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Numerical Simulation of Unsteady Heat Transfer in Canned Mushrooms in Brine During Sterilization Processes

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ABSTRACT

The proposed numerical model determines the temperature distribution in convectively heated liquid brine and in conductively heated mushroom solid during in-can sterilization. The convective heat transfer in the brine is described by the regular regime equation and the conductive heat transfer in the mushrooms which have an irregular shape by the generalized equation of heat conduction. The predicted results obtained by using this method are compared with experimental results. The proposed approach may also be used for simulating the temperature in heating and cooling of heterogeneous foodstuffs.

NOTATION

- a Thermal diffusivity of mushroom flesh ($m^2\ s^{-1}$)
- b Heating/cooling rate of the medium ($°C\ s^{-1}$)
- c Specific heat capacity of mushroom ($J\ kg^{-1}\ K^{-1}$)
- D Average (for the process) relative deviation between the temperatures determined theoretically and by experiments (%)
- E Thermal inertia coefficient of the brine (s)
- DF Relative deviation between the F-effects computed on the basis of the theoretically and experimentally determined temperatures (%)
- F Sterilization F-effect (min or s)
- G Shape factor: $G = S \cdot R / V - 1$
- h Convective heat transfer coefficient at the mushroom surface ($W\ m^{-2}\ K^{-1}$)
- k Thermal conductivity of the mushroom ($W\ m^{-1}\ K^{-1}$)
- m Number of experimentally measured temperatures during the process

n Number of discretization intervals of dimension R
R The smallest characteristic dimension of the mushroom body presenting the shortest distance between the thermal centre and the surface (m)
S Heat transfer surface of the mushroom (m^2)
t Current time (s or min)
T Current temperature (°C)
V Volume of the mushroom (m^3)
x Generalized current coordinate of the mushroom solid (m) as $0 < x < R$
z Temperature sensitivity of the inactivated microorganism: $z = 10$°C

ρ Density of mushroom flesh ($kg\ m^{-3}$)

Subscripts
a Temperature of asymptote
i Second index of the position of a node in the finite difference scheme on the axis $x = i \cdot R/n$ as $i = 0, 1, 2, \ldots, n$. For the thermal centre $i = 0$ and for the solid surface $i = n$
l Liquid brine
m Heating/cooling medium
o Initial temperature of the mushroom: T_o = const.
s Mushroom solid
st Standard temperature: $T_{st} = 121{\cdot}1$°C

Superscripts
e Experimental
o Initial
t Theoretical
v Volume average

Functions
Int(x) Integral part of the argument x
Sgn(x) = 1, 0, −1 when the argument $x > 0$, $x = 0$ and $x < 0$, respectively

INTRODUCTION

Sterilization is a widespread industrial process for preserving foodstuffs. For health and safety reasons and for process optimization of the unsteady-state sterilization process it is very important to model the temperature distribution in canned foods.

Canned mushrooms contain brine which is a low-viscosity liquid and is heated by convection. The temperature distribution in this liquid is assumed to be uniform in this volume and its change during the thermal process is described by the regular regime differential equation (Kondratev, 1954; Poshtov *et al.*, 1963; Molodetzki, 1968; Videv, 1972; Bimbenet & Michiels, 1974).

Mushrooms have bodies of rotation with an irregular cross-sectional contour and are heated by conduction. The temperature distribution of irregular shaped foodstuffs was computed by means of the three- or two-dimensional differential equation of heat conduction and by using the finite element method (e.g. Naveh *et al.*, 1983) or the finite difference method (e.g. Manson *et al.*, 1974; Sheen & Hayakawa, 1991). Sastry *et al.* (1985) simulated the temperature distribution in canned mushrooms using a three-dimensional finite element model and the ambient temperature of brine was obtained from experimental heating data. The generalized finite difference method (Fikiin & Fikiin, 1989; Akterian & Fikiin, 1994) allows the reduction of this three-dimensional problem to a one-dimensional one.

Bimbenet and Duquenoy (1974) and Sawada and Merson (1986) modelled the temperature distribution of heterogeneous canned foods (including liquid and solid fractions) by a system of two differential equations: one for the regular regime which concerns the liquid part and the other for the heat conduction in the solid foodstuff. But these authors solved this system only for regular shaped foodstuffs and at a constant ambient temperature.

The object of this paper is to develop and verify experimentally an engineering numerical approach for determining the temperature distribution in canned mushrooms in brine undergoing sterilization at variable ambient temperature.

THEORETICAL BASIS

The temperature T_l of liquid in convectively heated cans is described by the regular regime differential equation (Videv, 1972):

$$E \times \frac{dT_l}{dt} = T_m - T_l \quad \text{or} \quad \frac{d}{dt}[\ln(T_m - T_l)] = -\frac{1}{E} \tag{1}$$

The coefficient E of thermal inertia (Fig. 1) characterizes the temperature lag of the liquid (T_l) from the medium (T_m).

During a linear change in the medium temperature $T_m = T_m^o + bt$, the liquid temperature is calculated by the following analytical relationship (Videv, 1972):

$$T_l = T_m - bE - (T_m^o - T_l^o - bE) \cdot \exp(-t/E) \tag{2a}$$

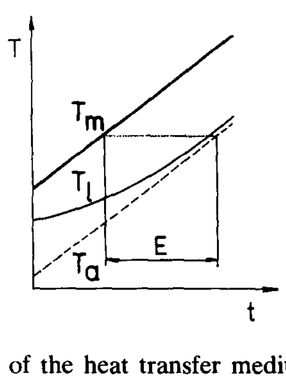

Fig. 1. Temperature curves of the heat transfer medium (T_m) and liquid (T_l) during heating.

or

$$\ln \frac{T_a - T_l}{T_a^o - T_l^o} = -\frac{t}{E} \quad \text{or} \quad \ln \frac{T_m - T_l}{T_m^o - T_l^o} = -\frac{t}{E} \tag{2b}$$

when $b=0$ (T_m = const.) where $T_a = T_m - bE$ is the temperature of the asymptote, $T_m^o = T_m$, $T_l^o = T_l$, $T_a^o = T_a$ at the beginning of a linear phase.

During the regular regime the temperature curve of the liquid appears as a straight line when plotted as $\ln(T_m - T_l - bE)$ versus t.

The temperature T_s in a mushroom (a body with an irregular shape) is modelled by the generalized differential equation of heat conduction (Fikiin & Fikiin, 1989; Akterian & Fikiin, 1994):

$$\frac{\partial T_s}{\partial t} = a\left(\frac{\partial^2 T_s}{\partial t^2} + \frac{G}{x} \cdot \frac{\partial T_s}{\partial x}\right) \tag{3}$$

The temperature distribution of the generalized coordinate x (the shortest route for heat penetration) is of greatest interest for engineering investigations. The corresponding initial and convective boundary condition of the third kind are:

$$T_s = T_o \quad \text{at} \quad t=0 \tag{4}$$

$$\frac{\partial T_s}{\partial x} = -\frac{h}{k}(T_s - T_l) \quad \text{for mushroom surface } (x=R) \tag{5}$$

$$\frac{\partial T_s}{\partial x} = 0 \qquad \text{for mushroom centre } (x=0) \tag{6}$$

The average volume temperature T_s^v of a mushroom is evaluated by means of the following relationship (Akterian & Fikiin, 1994):

$$T_s^v = \frac{G+1}{3 \times n^{G+1}} \Bigg\{ 4 \times \sum_{j=1}^{n^*/2} (2 \times j - 1)^G T_{s,2\times j-1} + 2 \times \sum_{j=1}^{n^*/2-1} (2 \times j)^G T_{s,2\times j} + n^G T_{s,n}$$
$$+ [1 - \mathrm{Sgn}(G)] T_{s,o} + (n - n^*)[2{\cdot}5(n-1)^G T_{s,n-1} + 0{\cdot}5 n^G T_{s,n}] \Bigg\} \tag{7}$$

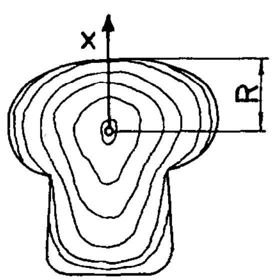

Fig. 2. Equipotential temperature curves across a front section of a mushroom and the generalized coordinate axis x.

where $n^* = 2\mathrm{Int}(n/2)$; $n \geq 4$ and 1 – index of the summations (Σ).

On the basis of this theoretical model and the finite differences algorithms (Akterian & Fikiin, 1994) a suitable computer program was developed for integrating eqn (3) for uniqueness conditions (4)–(6). This program simulates, simultaneously, the convective heating of the liquid brine, the conductive transfer in solid mushrooms and predicts the temperature distribution in the cans of mushrooms during sterilization with a linear pattern of heating and cooling medium temperature change (gradients).

MATERIALS AND METHODS

For experimental verification of the developed model 10 experimental time–temperature curves (Tolba, 1993) were used. These curves were measured during the sterilization of brined mushrooms packed in glass 'Omnia' 0–8 jars. The dimensions of this jar are diameter 105 mm, height 130 mm, volume 800 cm^3 by the Bulgarian State Standard (BDS 16 121-85). Each jar contained 300 g whole mushrooms vanity *Agaricus campestris* and the remainder was 1% brine.

The experimental thermal process was carried out in a pilot retort (autoclave). The heating was done by an electric heater and the cooling by adding cool water to produce a linear pattern of temperature change as shown in Figs 3 and 4.

The temperature distribution during sterilization was measured by means of copper–constantan thermocouple probes each of which was fitted in a stainless needle with a diameter of 1 mm. The error of the calibrated thermocouples

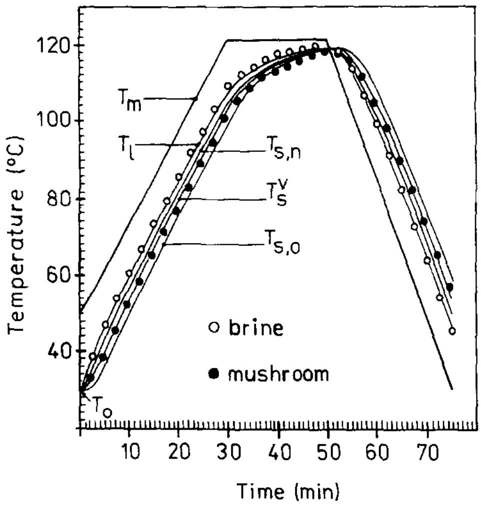

Fig. 3. Theoretical (lined) and experimental (marked) time–temperature curves during sterilization of brined mushrooms. Theoretical curves are computed using the individual geometric and thermophysical characteristics of the investigated canned food. The deviations between theoretical and experimental curves of the brine are $D = 0{\cdot}95\%$, $DF = -17{\cdot}7\%$ and in the mushroom are $D = 1{\cdot}96\%$, $DF = 41{\cdot}0\%$.

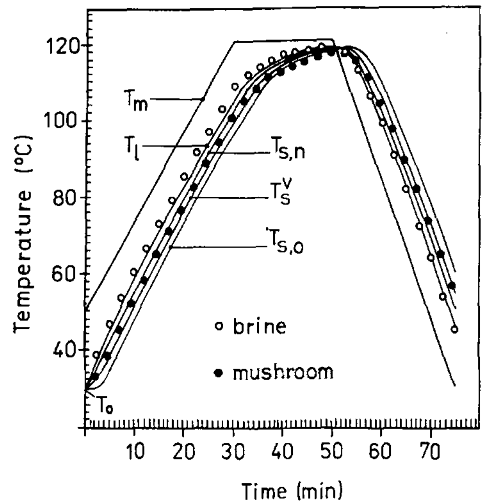

Fig. 4. Theoretical (lined) and experimental (marked) time–temperature curves during sterilization of brined mushrooms. Theoretical curves are computed using the average geometric and thermophysical characteristics of all the studied canned mushrooms. The deviations between theoretical and experimental curves of the brine are $D = 1{\cdot}75\%$, $DF = -17{\cdot}2\%$ and for the mushroom are $D = 0{\cdot}84\%$, $DF = 39{\cdot}6\%$.

compared to a standard thermometer is less than 0·5°C. The first thermocouple measured the temperature of the heating/cooling medium in the retort and the second the temperature of liquid brine. The third thermocouple was fitted along the axis of the mushroom stump and the needle end was located in the thermal centre of the mushroom (in the middle of the mushroom hood).

The following indices are used to evaluate the accuracy and applicability of the theoretical results.

(i) Average relative deviation between the temperatures determined theoretically and experimentally:

$$D = \frac{1}{m}\sum_{1}^{m}\frac{|T_s^t - T_s^e|}{T_s^e}\,100\,(\%) \tag{8}$$

When $D \leq 10\%$ it can be assumed according to Grubov (1971) that the applied approach meets the engineering requirements.

(ii) Relative deviation of the F-effects was computed respectively on the basis of theoretically and experimentally determined temperatures:

$$DF = \frac{F^t - F^e}{F^e}\,100\,(\%) \tag{9}$$

where the food safety index F-effect is determined by the well-known formula:

$$F = \int_0^t 10^{(T_s(t) - T_{st})/z}\,\mathrm{d}t \tag{10}$$

RESULTS AND DISCUSSION

For predicting the temperature of liquid brine by eqns (1) and (2) it was necessary to have previously determined the values of the coefficient E. For this the experimental temperature curves of the liquid brine and the heating medium were used. These curves were divided into three phases with linear increase, holding and linear decrease of the medium temperature T_m.

The appropriate values of the coefficient E in each phase were determined by scanning so that the differences D (eqn (8)) and DF (eqn (9)) between experimental and theoretical (eqn (2)) curves are minimal.

The range and average (underlined) values of the identified coefficients E referred to the studied curves are:

(a) for the increasing temperature phase $E_1 = 321 - \underline{359} - 384$ s
(b) for the holding phase $E_2 = 438 - \underline{514} - 570$ s (11)
(c) for the decreasing temperature phase $E_3 = 252 - \underline{276} - 300$ s

The range and average (underlined) values of the differences D and $|DF|$ are $0{\cdot}9-\underline{1{\cdot}03}-1{\cdot}2\%$ and $0{\cdot}7-\underline{9{\cdot}2}-18{\cdot}1\%$, respectively.

To compute the temperature distribution in the mushrooms by eqns (3)–(7) it was necessary to determine some geometric and thermo physical characteristics of the mushrooms. Using 50 mushrooms the range and average (underlined) values of the geometric characteristics were:

(a) shape factor: $G = 0{\cdot}93-\underline{1{\cdot}15}-1{\cdot}39$
(b) the smallest characteristic dimension: $R = 9-\underline{10{\cdot}5}-12{\cdot}5$ mm (12)

The thermophysical characteristics of mushroom flesh were determined experimentally or calculated by theoretical relationships and assumed to be constant with temperature: $\rho = 744$ kg m^{-3}, $c = 3965$ J kg^{-1} K^{-1}, $k = 0{\cdot}35$ W m^{-1} K^{-1}, $a = k/(c \cdot \rho) = 118 \times 10^{-9}$ m^2 s^{-1}. The heat transfer coefficient $h = 100$ W m^{-2} s^{-1} was evaluated as a mean integral effective value for the entire process and along the whole heat transfer surface of canned mushrooms by experimental determination of the surface heat flux density.

It is necessary to discuss what the temperature reading of the thermocouple in the mushroom represents. The thermal conductivity (k) of mushrooms is 50 times less than of the thermocouple probe. On the other hand the heat capacity ($c \cdot m$) of mushroom is 400 times higher than that of the probe. Moreover, even the small shrinkage of the mushroom leads to filling of the thermocouple hole with the easy-conductive brine. Therefore, the temperature reading does not correspond with that of the thermal centre. This is confirmed by the results of Sastry *et al.* (1985) where a similar lag in the thermocouple temperature from the computed one in the mushroom centre is observed. In the present case, in view of the large contact between the mushroom and the temperature probe it was assumed that the temperature reading corresponds to the average volume temperature of the mushroom.

In Figs 3 and 4 typical experimental (marked) and theoretical (lined) time–temperature curves of mushroom and brine are given. In Fig. 3 the theoretical curve was computed using the individual geometric characteristics ($R = 10$ mm, $G = 1{\cdot}08$) of the mushroom investigated and the thermal inertia coefficients ($E_1 = 321$ s, $E_2 = 570$ s, $E_3 = 276$ s) of canned mushrooms. The analogous curve in Fig. 4 was computed by using the average geometry characteristics (12) and the average values of the thermal inertia coefficients (11).

The range and average (underlined) values of the difference D were 0·9–4·3–8·7% while the values for $|DF|$ were 14–28–42%.

CONCLUSION

The proposed approach predicts with high accuracy the engineering computations of temperature distribution in brine liquid and mushroom solid.

It should be noted that the solution ensures satisfactory precision even under unfavourable conditions of modelling such as using average values of geometric and thermophysical characteristics.

The presented model could also be used to simulate the temperature distribution in other kinds of heterogeneous canned whole or cut fruit, vegetables, meat, fish, sausages, etc., in liquid undergoing the thermal/cooling processes.

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