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Estimation of the Critical Velocity in Pipeline Flow of Slurries

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SUMMARY

A detailed examination of the critical velocity in pipeline flow of non-colloidal slurries was carried out. Published critical velocity correlations were collected and were recast into a standard form so that they could be compared with each other, and they were also tested against a broad collection of experimental critical velocity data. Altogether, a total collection of 864 experimental critical velocity data, representing a broad variety of solid materials and pertaining to wide ranges of the variables involved, were used in these tests. This rather substantial body of experimental data was also used as the basis for developing a set of improved critical velocity correlations, which were established by fitting the data to various forms of the standard equation.

Among the principal results of this work are the following: The dependence of the critical velocity on pipe diameter is very nearly equal to $D^{1/2}$*, while its dependence on particle size, for slurries of non-colloidal particles, is very nearly equal to* d^0*. The analytical result due to Oroskar and Turian indicates that* v_C *depends on pipe diameter as* $D^{0.6}$*, and on particle size as* d^0*, while the best empirical fits to the data suggest that the dependence on pipe diameter is approximately* $D^{0.5}$*, and that on particle size is at most* $d^{0.06}$*. In addition, both the newly established empirical correlations and the analytical correlation due to Oroskar and Turian predict a maximum in the* v_C *vs. C curve, the maximum occurring at solids volume concentration of 0.25 to 0.30. The comparisons with experimental data further established that the analytical result by Oroskar and Turian, and the empirical correlations developed in this work, do a superior overall job of predicting the data than other published correlations.*

INTRODUCTION

This paper is concerned with the critical velocity in the flow through pipes of slurries composed of non-colloidal solids. The critical velocity v_C is defined as the minimum velocity demarcating flows in which the solids form a bed at the bottom of the pipe from fully suspended flows. It is also referred to as the minimum carrying or the limiting deposit velocity, and is perhaps the most important transition velocity in slurry transport. It is very difficult to determine experimentally, because the critical condition is difficult to discern, and because the flow becomes unstable near the critical condition.

Among the important technical problems in pipeline flow of slurries are the determination of the pressure drop–throughput relationship, the prediction of the prevailing flow regime, and the estimation of the critical velocity. These problems have been the subjects of extensive research, and have come to acquire a considerable literature. The literature contains many correlations for prediction of pressure drop, a few schemes for delineating slurry flow regimes, and many correlations for estimation of the critical velocity. There are wide discrepancies among the predictions from the various correlations. These difficulties will be examined in detail in this paper with respect to correlations for prediction of the critical velocity. In this paper, we review published correlations for the critical velocity, we compare them with each other by recasting them into a standard form, we test them against a broad collection of published experimental critical velocity data, and we present a new set of critical velocity correlations based on fitting various forms of the standard equation to the experimental data.

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PRESSURE DROP CORRELATIONS AND REGIME DELINEATION

There have been many attempts to develop correlations for predicting the pressure drop for the flow of slurries in pipelines. Beginning with the work of Blatch [1], some of the subsequent correlations include those by Howard [2], Wilson [3], Newitt *et al.* [4], Durand and co-workers [5 - 8], Zandi and Govatos [9], and Turian and Yuan [10]. The work of Durand and co-workers in the early fifties is significant because it involved examination of a very wide range of the pertinent variables in slurry transport. Sand and gravel slurries, with particle sizes ranging from 0.2 to 25 mm, in pipes ranging in diameter from 3.8 to 58 cm and solids concentrations up to 60% by volume were investigated. Durand's correlation is given by

$$\frac{i - i_w}{i_w C} = K\left[\frac{v^2}{gD(s-1)} C_D^{0.5}\right]^m \tag{1}$$

The adjustable constants K and m were taken to have the values 84.9 and −1.5, respectively, for the sand and gravel slurries investigated by Durand and co-workers. The drag coefficient C_D is that for settling of the particle at its terminal velocity in the quiescent, unbounded liquid. For the equivalent spherical particle of diameter d, it is given by

$$C_D = \frac{4}{3}\,\frac{gd(s-1)}{v_\infty^2} \tag{2}$$

Zandi's and Govatos' [9] work represents the next important contribution on pressure drop prediction. They proposed the following improved modification for prediction of slurry pressure drop:

$$\frac{i - i_w}{i_w C} = 280\psi^{-1.93} \qquad \text{for } \psi < 10 \tag{3}$$

$$\frac{i - i_w}{i_w C} = 6.30\psi^{-0.354} \qquad \text{for } \psi > 10 \tag{4}$$

in which

$$\psi = \frac{v^2}{gD(s-1)} C_D^{0.5} \tag{5}$$

Equations (3) and (4) are valid when the transition number, designated by Zandi and Govatos by N_I, obeys the inequality

$$N_I = \frac{v^2 C_D^{0.5}}{CDg(s-1)} > 40 \tag{6}$$

According to Zandi and Govatos [9], data characterized by $N_I < 40$ do not belong to the fully suspended regime of flow.

Turian and Yuan [10] developed an extended pressure drop correlation scheme which takes into account the fact that various flow regimes are observed to prevail in slurry transport depending upon flow conditions. Their correlation is given by the following relationships:

Flow with a stationary bed (regime 0):

$$f - f_w = 12.127 C^{0.7389} f_w^{0.7717} C_D^{-0.4054} \times \left[\frac{v^2}{Dg(s-1)}\right]^{-1.096} \tag{7}$$

Saltation flow (regime 1):

$$f - f_w = 107.09 C^{1.018} f_w^{1.046} C_D^{-0.4213} \times \left[\frac{v^2}{Dg(s-1)}\right]^{-1.354} \tag{8}$$

Heterogeneous flow (regime 2):

$$f - f_w = 30.115 C^{0.8687} f_w^{1.200} C_D^{-0.1677} \times \left[\frac{v^2}{Dg(s-1)}\right]^{-0.6938} \tag{9}$$

Homogeneous flow (regime 3):

$$f - f_w = 8.538 C^{0.5024} f_w^{1.428} C_D^{0.1516} \times \left[\frac{v^2}{Dg(s-1)}\right]^{-0.3531} \tag{10}$$

in which f and f_w are the friction factors for the slurry and for water, respectively, at the same mean velocity. Both f and f_w are defined in terms of the carrier liquid density ρ. Thus

$$f = \frac{1}{2}\,\frac{D}{\rho v^2}\left(-\frac{dp}{dz}\right) \tag{11}$$

where (dp/dz) is the pressure gradient.

A detailed evaluation of the various pressure drop correlations, including those cited above, has been given by Turian and Yuan [10]. The number of pressure drop correlations for pipeline transport of slurries is quite large. Kazanskij [11] cites 37 such correlations in his review. Virtually all these correlations are empirical.

Turian and Yuan [10] used their extended pressure drop correlation, given by eqns. (7) to (10), to develop a comprehensive regime delineation scheme. It is shown that maps of the various flow regimes may be drawn using relationships for what Turian and Yuan refer to as the *regime transition number* R_{ij}, which pertains to transition between regimes i and j. The expressions for the regime transition numbers R_{01}, R_{02} and R_{03}, which pertain to transition between the flow with a stationary bed regime (0) and each of the following three: the saltation flow (1), heterogeneous flow (2) and homogeneous flow (3) regimes, respectively, will be needed in our examination of the problem relating to the critical velocity given below. The relationships for these regime transition numbers are given by the following equations:

$$R_{01} = \frac{v^2}{4679.5\,C^{1.083} f_w^{\;1.064} C_D^{\;-0.0616} Dg(s-1)} \tag{12}$$

$$R_{02} = \frac{v^2}{0.1044\,C^{-0.3225} f_w^{\;-1.065} C_D^{\;-0.0616} Dg(s-1)} \tag{13}$$

$$R_{03} = \frac{v^2}{1.604\,C^{0.3183} f_w^{\;-0.8837} C_D^{\;-0.7496} Dg(s-1)} \tag{14}$$

More detailed reviews of published pressure drop correlations are given by Zandi and Govatos [10], Zandi [12], Turian and Yuan [10], and Kazanskij [11]. A review of flow regime delineation and a detailed description of the extended scheme referred to in the foregoing is given by Turian and Yuan [10].

THE CRITICAL VELOCITY — COMPARISON OF PUBLISHED CORRELATIONS

The critical velocity is a primary parameter in the design of slurry pipelines. Many correlations for predicting the critical velocity have been proposed. Different researchers have used different variables and variable groupings in their correlations. In order to compare and to evaluate these correlations, we will first recast them into a standard form.

Under restricted conditions, which include uniformly sized spherical particles and smooth pipe walls, the critical velocity dependence can be expressed by

$$v_C = \bar{f}\,[d, D, C, (\rho_s - \rho)g, \rho, \mu] \tag{15}$$

Inclusion of the variable grouping $[(\rho_s - \rho)g]$ implies that we view the effect of gravity to be only manifested through net settling forces on the particles. In non-dimensional variables eqn. (15) can be written in the form

$$\frac{v_C}{[2gD(s-1)]^{0.5}} = f\left[\left(\frac{d}{D}\right), \frac{D\rho[gD(s-1)]^{0.5}}{\mu}, C\right] \tag{16}$$

Use of the reference 'velocity' $[2gD(s-1)]^{0.5}$ in eqn. (16) is motivated by the fact that it is generally assumed that the critical velocity dependence on pipe diameter is approximately according to $D^{1/2}$.

Using an analysis based on balancing the energy required to suspend the particles with that derived from dissipation of an appropriate fraction of the turbulent eddies, Oroskar and Turian [13] derived the following critical velocity correlation:

$$\frac{v_C}{[2gD(s-1)]^{1/2}} = \frac{5^{8/15}}{2^{1/2}}\,[C(1-C)^{2n-1}]^{8/15} \times \left[\frac{D\rho[gD(s-1)]^{1/2}}{\mu}\right]^{1/15} \tag{17}$$

where the exponent n is the hindered settling exponent in multi-particle sedimentation as given by Richardson and Zaki [14] and by Garside and Al-Dibouni [15]. The values of n fall approximately in the range $2 < n < 5$, the upper limit corresponding to the low Reynolds Stokes' law region. The numerical coefficient in eqn. (17) depends upon what fraction of the turbulent energy, intrinsic to the flow, is assumed to be required in suspending the particles at incipient suspension. If this fraction is taken to be α, then the numerical coefficient would be $[(5/6\alpha)^{8/15}/2^{1/2}]$. The value $(5^{8/15}/2^{1/2})$ in eqn. (17) corresponds to $\alpha = 1/6$.

TABLE 1

Comparison of published critical velocity correlations in slurry transport

$$\frac{v_C}{[2gD(s-1)]^{0.5}} = f(C, s)C_D^k \left[\frac{D\rho[gD(s-1)]^{0.5}}{\mu}\right]^l \left(\frac{d}{D}\right)^m$$

Author	$f(C, s)$	k	l	m	$\bar{D}$[a]; RMS[a]
Wilson [3]	$1.1322[Cs/(Cs+1-C)]^{0.3636}(s-1)^{-0.3636}$	−0.1818	0.09818	0.0818	49.51; 0.6206
Durand and Condolios [6]	0.7 - 1.3	0	0	0	36.53; 0.4074 for $f(C, s)$ = 1.2
Craven [19]	$2.2508/(1-C)$	0	0	0.5	77.59; 0.8671
Knoroz (reported by Gorjunov [20])	$0.8165[sC/(1-C)]^{1/6}$	−0.5	0	−1/12	51.01; 0.6695
Newitt *et al.* [4]	13.8804	−1/2	0	1/2	93.43; 1.3100
Spells [21]	$0.03492[C(s-1)+1]^{0.6327}$	0	0.6327	0.8163	68.21; 1.5012
Cairns *et al.* [22]	$2.7074C^{0.1765}/(s-1)^{0.2353}$	0	0.1765	0.5882	58.17; 0.7129
Thomas [23]	21.3635	−0.1583	−0.6834	−0.6680	65.03; 0.7461 dilute, flocculated particles
Thomas [24]	$1.3762(s-1)^{0.1878}$	−0.4082	−0.1837	−0.4082	29.95; 0.5044 dilute, non-flocculated
Reported by Schulz [25]	$1.3622(s-1)^{2/7}$	0	1/7	0	750.40; 7.0877
Brauer and Kriegel [26]	$0.5115C^{0.4138}(s-1)^{0.2759}$	−0.6207	0.5172	0.6207	224.64; 6.1765
Zandi and Govatos [9]	$(20C)^{1/2}$	−1/4	0	0	50.02; 0.6521
Wiedenroth [27]	$0.4559(s-1)^{-0.25}$	−0.25	0	0	69.95; 0.8125
Larsen [28]	$2.9813C^{1/4}$	−1/4	0	0	45.22; 0.6090
Rose and Duckworth [29]	$9.6544[C/(1-C)]^{0.4}(s-1)^{0.5}s^{-1}$	−1	0	−0.2	172.35; 2.8223
Shook [30]	$4.47C^{1/2}$	−0.25	0	0	49.99; 0.6518
Shook [30]	$2.43C^{1/3}$	−0.25	0	0	34.50; 0.5059
Babcock [31]	$2.2361C^{1/2}$	−0.25	0	0	46.68; 0.6100
Bain and Bonnington [32]	$2.4607C^{1/3}$	−0.25	0	0	34.48; 0.5055
Novak and Nalluri [33]	1.3137	0	0	−0.05	90.11; 0.8254
Robinson and Graf [34]	$0.901C^{0.106}$	0	0	0	26.49; 0.4532
Kao and Wood [35]	$(2/3)^{1/2}g(n)$[b]	0	0	$1/2-1/n$	69.01; 0.7939 for n = 10
Carleton and Cheng [16]	$3.0137[16.7(1-C)^{4.6}/K]^{4/7}$	0	1/7	4/7	71.34; 0.9810 for ≤0.24 K = 1
	$3.0137\left\{\frac{2.67(1-C)^{4.6}}{K[(C_0/C)^{1/3}-1]^2}\right\}^{4/7}$	0	1/7	4/7	for $C > 0.24$, $C_0 = 0.65$
Turian and Yuan [10]	$8.948C^{0.4779}$	−0.0272	−0.1174	0	39.97; 0.4636 $R_{02}, R_{03} \leq 1$[c]
	$0.9125C^{-0.1860}$	−0.3406	0.1536	0	$R_{01}, R_{03} \leq 1$
	$3.251C^{0.1789}$	−0.4213	0.1242	0	$R_{01}, R_{02} \leq 1$
Wasp *et al.* [36]	$3.399C^{0.2156}$	0	0	1/6	26.68; 0.3750
Thomas [17]	4.9937	0	−5/21	0	60.14; 0.7125
Toda *et al.* [37]	$0.8054C^{0.20}$	−0.25	0	−0.35	166.52; 1.7486
Oroskar and Turian [13]	$2^{-1/2}[5C(1-C)^{2n-1}]^{8/15}$	0	1/15	0	25.94; 0.4331 for n = 2

[a] The absolute average per cent deviation $\bar{D}$ and the root mean square deviation RMS are defined by eqns. (25) and (26).

[b] $$g(n) = \frac{2n^2}{(2n+1)(n+1)}\left[\frac{(n+1)(n+2)}{n \times 2^{2/(n+2)}}\right]^{1/2}$$

[c] Use eqns. (12) to (14) to calculate R_{01}, R_{02} and R_{03}.

Published critical velocity relationships, and eqn. (17), suggest that most available correlations can be subsumed under the form

$$\frac{v_C}{[2gD(s-1)]^{1/2}} = f(C,s)C_D{}^k\left\{\frac{D\rho[gD(s-1)]^{1/2}}{\mu}\right\}^l\left(\frac{d}{D}\right)^m \quad (18)$$

in which $f(C, s)$ is mainly a function of particle concentration, and k, l and m take different numerical values for different correlations. It needs to be pointed out that the form in eqn. (18) is a special case of eqn. (16) even though the particle drag coefficient C_D does not appear explicitly among the dimensionless groups in eqn. (16). For a particle settling in an unbounded quiescent liquid, the quantity $\Lambda = C_D{}^{1/2}N_{Re}$ can be obtained from the drag coefficient–particle Reynolds number relationship. Furthermore, Λ is related to the first two groups on the right-hand side of eqn. (16) by

$$\Lambda = C_D{}^{1/2}N_{Re} = \left(\frac{4}{3}\right)^{1/2}\left\{\frac{D\rho[gD(s-1)]^{1/2}}{\mu}\right\}\left(\frac{d}{D}\right)^{3/2} \quad (19)$$

Accordingly, the dimensionless groups in eqn. (16) are sufficient to define C_D through the drag coefficient relationship.

Table 1 lists published critical velocity correlations which have been recast in the format of eqn. (18). The critical velocity correlations due to Carleton and Cheng [16], Turian and Yuan [10] and Thomas [17], in their original forms, contain the Fanning friction factor for the suspending liquid f_w defined by eqn. (11). In order to eliminate f_w in such cases, and recast the results in standard format, we use the Blasius [18] expression for the friction factor based on the (1/7)th power turbulent velocity profile:

$$f_w = 0.0791\tilde{N}_{Re}{}^{-1/4} \quad (20)$$

in which

$$\tilde{N}_{Re} = \frac{\rho D v}{\mu} \quad (21)$$

Equations (20) and (21) with $v = v_C$ are substituted for f_w for the critical velocity correlation under consideration, and v_C is then solved for.

The critical velocity expression based on eqn. (12) of Turian and Yuan [10] given above is

$$\frac{v_C}{[2gD(s-1)]^{0.5}} = \frac{1}{2^{0.5}}\{4679.5C^{1.083}f_w{}^{1.064}C_D{}^{-0.0616}\}^{0.5} \quad (22)$$

Substituting for f_w from eqns. (20) and (21) with $v = v_C$, we obtain after solving for v_C the relation

$$\frac{v_C}{[2gD(s-1)]^{0.5}} = 8.948C^{0.4779}C_D{}^{-0.0272} \times\left\{\frac{D\rho[gD(s-1)]^{0.5}}{\mu}\right\}^{-0.1174} \quad (23)$$

which is one of the entries in Table 1. In order to use the critical velocity correlation based on Turian's and Yuan's [10] regime delineation results, the values of the regime transition numbers R_{01}, R_{02} and R_{03} need to be calculated using eqns. (12) to (14).

Most of the critical velocity correlations listed in Table 1 are empirical, although some, like those due to Kao and Wood [35], Carleton and Cheng [16], and Oroskar and Turian [13], are based on an assumed model for the particle suspension process. A number of the critical velocity correlations which have appeared in the literature could not be transformed into the standard form given by eqn. (18). Some of these are given in Table 2. The correlation due to Thomas [24] in Table 2 comprises a sum of two terms each having the standard form.

Results of tests of the correlations listed in Tables 1 and 2 against experimental critical velocity data will be presented in the next section. It is obvious upon comparing the various expressions in Tables 1 and 2 that the qualitative disparities among the various correlations which have been proposed are quite broad. Most correlations, and the experimental data on which they are based, indicate that the critical velocity has approximately a square-root dependence on pipe diameter ($\sim D^{1/2}$), and a rather weak dependence on particle diameter ($\sim d^0$), especially for larger non-colloidal particles. Accordingly,

TABLE 2

Critical velocity correlations which cannot be cast in standard form (eqn. (18))

Yufin [38]:[a]

$$v_C = 14.23 d^{0.65} D^{0.54} \exp\{1.36[1 + C(s-1)]^{0.5} d^{-0.13}\}$$

$\bar{D}^b = 35.77$; $\mathrm{RMS}^b = 0.4470$

Thomas [24]:

$$\frac{v_C}{[2gD(s-1)]^{0.5}} = 1.3762 C_D^{-0.4082} \left\{\frac{D\rho[gD(s-1)]^{0.5}}{\mu}\right\}^{-0.1837} \left(\frac{d}{D}\right)^{-0.4082} (s-1)^{-0.1878}$$
$$+ 6.9877 C^{0.5714} C_D^{-0.4626} \left\{\frac{D\rho[gD(s-1)]^{0.5}}{\mu}\right\}^{-0.07483} \left(\frac{d}{D}\right)^{-0.08163} (s-1)^{-0.1252}$$

$\bar{D} = 75.02$; RMS = 0.9213

Yufin [39]:[c]

$$v_C = 9.8 D^{1/3} v_s^{1/4} [C(s-1) + 0.6]$$

$\bar{D} = 51.26$; RMS = 0.7844

Yufin and Lopasin [40]:[d]

$$v_C = 8.3 D^{1/3} (C\psi)^{1/6} \text{ with } \psi = C^{-0.75}$$

$\bar{D} = 32.54$; RMS = 0.4406

Bonapace [41]:

$$\frac{v_C}{[2gD(s-1)]^{0.5}} = 2^{0.5} \left(\frac{d}{D}\right)^{0.5} \left(1 - \frac{d}{D}\right)^{3.5} \left(\frac{s-1}{s_{\mathrm{sand}}-1}\right)^{1/3} \left(\frac{f_w}{8}\right)^{-0.5}$$

$\bar{D} = 424.75$; RMS = 5.0736

[a] Quoted in ASCE Task Committee Report [42]; units in ft and s.
[b] The absolute average per cent deviation $\bar{D}$ and the root mean square deviation RMS are defined by eqns. (25) and (26).
[c] Quoted by Sasic and Marjanovic [43]; units in ft and s.
[d] Quoted by Wiedenroth and Kirchner [44]; units in ft ans s.

the correlations of Wilson [3], Newitt *et al.* [4], Spells [21], Cairns *et al.* [22], Thomas [24], Brauer and Kriegel [26], Rose and Duckworth [29], Carleton and Cheng [16], and Turian and Yuan [10] are in conflict to varying degrees with experimental evidence.

Recently, Parzonka *et al.* [45] have presented a substantial body of experimental data which suggests that the critical velocity attains a maximum with concentration. The correlation of Oroskar and Turian [13], given by eqn. (17) and in Table 1, gives a maximum for v_C at $C = 1/2n$. It is clear from an examination of details of the physical model upon which eqn. (17) is based, that the maximum in the critical velocity–concentration relationship occurs because of increased particle interaction with concentration. The critical velocity initially increases with concentration, but as more particles are added, hindered settling effects become increasingly influential and counteract particle settling, which explains the reversal in the v_C *vs.* C relationship.

The drag coefficient C_D appears in a number of the critical velocity correlations. For large and heavy particles, which constitute the main type in coarse-particle slurry transport, C_D is virtually independent of particle Reynolds number N_{Re}. In the high Reynolds number, Newton's law, region ($N_{Re} > 500$), $C_D \cong 0.44$. The value of the drag coefficient for intermediate and low values of the particle Reynolds number may be estimated from correlations given by Bird *et al.* [46], by Davies [47] or by Turian and Yuan [10].

COMPARISON WITH EXPERIMENT

The correlations presented in Tables 1 and 2 were tested against experimental critical velocity data. The data used were collected from the literature. Table 3 lists the sources of these data, the materials comprising the slurries, and the ranges of the pertinent variables involved. The data were screened for incompleteness, redundancies and evident inaccuracies, resulting in a total collection of 864 data points as depicted in Table 3. For mixed particle sizes, the particle diameter d_{50}, corresponding to that above which 50% of the particles by weight lie, was taken as the equivalent particle diameter.

In order to compare the various correlations with the experimental data, we calculated the per cent deviation for each data point and correlation as defined by

$$\%\text{dev} = D_{\text{v}} = \left[\frac{v_{\text{C(calc)}} - v_{\text{C(exp)}}}{v_{\text{C(exp)}}}\right] \times 100 \qquad (24)$$

From the calculated values of the per cent deviation D_{vi} for each data point and correlation, an overall absolute average per cent deviation, and an overall root mean square deviation were calculated for each correlation. These are defined by

$$\bar{D} \equiv \text{Abs. avg. \%dev.} = \sum_{i=1}^{N} \frac{|D_{\text{vi}}|}{N} \qquad (25)$$

$$\text{RMS} \equiv \left\{\sum \frac{[v_{\text{C(calc)}} - v_{\text{C(exp)}}]^2}{N}\right\}^{0.5} \qquad (26)$$

The results of these calculations are given in the last column of Tables 1 and 2. These comparisons with experimental data indicate that the correlations proposed by Robinson and Graf [34], Oroskar and Turian [13] and Wasp *et al.* [36] do a better job of predicting the critical velocity than the remaining relationships listed in Tables 1 and 2. The correlations of Robinson and Graf and of Wasp *et al.* are purely empirical as stated earlier. Additional details relating to the comparison between experiments and correlations will be presented below.

CORRELATIONS BY REGRESSION

Aside from the previously published correlations listed in Tables 1 and 2, we developed an additional set of correlations using the following form of eqn. (18):

$$\frac{v_{\text{C}}}{[2gD(s-1)]^{0.5}} = \chi_1 C^{\chi_2}(1-C)^{\chi_3}\left\{\frac{D\rho[gD(s-1)]^{0.5}}{\mu}\right\}^{\chi_4}\left(\frac{d}{D}\right)^{\chi_5} \qquad (27)$$

Various forms of eqn. (27) were considered, and the corresponding values of the adjustable constants χ_i were determined by fitting the 864 critical velocity data using multilinear regression, in which the linearized log form of the equation is fitted. The estimates of these adjusted parameters, together with the corresponding standard errors of the estimates, are given in Table 4. Five different cases, corresponding to equations in which various χ_i are assumed to be zero, are depicted in Table 4. For each case considered, all 864 data points were used to obtain the estimates for the parameters χ_i.

The results in Table 4 lead to a number of conclusions. Based on the values of the absolute average per cent deviations and also the per cent root mean square deviations given in the table, it is evident that the correlations corresponding to all five cases are virtually equally effective in predicting the critical velocity. The results for Cases 1 and 3 suggest that the dependence of the critical velocity on particle diameter is quite weak, which is approximately $d^{0.06}$. Comparison of the results for Cases 1 to 3 demonstrates that the overall exponent of D on the right-hand side of eqn. (27) is about -0.06, which is clearly very small. Accordingly, this demonstrates that the critical velocity dependence on pipe diameter is very nearly $D^{1/2}$. For Oroskar's and Turian's [13] correlation, given by eqn. (17), the critical velocity dependence on particle diameter is given by d^0, and that on pipe diameter is $D^{0.6}$. Finally, comparison of the results for all five cases presented in Table 4 demonstrates that the critical velocity dependence on the factor $(1-C)$ is important. Indeed, exclusion of either of the two factors C^{χ_2} and $(1-C)^{\chi_3}$ would preclude the possible attainment of a

TABLE 3

Experimental critical velocity data

Source	Slurry	Solid density $(kg/m^3) \times 10^{-3}$	Fluid density $(kg/m^3) \times 10^{-3}$	Fluid viscosity $(kg/(m\,s)) \times 10^3$	Particle size $(m \times 10^6)$	Pipe diameter $(m \times 10^2)$	Concentration (vol.%)	Number of points
Present work	Bottom ash/water	1.834	0.998	1.0	130	7.62 - 10.16	2.8 - 56.1	8
Present work	Coal/brine	1.67	1.325	130 - 190	2200	5.08 - 10.16	12.3 - 39.8	13
Silin[a] (1962, 1971, 1973)	Coal/water	1.63 - 1.90	0.998	0.978	1000 - 2000	10.3 - 25.5	6.2 - 37.5	17
Present work	Coal/water	1.6	0.999	1.126	420	5.03 - 10.16	9.9 - 40.4	11
Sinclair[a] (1962)	Coal/water	1.4	0.998	0.978	2200 - 2260	1.27 - 2.54	1 - 18	24
Schriek *et al.* [48]	Coal/water	1.30 - 1.40	0.998 - 1.0	0.978 - 1.15	200 - 370	5.22 - 31.5	20 - 50	50
Goedde [49]	Coal/water	1.53	0.998	1.0	1830 - 8680	4.0 - 20.8	5.6 - 24.3	7
Present work	Copper tailings/ water	2.73	0.997 - 0.998	1.0	500	5.08 - 10.16	8.3 - 29	15
Silin[a] (1962, 1971, 1973)	High density material/water	3.36	0.998	0.978	300	10.3	2.0 - 30.8	7
Vodolozski[a] (1971)	High density material/water	3.0 - 4.0	0.998	0.978	60 - 90	14.9 - 20.7	1 - 14	28
Wolanski[a] (1972)	High density material/water	2.69	0.998	0.978	140	10 - 20.7	2.2 - 32	17
Sinclair[a] (1959, 1962)	Iron/[b]	7.475	0.77	1.1	55	2.54	12	1
Wasp *et al.* [50]	Iron/kerosene	5.245	0.9	1.9 - 2.0	138	1.905	1 - 18	12
Schriek *et al.* [51]	Iron ore/water	5.25	0.998	0.978	50	5.2 - 26.3	13 - 30	5
Present work	Iron powder/water	7.40	0.998 - 0.999	1.0 - 1.1	220	5.08 - 10.16	1.9 - 5.2	6
Present work	Iron trailings/water	5.245	0.997	1.0	20	5.08 - 10.16	1.9 - 5.2	6
Goedde [49]	Ore/water	4.55	0.998	1.0	150 - 3350	4.0 - 20.8	5.5 - 34.7	29
Thomas [17]	Limonite/[b]	4.470	1.0 - 1.16	4.75 - 0.8	130 - 170	1.89 - 10.5	12	7
Schriek *et al.* [52]	Limestone/water	2.755	1.0	0.98 - 1.12	100 - 415	5.245 - 13.77	12 - 40	15
Present work	Nickel shot/water	8.90	0.997 - 0.999	1.0 - 1.1	1600	7.62 - 10.16	0.9 - 11	4
Goedde [49]	Plastic/water	1.15	0.998	1.0	2870	4.0	7.6 - 37.7	4
Smith *et al.* [53]	Potash/brine	1.984	1.13 - 1.15	1.14 - 1.20	300 - 400	5.22 - 26.31	30 - 50	15
Shook *et al.* [54]	Sand/ethylene glycol	2.65	1.1 - 1.35	5.6 - 38.1	200 - 500	5.24	5 - 42	90
Craven[a] (1951)	Sand/water	2.65	0.998	0.978	250	5.15	0.5	2
Durand and Condolios[a] (1952)	Sand/water	2.65	0.998	0.978	440 - 2040	15.0	2.5 - 15	4
Smith[a] (1955)	Sand/water	2.65	0.998	0.978	190 - 850	5.08 - 7.69	2.5 - 21.5	23
Spells [21]	Sand/water	2.60	0.998 - 1.0	1.0	240 - 380	7.6 - 30.0	0.8 - 27.7	13
Sinclair[a] (1962)	Sand/water	2.65	0.998	0.978	600	12.7	1 - 9.5	8
Silin *et al.*[a] (1962, 1971, 1973)	Sand/water	2.65	0.998	0.978	250 - 420	10.3 - 80	1.8 - 35.8	63
Votsukara[a] (1963)	Sand/water	2.65	0.998	0.978	230 - 1150	10.8	5 - 20	9
Bielova[a] (1968)	Sand/water	2.65	0.998	0.978	1350 - 19000	40.7	4 - 17.5	5
Wasp *et al.* [36]	Sand/water	2.65	1.0	0.98	250 - 2040	2.67 - 13.97	1 - 25	45

Hayden and Stelson [55]	Sand/water	2.65	0.998	0.978	180 - 3700	5.08	4 - 14.8	8
Wicks[a] (1971)	Sand/water	2.65	0.998	0.978	250	12.7	0.5	1
Charles[a] (1972)	Sand/water	2.65	0.998	0.978	100 - 650	2.61	13.6 - 27.5	9
Pokrovskaya[a] (1972)	Sand/water	2.65	0.998	0.978	200	20.2	3.8 - 28.5	5
Schriek *et al.* [56]; Shook *et al.* [54]	Sand/water	2.65	0.98 - 1	0.5 - 1	170 - 500	5.22 - 31.52	14 - 50	144
Wilson and Watt [57]	Sand/water	2.65	0.998	0.978	750	5.25 - 8.85	1.0 - 13	14
Goedde [49]	Sand/water	2.65	0.998	1.0	280 - 330	4.0 - 15.20	0.1 - 40.0	101
Thomas [17]	Sand/[b]	2.65	1.0 - 1.3	0.8 - 5.6	130 - 900	5.25 - 10.5	12	3
Present work	Sand/water	2.60 - 2.65	0.999 - 1.0	1.1 - 1.3	300 - 460	5.08 - 10.16	5 - 29.2	16

[a] Extracted from the paper by Parzonka *et al.* [45].
[b] Carrier liquid not identified in published source, Thomas [17].

TABLE 4

Critical velocity correlations using regression

$$\frac{v_C}{[2gD(s-1)]^{0.5}} = \chi_1 C^{\chi_2}(1-C)^{\chi_3}\left\{\frac{D\rho[gD(s-1)]^{0.5}}{\mu}\right\}^{\chi_4}\left(\frac{d}{D}\right)^{\chi_5}$$

Parameter	Estimate	Standard error of estimate
Case 1 : Estimate all χ_i		
χ_1	1.7951	1.0878
χ_2	0.1087	0.01610
χ_3	0.2501	0.09870
χ_4	0.00179	0.00767
χ_5	0.06623	0.00958
$\bar{D}^a$ = 20.53; RMS[a] = 0.3416		
Case 2: Assume $\chi_5 = 0$		
χ_1	1.8471	1.0901
χ_2	0.1126	0.01652
χ_3	0.03421	0.1004
χ_4	−0.03093	0.00621
$\bar{D}$ = 21.54; RMS = 0.3447		
Case 3: Assume $\chi_4 = 0$		
χ_1	1.8176	1.0665
χ_2	0.1086	0.01608
χ_3	0.2525	0.09812
χ_5	0.06486	0.00754
$\bar{D}$ = 20.57; RMS = 0.3412		
Case 4: Assume $\chi_4 = 0$ and $\chi_5 = 0$		
χ_1	1.3213	1.0564
χ_2	0.1182	0.01671
χ_3	0.3293	0.1018
$\bar{D}$ = 21.04; RMS = 0.3552		
Case 5: Assume $\chi_3 = 0$, $\chi_4 = 0$ and $\chi_5 = 0$		
χ_1	1.1228	1.0223
χ_2	0.07367	0.00954
$\bar{D}$ = 21.35; RMS = 0.3559		

[a]The absolute average per cent deviation $\bar{D}$ and the root mean square deviation RMS are defined by eqns. (25) and (26).

maximum in the v_C *vs.* C relationship, in contradiction to a substantial body of experimental evidence which demonstrates that such a maximum occurs [45].

The occurrence of a maximum in the v_C *vs.* C relationship may, as previously stated, be explained on the basis of hindered settling which becomes more pronounced as concentration increases. For the critical velocity correlations corresponding to Cases 1 through 4 in Table 4, the maximum in the v_C *vs.* C curve occurs at the concentration C_{max} given by

$$C_{max} = C\Big|_{\frac{\partial v_C}{\partial C}=0} = \frac{\chi_2}{\chi_2 + \chi_3} \tag{28}$$

Based on the values of χ_2 and χ_3 given in Table 4 for Cases 1 through 4, the value of C_{max} ranges between 0.25 and 0.30. According to Oroskar's and Turian's [13] relation, given by eqn. (17), the concentration $C_{max} = 1/2n$, and the values of $C_{max} = 0.25$ and 0.30 correspond to $n = 2$ and 1.67, respectively.

In order to provide a somewhat finer assessment of the relative predictive qualities of the empirical regression correlations in Table 4 and also those by Robinson and Graf [34], Wasp *et al.* [36], and Oroskar and Turian [13], which were found to be most promising, we present more detailed information on deviations from experimental data. Table 5 presents a listing of the maximum deviation and also the numbers of data points which lie in various deviation bands for each of these correlations. It must be emphasized that the relationships due to Robinson and Graf [34] and Wasp *et al.* [36], aside from being purely empirical, are incapable of predicting a maximum in the v_C *vs.* C relationship. This limitation, and the detailed comparisons presented in Table 5 suggest that the correlation due to Oroskar and Turian [13], given by eqn. (17), and the empirical regression results corresponding to Cases 1 and 3 in this work do the best overall job of predicting the critical velocity. It is, however, clear from an examination of all cases that the dependence of v_C on particle size is quite weak.

CONCLUSIONS

The analysis presented in this work, and the evaluations of published critical velocity relationships for pipeline flow of slurries are based on a substantial body of experimental data pertaining to a wide range of the pertinent variables. Critical velocity data are very difficult to get, and are often uncertain and equivocal because the critical condition is very difficult to ascertain. Indeed, the instability in the flow, as the critical condition is approached, results in wide variations in measured variables, particularly the concentration (see, for example, Thomas [17]).

TABLE 5

Detailed comparison with experimental data

$\frac{v_C}{[2gD(s-1)]^{0.5}}$

		No. of points in deviation band				Max. %dev.
		⩽20%	20 - 50%	50 - 100%	>100%	
Robinson and Graf [34]	$0.901C^{0.106}$	339	458	67	0	80
Wasp *et al.* [36]	$3.399C^{0.2156}\left(\frac{d}{D}\right)^{1/6}$	380	388	84	12	206
Oroskar and Turian [13]	$1.6683C^{0.5333}(1-C)^{1.6}\left\{\frac{D\rho[gD(s-1)]^{0.5}}{\mu}\right\}^{1/15}$	437	302	118	7	164
Regression results:	$\chi_1 C^{\chi_2}(1-C)^{\chi_3}\left\{\frac{D\rho[gD(s-1)]^{0.5}}{\mu}\right\}^{\chi_4}\left(\frac{d}{D}\right)^{\chi_5}$					
Case 1: $\chi_1 = 1.7951; \chi_2 = 0.1087; \chi_3 = 0.2501; \chi_4 = 0.00179; \chi_5 = 0.06623;$		535	260	65	4	123
Case 2: $\chi_5 = 0; \chi_1 = 1.8471; \chi_2 = 0.1126; \chi_3 = 0.3421; \chi_4 = -0.03093;$		506	283	70	5	130
Case 3: $\chi_4 = 0; \chi_1 = 1.8176; \chi_2 = 0.1086; \chi_3 = 0.2525; \chi_4 = 0.06486;$		533	262	65	4	123
Case 4: $\chi_4 = \chi_5 = 0; \chi_1 = 1.3213; \chi_2 = 0.1182; \chi_3 = 0.3293;$		522	270	65	7	132
Case 5: $\chi_3 = \chi_4 = \chi_5 = 0; \chi_1 = 1.1228; \chi_2 = 0.07367;$		511	281	64	8	132

This is an inherent limitation which must be weighed in assessing the results of tests between correlation and experiment. Nonetheless, a number of conclusions can be drawn from the analysis carried out here, and these are as follows:

(1) The dependence of the critical velocity on pipe diameter is very nearly equal to $D^{1/2}$.

(2) For slurries comprised of large non-colloidal particles, the critical velocity is virtually independent of particle size. The analytical result due to Oroskar and Turian [13] indicates that v_C is independent of d, while analysis of the experimental data suggests that v_C is a very weak function of d. Empirical fits to the data result in a critical velocity–particle size dependence which is approximately equal to $d^{0.06}$.

(3) A substantial body of experimental data suggests that the critical velocity–concentration relation possesses a maximum. Accordingly, correlations which account for the occurrence of a maximum in the v_C *vs.* C relationship are consistent with the experimental evidence. Both the analytical result of Oroskar and Turian [13] and the empirically fitted correlations given in Table 4 (Cases 1 through 4) indicate that the concentration at which the maximum in v_C occurs is between 0.25 and 0.30, which are somewhat higher than the value pertaining to the experimental data presented by Parzonka *et al.* [45].

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LIST OF SYMBOLS

C	concentration, volume fraction
C_D	$(4/3)gd(s-1)/v_\infty^2$, drag coefficient for free-falling sphere
D	inside diameter of pipe, m
$\bar{D}$	absolute average per cent deviation (eqn. (25))

D_v per cent deviation (eqn. (24))
d diameter of solid particle, m
f $(-\Delta P/L)(D/2\rho v^2)$, Fanning friction factor for pipe flow
g gravitational acceleration, m/s^2
i head loss for pipe flow, in feet of water per foot of pipe
K constant (eqn. (1))
L pipe length, m
N number of data points
N_I $v^2 C_D^{1/2}/[CDg(s-1)]$, eqn. (6))
N_{Re} $d\rho v_\infty/\mu$, particle Reynolds number for free-falling spheres
$\tilde{N}_{Re}$ $D\rho v/\mu$, Reynolds number for pipe
$-\Delta P$ pressure drop in length L of pipe, Pa
R_{ab} regime number (eqns. (12), (13), (14))
RMS per cent root mean square deviation (eqn. (26))
s ρ_s/ρ, solid to liquid density ratio
v mean velocity, m/s
v_C critical velocity of slurry, m/s
v_s hindered settling velocity of particles, m/s
v_∞ terminal velocity of sphere settling in an unbounded fluid, m/s

Greek symbols
Λ $N_{Re}C_D^{1/2}$
μ viscosity of liquid, kg/(m s)
ν kinematic viscosity of liquid, m^2/s
ρ density of liquid, kg/m^3
ρ_s density of solid, kg/m^3
ρ_{sand} density of sand, kg/m^3
χ_i constants (i = 1, 2, ..., 5) (eqn. (25))

Subscripts
calc calculated
exp experimental
w related to water or liquid
0 regime of flow with a stationary bed
1 saltation flow regime
2 heterogeneous flow regime
3 homogeneous flow regime

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