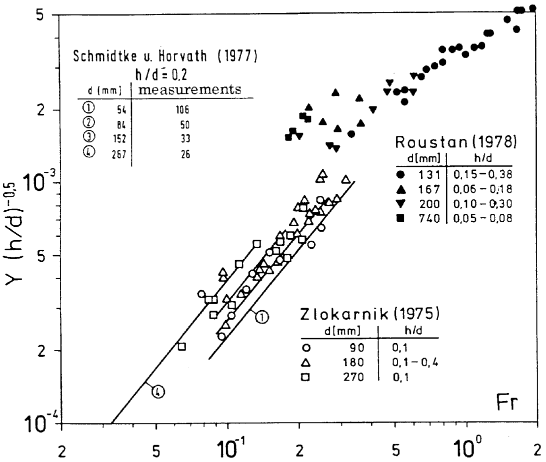

Fig. 3. Sorption characteristics of a Rushton turbine as a surface areator. Incorrect presentation in the space $Y \equiv f(Fr)$. Taken from Zlokarnik (1984).

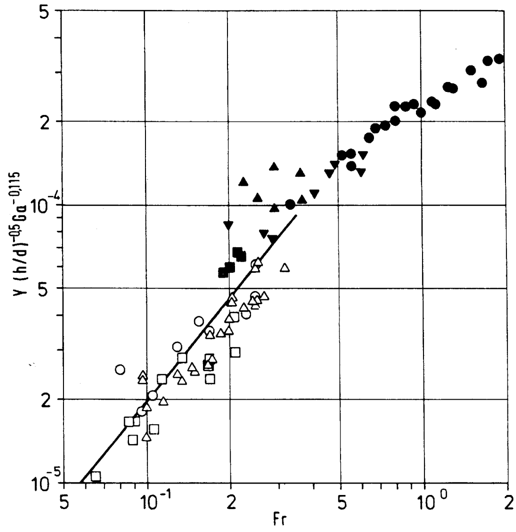

Fig. 4. Sorption characteristics of a Rushton turbine as a surface areator. Improved presentation in the space $Y\ Ga^{-0.115} = f(Fr)$. Taken from Zlokarnik (1984).

This finding (Fig. 3) led to the devastating result that the efficiency E (kg O_2/kWh) of surface aerators diminishes proportionally to the square root of the scale-up factor:

$$E_T = E_M \mu^{-1/2}.$$

In case of a (absolutely realistic) scale-up factor of $\mu = 10$—typical surface aerators have diameters of $d \geqslant 3$m—the technical design would have only 32% of the model efficiency.

A comparative evaluation (Zlokarnik, 1984) of precise measurements performed by Schmidtke und Horvath (1977) revealed that the sorption characteristics are not given by $Y(Fr)$ but by $YGa^{-0.115}\ (Fr)$, Fig. 4. This results in a completely different scale-up rule for the efficiency of the mass transfer

$$E_T = E_M \mu^{-0.155}.$$

It implies that at $\mu = 10$ the industrial design retains 70% of the efficiency of the model.

7. MEASUREMENTS IN EQUIPMENT OF DIFFERENT SIZES

It should be pointed out that for clarification of some scale-up problems, it is indispensable to perform measurements on differently sized equipment.

7.1. *Mass transfer* G/L

Because of the intensity character of the target quantity $k_L a$ (see Section 3), the process variables have to be formulated as intensive parameters as well. Now the question arises, whether the operational parameters should be P/V and q/V (q —gas throughput) or whether it is accurate to formulate the latter parameter as superficial velocity $v_G = q/S$ as commonly done in bubble columns. The decision in favor of either representation is impossible if one holds on to a single laboratory equipment. In this case, a change in equipment size is indispensable.

Judat (1982) has made a comparative evaluation of nine publications on mass transfer in the coalescent material system water/air in mixing vessels equipped with turbine stirrers, where the liquid volume changed from 2.5 l to 906 m^3 ($\mu = 1:71$). His study verified that the process characteristics are given in the pi space

$$\{(k_L a)^*,\ P/V)^*,\ v_{G*}\}$$

$$(k_L a)^* \equiv k_L a(\nu/g^2)^{1/3},\quad (P/V)^* \equiv (P/V)/[\rho(\nu g^4)^{1/3}];$$

$$v_{G*} \equiv v_G(\nu g)^{-1/3}.$$

7.2. *Flooding point in gas dispersion by stirrers*

Without change in scale it would be impossible to prove that the flooding point characteristics are given by $Q(Fr, Ga)$ and not by Q(Fr). One could, of course, obtain this information also by changing the liquid viscosity in a vessel of given size, but then the flow behaviour would correspond to lower Reynolds numbers, which is not advantageous for scaling *up*.

7.3. *Mass transfer experiments*

The experiments for mass transfer in surface aeration had to be performed in differently scaled equipment (see Section 6) due to the fact, that decoupling the numbers *Re* and *Fr* or *Fr* and *Ga*, resp. would otherwise only be possible by changing the viscosity, because the gravitational force cannot be changed in mixing technology. (Froude, too, had to change scales in his studies of drag resistance of ships; see Zlokarnik, 1991).

7.4. *Physical properties*

When physical properties are unknown and cannot be measured, then the experiments have to be performed with the same (exactly specified) material system using differently scaled models. (*Examples*: Flotation, mechanical foam destruction, influence of shear rate on biological systems, etc.) Undoubtly the results of such investigations are only exactly valid for the specific material system used.

8. PROBLEMS IN THE GENERATION OF RELIABLE SCALE-UP RULES

It is an old-established practice to perform experiments in differently sized equipment in order to obtain process characteristics that represent reliable scale-up criteria. Alas, laboratory devices at universities normally do not allow a large change of scale and therefore sometimes do not deliver reliable scale-up rules for the mixing operation on a really large industrial scale. This situation was already addressed in the discussion of mixing time problems, Section 5.1. In the following, this problem will be consolidated in connection with the suspension of solid particles.

So far, no mixing operation has stirred more emotions and has been discussed more controversely in Germany than the suspension of solids. In this context it was noticeable that the authors without exception blamed the dimensional analysis and the resulting pi space for not being able to present their measurements in a form which would allow a reliable scale-up of this mixing operation.

This, of course, is absurd inasmuch as the dimensional analysis represents a method for producing dimensionless numbers. In no way it is a means to replace thinking or to decide which quantities should be regarded as relevant and incorporated into the relevance list.

After the 1-s-criterion has been replaced by the relative suspension layer criterion $h^* \equiv h_s/H$, the final settling velocity of the swarm w_{ss} has been additionally added into the relevance list (Einenkel, 1980). This quantity was chosen in analogy to flow behaviour in fluidized beds. Several formulas are at our disposal to calculate it. In addition, the mass ratio φ_m of solids to liquid has been replaced by the volume ratio φ_v of them.

With respect to the fact that the interdependence $v_{ss}/v_s = f(\varphi_v)$ exists and v_s, the sinking velocity of one particle, depends only on the particle diameter d_p and weight difference $g\Delta\rho$, the following must be stated. The introduction of v_{ss} into the relevance list was by no means necessary from the viewpoint of dimensional analysis. However, it is very well advantageous to physically interpret the sedimentation and the suspension process, resp. The sedimentation process can be described by w_{ss}/w_s and φ_v more clearly than with w_{ss}/w_s or φ_v alone. In publications of Einenkel (1980) and Kraume and Zehner (1995) w_{ss} and φ_v appear as a product $w_{ss}\,\varphi_v$, in the paper of Voit and Mersmann (1986) $w_{ss}^2\,\varphi_v$ occurs.

In view of the unambigous interdependence of $w_{ss}/w_s = f(\varphi_v)$ it was not necessary to add w_{ss}/w_s and w_{ss}, resp. into a relevance list which already contained φ_v, d_p and $g\Delta\rho$. However, it is advantageous with respect to experiments. In the scale-up of suspension processes, the requirement d_p/d or $d_p/D =$ idem has to be satisfied. This results in extremely small d_p values on the laboratory scale. In addition, the high Re values that occur in large industrial vessels ($D > 3$ m) are difficult to adjust to the laboratory scale.

The experimental procedure employed by Kraume and Zehner (1995)—vessel of $D = 0.1$ m, viscous liquid and a small w_{ss}: laminar flow range; vessel of $D = 1.0$ m, water and a high w_{ss}: turbulent flow range—disclose this problem and show that the incorporation of w_{ss}/w_s into the relevance list resulted in an advantage.

In conclusion, we have to comment on the conjecture that the turbulence field in the experimental space cannot be adequately described by either the Reynolds number or the Froude number alone. Kipke (1982) raised this suspicion in referring to the drag characteristics of a sphere and the controversy between Prandtl and Eiffel at the beginning of this century. Prandtl, however, showed that the laminar boundary layer around the sphere can be turned turbulent simply by a piece of equipment ('trip-wire'). The turbulent boundary layer tears off at a lower Reynolds number thus diminishing the drag coefficient.

To put it differently: This piece of equipment is not characterized by any dimensionless number. Rather, the drag characteristics of a sphere are unambigously described by the space $\{Eu, Re\}$. In mixing technology, this corresponds to the presence of baffles: Although they influence the flow behaviour just as strongly (see e.g. Fig. 1) they are captured merely as a boundary condition and do not enter any dimensionless number.

NOTATION

Dimensional parameters

A	area
d	stirrer diameter
D	vessel diameter
H	liquid height
n	rotational speed of the stirrer
q	throughput of the fluid
v	superficial velocity of the fluid
φ	ratio of solids to liquid (φ_v—by volume)
P	stirrer power in pure liquid
P_0	stirrer power in aerated liquid
$k_L a$	mass transfer coefficient in gas/liquid contacts
h	heat transfer coeff. at the inner wall of the vessel
S	surface

Greek letters

ρ	liquid density
η	liquid dynamic viscosity
ν	liquid kinematic viscosity ($\nu \equiv \eta/\rho$)

Dimensionless numbers

$Ar \equiv Re^2/Fr'$	Archimedes number
$Eu \equiv \Delta p/\rho v^2$	Euler number
$Fr \equiv n^2 d/g$	Froude number

$Fr' \equiv n^2 d\,\rho/g\Delta\rho$	Froude number, extended
$Ga \equiv Re^2/Fr$	Galileo number
$Gr \equiv \beta\Delta\vartheta\ Ga$	Grashof number
$Ne \equiv F/(\rho v^2 l^2)$	Newton number (force)
$Ne \equiv P/(\rho\, n^3 d^5)$	Newton number (power)
$Ne_0 \equiv Ne(Re = 1)$	
$R \equiv n\,d^2/\nu$	Reynolds number
$Q \equiv q/n\,d^3$	throughput number

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