

A MATHEMATICAL SIMULATION MODEL FOR THERMOSTATIC EXPANSION VALVES

MANUEL R. CONDE and PETER SUTER

Energy Systems Laboratory, Swiss Federal Institute of Technology, CH-8092 Zürich, Switzerland

(Received in revised form 23 October 1991)

Abstract—A mathematical model to simulate the steady-state operation of thermostatic expansion valves, in the range up to 30 kW refrigeration capacity, is described. Such a model is necessary in the study of component matching, over wide ranges of operating conditions, in heat pumps and refrigeration machines. The model treats the mutual dependence between the valve's throttling and control functions, and requires a few experimentally determined parameters, besides the accurate description of the geometry of the throttling section. An algorithm describing the solution of the mathematical model, as implemented in the computer program *HPDesign* for simulation of heat pumps, is presented. The paper also shows that the data required may be obtained without the participation of the manufacturers, but stresses that such participation would ease the effort, and eventually reduce the uncertainty, in the determination of some parameters.

NOMENCLATURE

A	area
D	diameter
$D_{H,T}$	hydraulic diameter of the valve throat
F	force
K	spring constant
\dot{M}	refrigerant mass flow rate
P	refrigerant pressure at the throat
T	temperature
T_r	reduced temperature
u	velocity
X	valve opening displacement
ΔT	temperature difference
ΔP	pressure difference
θ	taper angle of the poppet
λ_i	i th parameter of the model
ρ	density
ϕ	angle of superheating adjustment

Subscripts

a	adjusted
b	bulb, phial
diaph	diaphragm
dy	dynamic
ev	evaporation
H	hydraulic
p	phial, poppet
r	refrigerant (operating fluid)
s	spring, saturation
sat	at saturation
st	static (no flow)
T	throat
0	default (factory set)

1. INTRODUCTION

Thermostatic expansion valves (TXVs) are used as throttling devices in vapour compression machines for both refrigeration and heat pump applications. The TXV, in a vapour compression machine, controls the supply of refrigerant to the evaporator, while maintaining a certain degree of superheating at the evaporator outlet. Although new technologies to control the supply of refrigerant to the evaporator are already in the market, e.g. electronic expansion valves, TXVs still hold advantages in many applications, in particular regarding costs. Heat pumps, particularly of

the air-source type, operate within a wide range evaporating temperatures, requiring the characteristics of all components to match all over this range of conditions. The matching of all components should be perfect over the whole operating range, in order to obtain the best possible performance. The study, or verification, of component matching, cannot be done using the data usually supplied by the components' manufacturers alone, as it is not possible to analyse the interplay of the characteristics of the various components dynamically. The matching of components requires the mutual accommodation of their characteristics, and this can only be studied in either of two ways: experiment or simulation. Although testing represents the best possible assessment of performance, it is very expensive, especially when a wide range of operating conditions for various machine capacities are to be tested. On the other hand, the flexibility of the simulation approach does not need to be stressed. However, it is effective only when the simulation models describe the individual components with enough accuracy. In an effort to provide the heat pump manufacturers with a tool to analyse the performance of their products over the whole operating range, mathematical models of a number of component types, used to assemble low- and medium-capacity heat pumps, have been developed. These models are implemented in the computer program *HPDesign* for heat pump simulation [1]. The simulation model of the TXVs reported in this paper is implemented there as well. In the following, we describe the operating principle of TXVs, and give details of a methodology to develop a mathematical model for them. Further, an algorithm to solve the model, as used in the simulation program *HPDesign*, is described.

2. PHYSICAL DESCRIPTION AND OPERATION OF THERMOSTATIC EXPANSION VALVES

Figure 1 depicts schematically a typical thermostatic expansion valve. The thermostatic bulb, or phial (4) is attached to the evaporator outlet tube, thus sensing the temperature of the refrigerant at the evaporator outlet. The pressure generated inside the phial acts upon the upper face of the diaphragm (3), and is a function of the evaporator outlet temperature, and of the type of charge in the bulb. Under the diaphragm acts the actual evaporation pressure.* Thus, the diaphragm "computes" the evaporator outlet superheating by the play of these two pressures. The spring (7) under the diaphragm ensures that the phial pressure is always higher than the evaporation pressure. The tension on this spring may be adjusted by turning the screw (8). The TXV may be adapted to the actual evaporator using this screw, which sets the opening superheating (ΔT_s in Fig. 2), in fact promoting a translation of the valve characteristic. It also helps in setting the TXV to satisfy the stability criteria, Huelle [2]. The diaphragm acts upon the valve poppet (5) through the actuator (2), displacing it and thus generating a wider or narrower flow section. A TXV performs several functions at once in a vapour compression refrigeration or heat pump machine: it throttles the refrigerant from the condensation to the evaporation pressure while maximizing the use of the evaporator heat transfer area, and protecting the compressor from liquid surges. These functions may be divided by their nature into control and throttle functions. However, they are not independent, and a simulation model should reflect their mutual influence.

Viewing the TXV as a throttling device, it may be described as a variable free-flow area orifice, with the fluid at the inlet in the liquid phase, and at the outlet as a two-phase liquid-vapour mixture.† This free-flow area may be generated in many ways. The most common is a tapered plug [poppet, Fig. 1 (5)] moving against a fixed diameter orifice plate or tube. Higher capacity TXVs may have several ports, or the poppet may be so large that it appears more like a disk. The throttling of the liquid refrigerant through the very narrow passage generated by the displacement of the poppet induces an extreme acceleration upon the fluid. This acceleration reduces the local pressure to values well under the saturation at the current temperature. When emerging from the throat to a much larger flow area, the superheated liquid flashes into a two-phase liquid-vapour mixture flow. The acoustic velocity of such a mixture is so low (typically under 40 m/s for HCFC22)

*For evaporator systems with a small pressure drop, this pressure may be the TXV outlet pressure, taken inside the valve (internally equalized valve), while for evaporators with a large pressure drop this pressure should be the refrigerant pressure at evaporator outlet (externally equalized valve).

†This is true for standard steady-state operation. Other conditions may occur, particularly at start-up, but are transient phenomena. Operation with two-phase liquid-vapour at inlet, for example, is unstable.

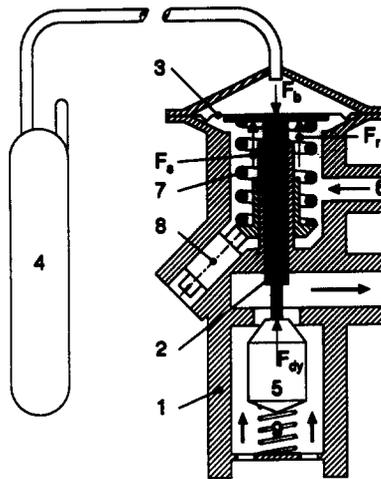


Fig. 1. Schematic of a typical thermostatic expansion valve: (1) valve body; (2) actuator; (3) diaphragm; (4) thermostatic bulb; (5) value poppet or plug; (6) external equilization port; (7) opening superheating setting spring; (8) opening superheating setting screw; (9) poppet supporting spring.

that the flow chokes. The choking makes the rate of flow across the throat of the valve practically independent of the downstream, evaporation pressure. Due to the flashing, there is no pressure recovery after the throat, as schematically shown in Fig. 3, and the fluid will tend to equilibrium conditions, at the downstream pressure, while maintaining the total enthalpy constant (the thermal losses in the throttling section are negligible).

As control devices TXVs may be described as proportional controllers with a time delay imposed by the evaporator with which they work. Figure 4 depicts a block diagram of the control function of a TXV. Although the static (no flow) opening characteristics of the valve are normally linear, as shown in Figs 9 to 11, the effect of the forces generated by the flow acceleration may modify these characteristics significantly. In order to model the steady-state operation of a TXV, it must be assumed that the stability criteria [2] are fulfilled. However, the model should be able to predict extreme operating conditions, e.g. maximum opening, which are an indication that those criteria

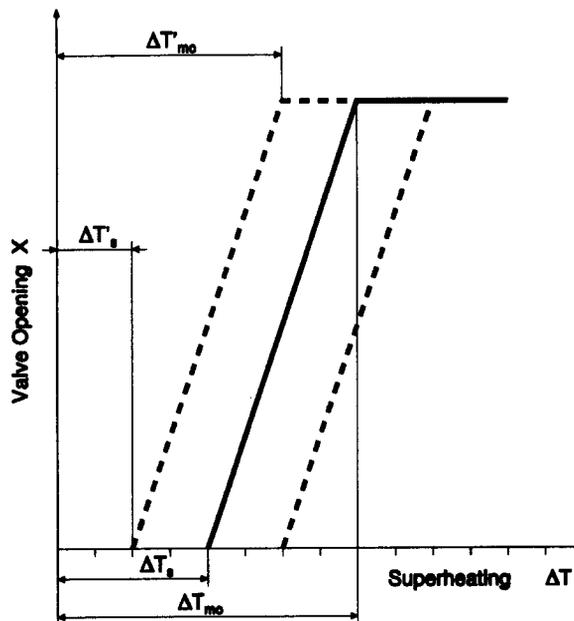


Fig. 2. Translation of the TXV opening characteristic by adjusting the superheating adjusting screw (Fig. 1 (8)). ΔT_s : static opening superheating (minimum opening superheating); ΔT_{mo} : superheating of maximum opening.

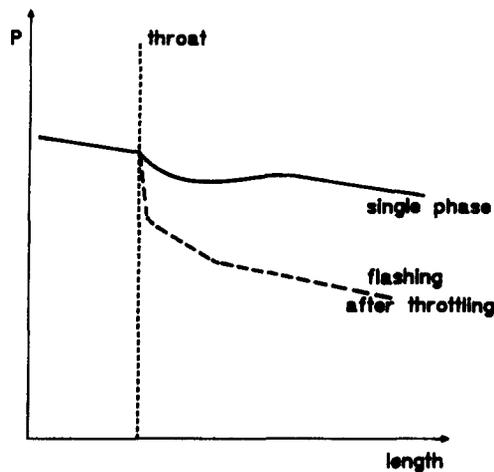


Fig. 3. Typical pressure variation across an orifice.

are not met. The model should also predict the minimum required adjustment of the valve static superheating settings necessary to avoid extreme operating conditions.

3. THE MATHEMATICAL MODEL

The present model has been developed for use in the simulation of vapour compression heat pumps, and the choice of input and output variables is that shown in the block diagram of Fig. 5 [1].

3.1. Theory

The steady-state operation of a TXV is reached when the equilibrium of the forces acting upon the diaphragm, Fig. 1, is attained. The equilibrium is mathematically expressed as

$$F_b = F_r + F_s + F_{dy}, \tag{1}$$

where F_b is the force generated by the bulb (phial) pressure on the upper face of the diaphragm; F_r is the force generated by the operating fluid (refrigerant) on the lower face of the diaphragm;

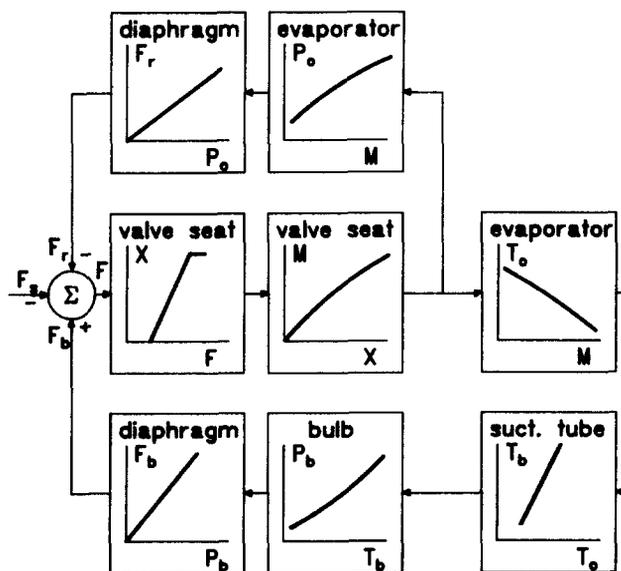
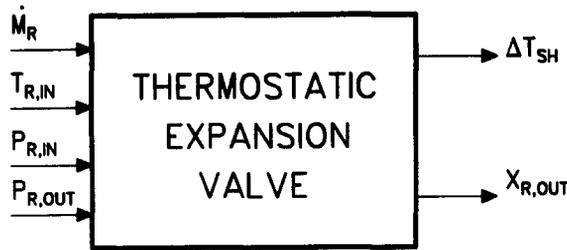


Fig. 4. Block diagram of the control function of a TXV.



Data Required

- Geometry
- Opening Characteristic
- Refrigerant Type

Fig. 5. Information block diagram of the TXV's model [1].

F_s is the sum of the forces exerted by both springs (7 and 9) (Fig. 1); and F_{dy} is a force resulting from the imbalance of pressures around the poppet, due to the acceleration of the flow.

The force F_b depends exclusively upon the current temperature at the thermostatic bulb (4) and on the kind of charge in it. The spring force F_s may be broken down into three components

$$F_s = F_{s,0} + F_a + K_s X, \quad (2)$$

where $F_{s,0}$ is the sum of the forces exerted by the both springs (7 and 9) with the valve completely closed, at the factory settings; F_a results from an eventual adjustment of the static opening superheating of the valve, by means of the screw (8); and $K_s X$ is the component due to the poppet displacement. K_s is assumed constant (linear springs).

There are several unknowns in this model that are difficult to determine explicitly. The calculation of F_b requires the knowledge of the pressure-temperature relationship of the phial charge, which is unknown. The calculation of F_s is virtually impossible: although K_s is easy to determine, the factory set tension $F_{s,0}$ cannot be easily measured. The relationship between the poppet displacement and the flow area at the valve throat may be calculated when the geometry of the poppet and of the orifice are known. From the geometry as shown in Fig. 6, the following results:

$$A_T = \pi X \left(D \sin \theta - \frac{X \sin^2 2\theta}{\cos \theta} \right) \quad (3)$$

and the hydraulic diameter at the throat is

$$D_{H,T} = \frac{4A_T}{\pi(2D - X \sin 2\theta)}. \quad (4)$$

Although this model seems at first impractical, it may be put to work if the difficulties mentioned are circumvented. One way to overcome them is to establish a direct relationship between the poppet displacement X and the measurable variables that determine it, on the one hand, and between the forces due to flow acceleration and the excess superheating required to equilibrate them, on the other.

Under static (no flow) conditions, it is possible to establish a relationship between the valve opening displacement X and the superheating applied to the phial to produce it. Within the physical limits of the valve—totally closed and fully open—this relationship may be assumed linear (Figs 9–11). The parameters describing the static opening characteristic—slope and intercept—depend upon the evaporation pressure, as expressed, in general, by equation (5). The form and the parameters of the functions f_1 and f_2 are determined through experiment.

$$X = f_1(P_{ev}) + f_2(P_{ev}) \Delta T_{SH, st} \quad 0 \leq X \leq X_{max}. \quad (5)$$

additional relationship between F_{dy} and the excess superheating necessary to equilibrate it at the diaphragm, may only be obtained from experiments, in the absence of manufacturers' specific data. Determining the actual pressure-temperature dependence of the phial charge is feasible, but requires a lot of experimental effort. The remaining possibility is to use measurements of the dynamic operation of the valve, in as wide a range as possible, and establish that relationship empirically with the help of the static model described above. The absolute values of the phial pressure cannot be determined with this method, but its gradient with the temperature may be related to that of the working fluid. Figure 7 depicts a typical dependence of these two pressures upon the temperature for a liquid cross-charge. Mathematically, their gradient with the temperature may be related as follows:

$$\left(\frac{\Delta P_{\text{phial, dy}}}{\Delta T_{\text{SH, dy}}}\right) = f(T_{s, p}) \left(\frac{dP_{\text{sat, r}}}{dT}\right)_{T_{s, p}} \quad (7)$$

This equation yields $\Delta T_{\text{SH, dy}}$, the excess superheating required to balance the forces produced at the diaphragm by the acceleration of flow across the throttling section, when $f(T_{s, p})$ is known. Other types of phial charges (gas cross, MOP, adsorption, etc.), may require a different formulation of this dependence. It is immediately evident that the knowledge of the actual pressure-temperature relationship of the phial charge would be useful. Plans are made to investigate further this relationship through experiment.

The first output variable of the model, the operating superheating, ΔT_{SH} , is then calculated. The second variable is the refrigerant vapour quality at the TXV outlet, which is easily calculated applying the continuity and energy equations, and assuming the flow at the TXV outlet to be homogeneous and in thermodynamic equilibrium. The details will not be given here.

3.2. Methods

The static (no flow) opening characteristics of TXVs were measured using the installation schematically depicted in Fig. 8. The evaporation pressure, P_{ev} , is generated by pressing low-viscosity oil in the cylinder (4), and the phial temperature is adjusted by means of the thermostatic bath (5). The pressure is kept constant at the desired value, which depends on the refrigerant for which the valve was designed. By changing the phial temperature, different values of the superheating are generated, and the valve opening displacement is measured by means of the indicator (2). This has been done for a number of valve brands and capacities. It is interesting to note here that hysteresis of the valve opening was only found for new valves. After some run

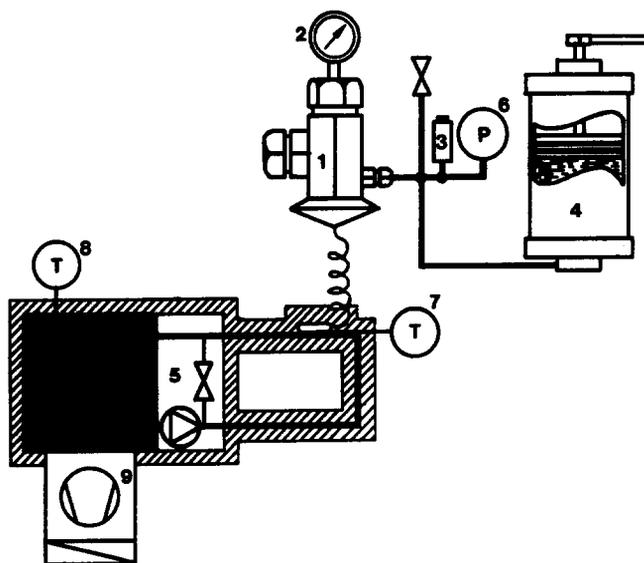


Fig. 8. Schematic of the set-up to measure TXVs opening characteristics: (1) TXV; (2) displacement meter; (3) pressure transducer (absolute); (4) pressure generator; (5) thermostatic bath; (6) manometer; (7) bulb temperature thermometer (Pt100); (8) bath temperature indicator; (9) heating/cooling device.

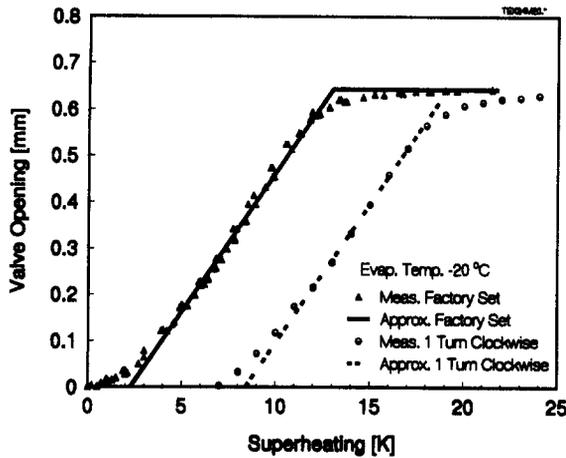


Fig. 9. Opening characteristic at -20°C evaporation temperature, and its displacement by adjusting the setting screw by one turn clockwise.

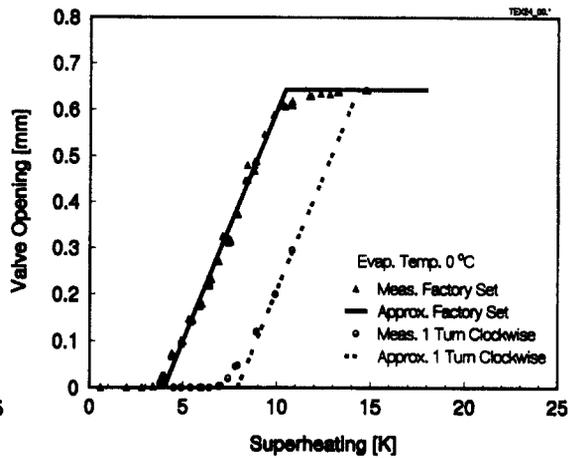


Fig. 10. Opening characteristic at 0°C evaporation temperature, and its displacement by adjusting the setting screw by one turn clockwise.

in, we did not find any significant hysteresis. Figures 9 to 11 show some results of measurements by this method for one brand. They also depict the translation of the static opening characteristic when the adjusting screw [Fig. 1(8)] was turned one full turn in the clockwise direction (increase of the static opening superheat). Figure 12 shows the slopes and intercepts of the static opening characteristic of the same valve brand, as a function of the reduced evaporation temperature, $*T_{s,p}$, of the operating refrigerant. The functions f_1 and f_2 describing the slope and intercept are, respectively:

$$f_1 = \lambda_0 Tr^{\lambda_1} \quad f_2 = \lambda_2 Tr^{\lambda_3} \quad (8)$$

The translation of the static opening characteristic by turning the adjusting screw one full turn in the clockwise direction, is described by equation (9) and is shown in Fig. 13 for the example TXV.

$$\Delta(\Delta T_{SH})_{\text{turn}} = \lambda_4 Tr^{\lambda_5} \quad (9)$$

The determination of the excess superheating to balance the dynamic forces is done from measurements with an air-to-water heat pump. The magnitudes required for this calculation are: (1) the refrigerant flow rate; (2) the temperature and pressure at valve outlet; and (3) the temperature and pressure at the phial location (the valve for which the measurements are shown here is externally equilibrated).

Using these measurements, together with the conservation equations and the static model, the additional pressure required on the diaphragm, $\Delta P_{\text{phial, dy}}$, is calculated. The excess superheating necessary, $\Delta T_{SH, dy}$, is obtained by subtracting that required to produce the throat area, with the static model, from that actually measured. The relationship between the two pressure gradients $(\Delta P_{\text{sat, r}}/dT)_{T_{s,p}}$ and $(\Delta P_{\text{phial}}/\Delta T_{SH, dy})$ is expressed by equation (10) and Fig. 14. For the example valve, the following values of the parameters were found: $\lambda_0 = 0.772$; $\lambda_1 = 6.658$; $\lambda_2 = -25.421$; $\lambda_3 = 13.739$; $\lambda_4 = 0.579$; $\lambda_5 = -6.261$; $\lambda_6 = 0.344$; $\lambda_7 = 19.78$.

$$\left(\frac{\Delta P_{\text{phial, dy}}}{\Delta T_{SH, dy}}\right) = \lambda_6 \left(\frac{T_{s,p}}{273.15}\right)^{\lambda_7} \left(\frac{dP_{\text{sat, r}}}{dT}\right)_{T_{s,p}} \quad (10)$$

3.3. Application

The intended application of this model is in the simulation of whole refrigeration or heat pump machines. Together with equal grade models of the other components, it permits the study of the performance of the whole machine under all possible working situations. It also allows variation of the TXV settings, and the determination of the consequences of that variation over the expected

*This is the saturation temperature at the evaporator outlet pressure.

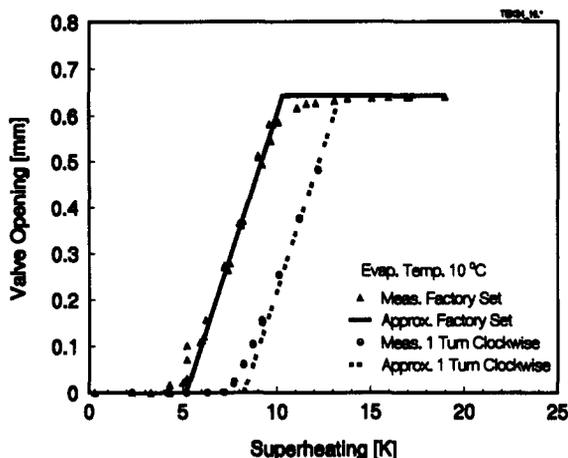


Fig. 11. Opening characteristic at 10°C evaporation temperature, and its displacement by adjusting the setting screw by one turn clockwise.

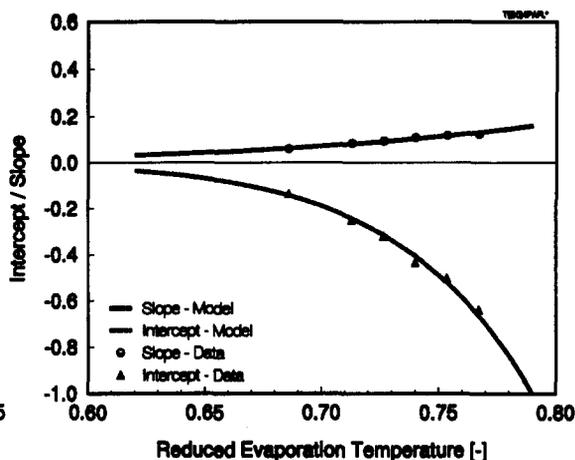


Fig. 12. Slope and intercept of a TXV opening characteristic.

operating range of the machines. The algorithm to solve the model in a computer program is described in the flow diagram shown in Fig. 15.

4. VERIFICATION OF THE MODEL

The simulation model described is compared to experimental data obtained on an air-to-water heat pump. The TXV used has a nominal capacity of 8.0 kW, at an evaporation temperature of 5°C, and a liquid inlet temperature of 28°C, with 4 K of subcooling. It is an externally equilibrated valve, designed for HCFC22. The experimental data set consists of 50 data points, where the values are averages of 60 scannings taken at 1-min intervals, in steady-state operation. Figure 16 shows a comparison of the measured operation superheating with that calculated from the model. The agreement is good. The scatter originates essentially from the dynamic part of the superheating, $\Delta T_{SH, dy}$, and is due to the method used in establishing the parameters λ_6 and λ_7 . While $dP_{sat,r}/dT$ is an absolutely smooth function of the temperature, $\Delta P_{phial, dy} / \Delta T_{SH, dy}$ is calculated for varying intervals $\Delta T_{SH, dy}$. Besides this, an indirect method as used in this study cannot avoid uncertainties in the measurements, and the eventual uncontrollable variations on a real piece of equipment.

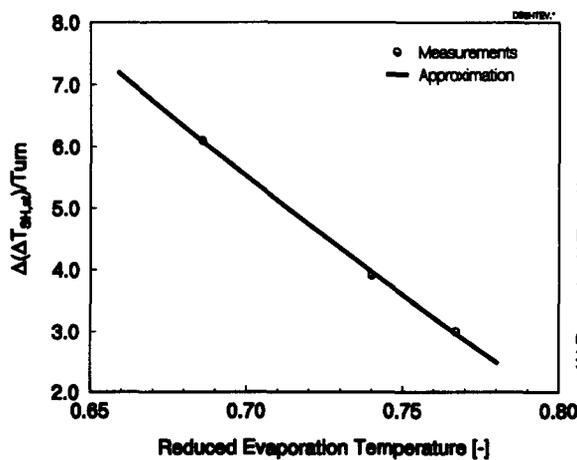


Fig. 13. Variation of the opening superheating with the evaporation temperature, for the example TXV.

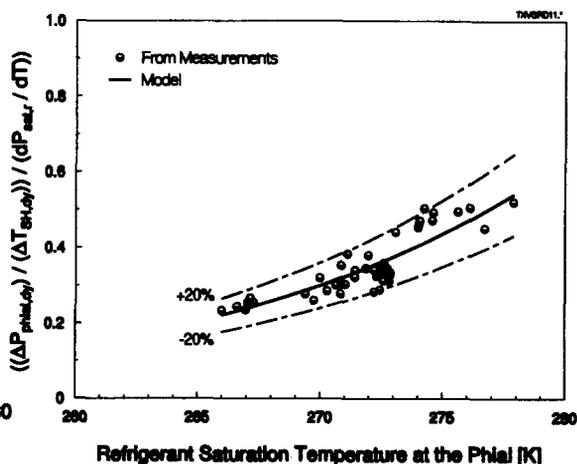


Fig. 14. Ratio of the charge to operating fluid pressure-temperature gradients.

ALGORITHM OF THE TXV MODEL

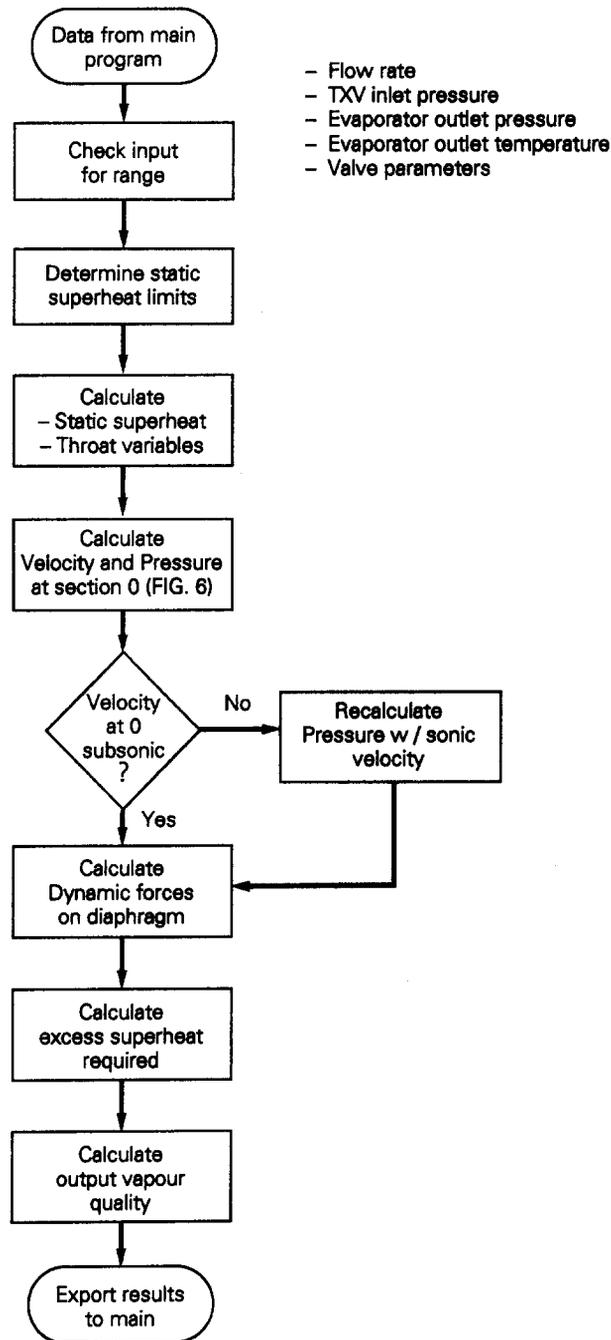


Fig. 15. Flow diagram of the algorithm applying the model for heat pump simulation in the simulation program *HPDesign* [1].

5. CONCLUSIONS

A mathematical simulation model for the steady-state operation of thermostatic expansion valves, with liquid cross-charge in the thermostatic bulb (phial), has been developed, based on the internal geometry of the valve, and on a few experimentally determined parameters. A simulation model of this type is required when studying the matching among the various components in refrigeration machines or heat pumps. Although the model is conceptually simple, the data it

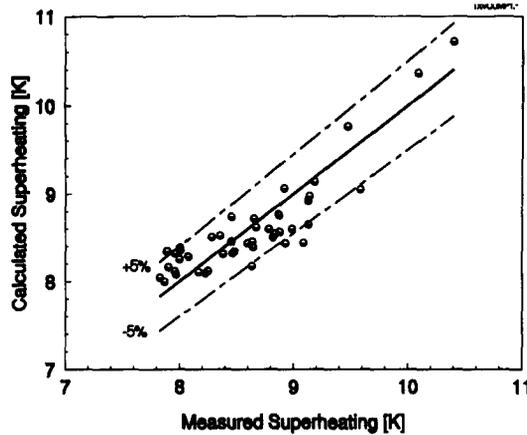


Fig. 16. Comparison of measured to calculated operating superheating.

requires are not readily available. The information usually provided by the manufacturers, in their catalogues and data sheets, is not sufficient to determine the parameters of the model. While some of these parameters are easily obtained by direct measurement, others require significant effort in time, means and skills.

The development of this model demonstrates that it is possible to gather all the data required without the assistance of the manufacturers. On the other hand, their assistance would reduce the investment in time and means, and would provide less uncertain data, and, contrary to the opinions they systematically express, needs not spread important proprietary data.

The application of this model, in the simulation of refrigeration machines and heat pumps, should enable their manufacturers to tune better the TXV settings, by design, in order to arrive at the best possible performance.

Further work is required to extend this modelling approach to other types of phial charge. In order to avoid extensive experimental work, more willing support of the manufacturers is desirable, believing, as we do, that the information necessary, when it exists, needs not be considered a corporate secret.

Acknowledgement—The support for this study, by the Swiss Federal Department of Energy and Transportation, is gratefully acknowledged.

REFERENCES

1. M. R. Conde and P. Suter, *HPDesign*—a computer program for the simulation of domestic heat pumps. *XVIIIth Int. Cong. Refrigeration*, Montréal (1991).
2. Z. R. Huelle, Matching of the evaporator and thermostatic expansion valve characteristics, in order to achieve system operation without hunting by using a digital computer. *Proc. XIIIth Int. Cong. Refrigeration*, Vol. II, 751–758 (1971).
3. E. E. Michaelides and K. L. Zisis, Velocity of sound in two-phase mixtures. *Int. J. Heat Fluid Flow* 4, 79–84 (1983).

APPENDIX: CALCULATION OF THE DYNAMIC FORCES APPLIED ON THE TVX'S DIAPHRAGM

The balance of forces due to the refrigerant flow, around the poppet of the valve, yields the excess force F_d , that the pressure in the phial-diaphragm system must equilibrate. Figure A1 shows the details of the poppet and actuator, and all the forces concerned. The force F_3 (see also Fig. 6), is calculated from the conservation equations applied to the control volume CV1, of the operating fluid (Figs 6 and A2).

Then, the additional forces applied to the poppet, F_5 and F_6 , and to the actuator F_4 , are calculated from the balance of the control volume CV2 (Figs 6 and A3). Several assumptions are required to write the conservation equations for these control volumes, namely: (1) the flow may be considered unidimensional; (2) the flow is incompressible while the fluid is in the liquid phase; (3) flashing of the superheated liquid occurs only after the throat; and (4) the two-phase mixture is in thermodynamic equilibrium at the section A_0 , and the flow is considered homogeneous.

The conservation of linear momentum applied to CV1, in the direction parallel to the \mathcal{Q}_L , yields

$$F_p - F_2 + F_3 \sin \theta - F_T \cos \theta = \dot{M}(u_T \cos \theta - u_1), \quad (\text{A1})$$

the continuity equation for the same control volume is

$$u_T = \frac{A_1}{A_T} u_1, \quad (\text{A2})$$

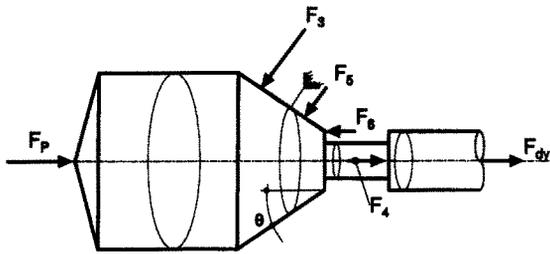


Fig. A1. Schematic of the TXV poppet and actuator, displaying the forces applied to them.

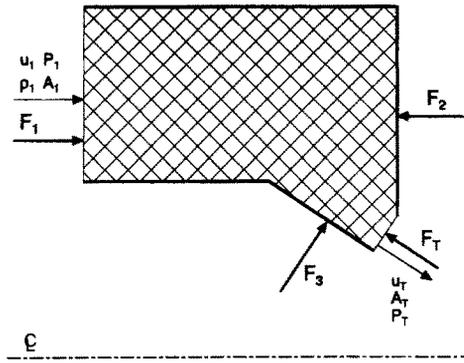


Fig. A2. Refrigerant control volume CV1 showing forces applied.

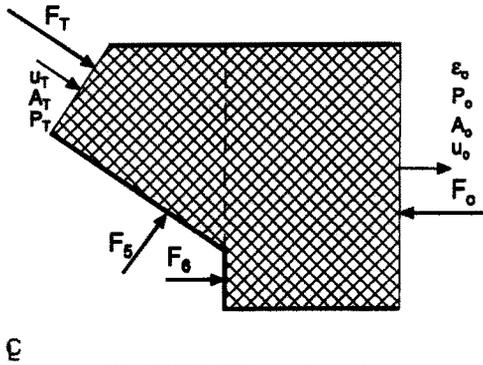


Fig. A3. Refrigerant control volume CV2 depicting forces applied.

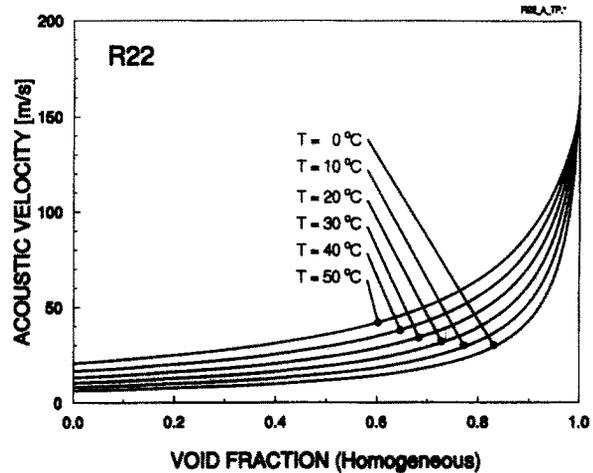


Fig. A4. Acoustic velocity of homogeneous two-phase liquid-vapour mixtures of HCFC22, calculated according to [3].

and the energy equation is the Bernoulli equation under the assumptions made,

$$P_1 + \frac{1}{2} \rho_1 u_1^2 = P_T + \frac{1}{2} \rho_1 u_T^2. \tag{A3}$$

The solution to this system of equations returns the flow area at the throat A_T , the pressure at the throat P_T , and the force F_3 . The static operating superheating is that required to produce A_T , and is determined from the static opening characteristic. The forces F_2 and F_3 are calculated as integrals of the pressure distribution on the respective surfaces (Fig. 6). As the flow accelerates converging to the throat, there is a significant variation of the local pressure (typical velocities at the throat are 60 m/s). Average velocities at each section in the convergent are considered.

The conservation of linear momentum applied to CV2 yields

$$F_T \cos \theta + F_5 \sin \theta + F_6 - F_0 = \dot{M} (u_0 - u_T \cos \theta), \tag{A4}$$

from where $F_5 \sin \theta + F_6$ is necessary to calculate F_{dy} , (Fig. A1).

$$F_{dy} = F_p - F_3 \sin \theta - F_5 \sin \theta - F_6 + F_4. \tag{A5}$$

Although it is not necessary to take into account what happens within CV2, it is necessary to be aware of the flow conditions at section A_0 , that it is necessary to know whether the flow chokes there, or whether it remains subsonic. The condition to remain subsonic is that the flow velocity, u_0 , be lower than the acoustic velocity corresponding to the thermodynamic state of the refrigerant at A_0 . It should be remembered that the acoustic velocity for a two-phase liquid-vapour mixture is substantially lower than that of the saturated vapour, (Fig. A4).

This done, the excess pressure required on the diaphragm is

$$\Delta P_{\text{phial, dy}} = \frac{F_{dy}}{A_{\text{diaph}}}, \tag{A6}$$

where A_{diaph} is the active area of the diaphragm. The excess superheating necessary to generate this excess pressure is now determined by calculating the variation of the temperature of the operating fluid that satisfies equation (10).

The actual operating superheating is the sum of the required to generate the throat area A_T (static model), with the excess superheating necessary to balance the dynamic forces, $\Delta T_{\text{SH, dy}}$.

$$\Delta T_{\text{SH}} = \Delta T_{\text{SH, st}} + \Delta T_{\text{SH, dy}}. \tag{A7}$$